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ON STEENROD'S PROBLEM FOR CYCLIC p-GROUPS

JAMES E. ARNOLD, JR.

1. Introduction. Let G be a finite group and A a $Z[G]$ module.

Definition (1.1). A simply connected CW complex X is of *type* (A, n) if G operates on X cellularly, and $\tilde{H}_i(X) = 0$, $i \neq n$, $H_n(X) \cong A$ as $Z[G]$ modules.

If A is a f.g. (finitely generated) $Z[G]$ module, we consider the following problems:

I. Is there a complex of type (A, n)?

II. Is there a finite complex of type (A, n)?

The second question was posed by Steenrod, and considered by R. Swan in [**5**]. In [**1**] we used an invariant of Swan denoted $\mathrm{Sw}(A)$ to obtain the following solution for $G = Z_p$, the cyclic group of prime order p:

THEOREM. *Let A be a f.g. $Z[Z_p]$ module. There are complexes of type (A, n), $n \geqq 3$, and there is a finite complex of type (A, n) if and only if* $\mathrm{Sw}(A) = 0$.

In this paper we obtain a similar result for G a cyclic p-group.

2. Preliminary definitions and lemmas.

Definition (2.1). A $Z[G]$ module M is a *signed permutation module* if M is free abelian with a set of generators permuted up to sign G.

Let $G_0(Z[G])$ denote the Grothendieck group of f.g. $Z[G]$ modules, and S the subgroup generated by the f.g. signed permutation modules.

Definition (2.2). Given a f.g. $Z[G]$ module A, $\mathrm{Sw}(A)$ is the class of A in the group $G_0(Z[G])/S$.

We will say that X is a *G-complex* if X is a CW complex and G operates effectively and cellularly on X. The cellular chain complex of X denoted $C_*(X)$ will then be a $Z[G]$ chain complex, and $C_n(X) = H_n(X^n, X^{n-1})$ is a signed permutation module for all $n \geqq 0$ (see [**1**]). If X is a finite G-complex,

$$\sum(-1)^i\mathrm{Sw}(H_i(X)) = \sum(-1)^i\mathrm{Sw}(C_i(X)) = 0.$$

Thus a necessary condition for there to be a finite complex of type (A, n) is that $\mathrm{Sw}(A) = 0$.

The following two lemmas from [**1**] are useful in constructing G-complexes. We include the proofs for completeness.

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LEMMA (2.3). *Let X and Y be G-complexes where*

a) $X = \bigvee_{\alpha \in A} S_\alpha^n$ *with G permuting the n-spheres S_α^n freely and fixing the base point x_0; and*

b) *Y is $n - 1$ connected with a 0-cell y_0 fixed by G.*

Then any $Z[G]$ homomorphism $h : H_n(X) \to H_n(Y)$ is realized by an equivariant cellular map $f : X \to Y$.

Proof. Let X_0 be the subcomplex of X consisting of one sphere from each orbit of n-spheres. Let $f_0 : (X_0, x_0) \to (Y, y_0)$ be a cellular map realizing the induced homomorphism

$$\pi_n(X_0, x_0) \subset \pi_n(X, x_0) = H_n(X) \xrightarrow{h} H_n(Y) = \pi_n(Y, y_0).$$

f_0 is then extended to $f : (X, x_0) \to (Y, y_0)$ by defining $fg(x) = gf_0(x)$ for all $g \in G$, $x \in X_0$.

LEMMA (2.4). *Let X and Y be G complexes where*

a) dim $(X) = n$ *and G permutes the n-cells of X freely; and*

b) *Y is $n - 1$ connected and G fixes a 0-cell y_0 of Y.*

Then any $Z[G]$ homomorphism $h : H_n(X) \to H_n(Y)$ which factors through a projective $Z[G]$ module is realized by a G equivariant cellular map $f : X \to Y$.

Proof. Let $h = \beta\alpha$ where $\alpha: H_n(X) \to P$, $\beta : P \to H_n(Y)$ and P is projective. Since P is weakly injective and $H_n(X)$ is a Z-summand of $C_n(X)$, α extends to $C_n(X) = H_n(X/X^{n-1})$. Let $h' : H_n(X/X^{n-1}) \to H_n(Y)$ denote the corresponding extension of h. By Lemma (2.3) there is a G-equivariant cellular map f': $X/X^{n-1} \to Y$ realizing h', and the composite $X \to X/X^{n-1} \xrightarrow{f'} Y$ realizes h.

As in [**1**], the proof of the main theorem relies on the construction of complexes satisfying the following:

Definition (2.5). X is *tractable of type* (A, n) if X is an n-dimensional G-complex of type (A, n) so that G permutes the n-cells of X freely and fixes a 0-cell.

Given G and an integer $N \geqq 2$, we consider the following properties:

$P(N)$: For any Z-torsion free f.g. $Z[G]$ module A, there is a G-complex X of type $(A, N - k)(k \geqq 0$ fixed$)$ such that dim $(X) = N$, G fixes a 0-cell of X, and X is finite if $\mathrm{Sw}(A) = 0$.

$P'(N)$: For any Z-torsion free f.g. $Z[G]$ module A, there is a G-complex X of type $(A, N - k)(k \geqq 0$ fixed$)$ as in $P(N)$, and so that G permutes the N-cells of X freely.

$Q(N)$: For any Z-torsion free f.g. $Z[G]$ module A, there are tractable complexes of type (A, N) (finite if $\mathrm{Sw}(A) = 0$).

$R(N)$: For any f.g. $Z[G]$ module A, there are complexes of type $(A, n)n \geqq N$ (finite complexes if $\mathrm{Sw}(A) = 0$).

LEMMA (2.6). *$P(N) \Rightarrow Q(N + 1)$, $P'(N) \Rightarrow Q(N)$, and $Q(N) \Rightarrow R(N)$.*

Proof. $P(N) \Rightarrow Q(N+1)$: Given k as in $P(N)$, choose an exact sequence C_* of the form $0 \to Z \to F_k' \to \ldots \to F_0' \to M \to 0$ with F_i' f.g. free, and M Z-torsion free. Such a sequence is determined for example, by part of a complete resolution for G (see [**2**]). If A is a f.g. Z-torsion free $Z[G]$ module, then $C_* \otimes_Z A$ with $g(x \otimes y) = gx \otimes gy$ defines an exact sequence of the form

$$0 \to A \to F_k \xrightarrow{\epsilon_k} F_{k-1} \to \ldots \xrightarrow{\epsilon_1} F_0 \xrightarrow{\epsilon_0} B \to 0$$

with F_i f.g. free and B Z-torsion free. Since the F_i are f.g. free $\mathrm{Sw}(B) = \pm\mathrm{Sw}(A)$.

Now let X be a G-complex of type $(B, N-k)$ as in $P(N)$. If $\mathrm{Sw}(A) = 0$, then $\mathrm{Sw}(B) = 0$ and X is finite. Let X_i be a wedge of $N - k + i$ spheres (freely permuted by G) of type $(F_i, N - k + i)$. By Lemma (2.3), there is an equivariant cellular map $f_0 : X_0 \to X$ realizing ϵ_0, and Cf_0 (the mapping cone of f_0) is of type (Kern (ϵ_0), $N - k + 1$). Iterating this argument with $X_i \xrightarrow{f_i} Cf_{i-1}$ realizing $\epsilon_i : F_i \to$ Kern (ϵ_{i-1}), we attach a finite number of cells to X and obtain a complex Y of type $(A, N+1)$. Since the attached cells are freely permuted by G, Y is tractable and we have $Q(N+1)$.

$P'(N) \Rightarrow Q(N)$: Given A, we proceed as in the previous argument using an exact sequence $0 \to A \to F_{k-1} \to \ldots \to F_0 \to B \to 0$. Since the N-cells of the complex X are permuted freely, we obtain a tractable complex of type (A, N) as in $Q(N)$.

$Q(N) \Rightarrow R(N)$: Let A be a f.g. $Z[G]$ module, and $0 \to B \xrightarrow{\alpha} F \to A \to 0$ an exact sequence with F f.g. free. By $Q(N)$ there is a tractable complex X of type (B, N) (finite if $\mathrm{Sw}(B) = -\mathrm{Sw}(A) = 0$). Let Y be a wedge of N-spheres of type (F, N). By Lemma (2.4) there is a G-equivariant cellular map $f : X \to Y$ realizing α. Cf is then of type (A, N), and finite if $\mathrm{Sw}(A) = 0$. Complexes of type $(A, n) n > N$ are obtained by suspension.

3. Proof of the main theorem. Let Z_{p^n} denote the cyclic group of order p^n (p prime) with generator t. Given a module M, we let M^k denote the direct sum of k copies of M. The main theorem relies on the following algebraic result whose proof we defer to § 4.

THEOREM (3.1). *Let A be a f.g. Z-torsion free $Z[Z_{p^n}]$ module. There is an exact sequence of $Z[Z_{p^n}]$ modules*

$$0 \to A \oplus P \to F \oplus Z[Z_{p^{n-1}}]^k \to B \to 0$$

with the following properties:

a) *P is a f.g. projective $Z[Z_{p^n}]$ module, and F is f.g. free;*

b) *B is a $Z[Z_{p^{n-1}}]$ module (i.e. $t^{p^{n-1}} \cdot x = x$ for all $x \in B$); and*

c) *if $\mathrm{Sw}(A) = 0$, P is free and $\mathrm{Sw}(B) = 0$ in $G_0(Z[Z_{p^{n-1}}])/S$.*

We now prove the main theorem modulo Theorem (3.1).

THEOREM (3.2). *Let A be a f.g. $Z[Z_{p^n}]$ module. There are complexes of type $(A, m) (m \geqq n + 2)$, and finite complexes of type (A, m) if and only if* $\mathrm{Sw}(A) = 0$.

Proof. By Lemma (2.6) it is sufficient to show that $P'(n+2)$ holds for Z_{p^n}. Specifically we prove that for A a f.g. Z-torsion free $Z[Z_{p^n}]$ module, there is an $n+2$ dimensional complex of type $(A, n+1)$ (finite if $\mathrm{Sw}(A) = 0$) such that Z_{p^n} permutes the $n+2$ cells freely and fixes a 0-cell. We prove this by induction on n.

$n = 1$: Let A be a f.g. Z-torsion free $Z[Z_p]$ module. Then $A = M \oplus Z^s$, and there is an exact sequence

$$0 \rightarrow F_1 \oplus Z^r \xrightarrow{\alpha} F \rightarrow M \rightarrow 0$$

with F and F_1 free (f.g. free if $\mathrm{Sw}(A) = 0$). This follows as in Lemma (3.1) of [**1**] replacing the sequences $0 \rightarrow \mathscr{B} \rightarrow \mathscr{B}_{\bar{\omega}} \rightarrow Z \rightarrow 0$ by the sequences constructed in (4.5) (this paper). Let X_1 be a tractable complex of type $(F_1 \oplus Z^r, 2)$, and Y a tractable complex of type $(F, 2)$ as constructed in [**1**]. By Lemma (2.4), there is a Z_p equivariant cellular map f realizing α, and Cf is of type $(M, 2)$ with 3-cells freely permuted by Z_p. Let $X = Cf \vee X_2$ where X_2 is tractable of type $(Z^s, 2)$. X is 3-dimensional of type $(A, 2)$ and satisfies the requirements of $P'(3)$.

$n - 1 \Rightarrow n$: Given an f.g. Z-torsion free $Z[Z_{p^n}]$ module A, let

$$0 \rightarrow A \oplus P \rightarrow F \oplus Z[Z_{p^{n-1}}]^k \xrightarrow{\beta} B \rightarrow 0$$

be the exact sequence in Theorem (3.1). By the inductive assumption there is an $n+1$ dimensional $Z_{p^{n-1}}$ complex Y of type (B, n) with fixed 0-cell. If $\mathrm{Sw}(A) = 0$, $\mathrm{Sw}(B) = 0$ and we choose Y to be finite. Let X_1 be a wedge of n-spheres permuted by Z_{p^n} of type $(F \oplus Z[Z_{p^{n-1}}]^k, n)$. By Lemma (2.3) there is a Z_{p^n} equivariant map $f_1 : X_1 \rightarrow Y$ realizing β. Cf_1 is then a Z_{p^n} complex of type $(A \oplus P, n+1)$ and dimension $n+1$ (finite if $\mathrm{Sw}(A) = 0$). If $\mathrm{Sw}(A) = 0$, P is free and we let X_2 be a wedge of $n+1$ spheres freely permuted by Z_{p^n} of type $(P, n+1)$. Otherwise, choose an exact sequence

$$0 \rightarrow P \rightarrow F_1 \xrightarrow{\epsilon} F_2 \rightarrow 0$$

with F_1 and F_2 free, and let X_2 be the mapping cone of an equivariant cellular map between tractable complexes which realizes ϵ. By Lemma (2.4), there is an equivariant cellular map $g : X_2 \rightarrow Cf$ realizing the inclusion $P \rightarrow A \oplus P$. Cg is then of type $(A, n+1)$ and satisfies the requirements of $P'(n+1)$.

This completes the induction, and we have the main theorem modulo (3.1).

4. The proof of Theorem (3.1).

Definition (4.1). A commutative diagram of rings and ring homomorphisms

$$\begin{array}{ccc} R & \xrightarrow{i_1} & R_1 \\ {\scriptstyle i_2}\downarrow & & \downarrow{\scriptstyle j_1} \\ R_2 & \xrightarrow{j_2} & \bar{R} \end{array}$$

is a *fibered product diagram* (or *pullback diagram*) if

$$R \simeq \{(r_1, r_2) | r_i \in R_i, j_1(r_1) = j_2(r_2)\} \subset R_1 \oplus R_2.$$

As an example, if I and J are ideals in R, the following is a fibered product diagram:

$$\begin{array}{ccc} R/I \cap J & \to & R/I \\ \downarrow & & \downarrow \\ R/J & \to & R/I + J \end{array} \tag{4.2}$$

Our main interest in fibered product diagrams is the following construction of projective modules due to Milnor (see [**3**]): Assume that we have a diagram as in (4.1) with at least one of j_1, j_2 onto. Then given P_i f.g. projective R_i modules $i = 1, 2$, and an isomorphism h: $\bar{R} \otimes_{R_1} P_1 \to \bar{R} \otimes_{R_2} P_2$, let $P = \{(p_1, p_2) \in P_1 \oplus P_2 | \alpha(p_1) = \beta(p_2)\}$ where $\alpha(p_2) = 1 \otimes p_2$, and $\beta(p_1) = h(1 \otimes p_1)$. In short, P is the pullback in the following diagram:

$$\begin{array}{ccc} P & \to & P_1 \\ \downarrow & & \downarrow \beta. \\ P_2 & \xrightarrow{\alpha} & \bar{R} \underset{R_2}{\otimes} P_2 \end{array}$$

P is then a f.g. projective R module with $(r_1, r_2) \cdot (p_1, p_2) = (r_1 p_1, r_2 p_2)$.

Now consider the principal ideals $I = (t^{p^{n-1}} - 1)$, $J = (\phi_{p^n}(t)) = (\phi_p(t^{p^{n-1}}))$ in $Z[Z_{p^n}]$ where $\phi_m(t)$ denotes the mth cyclotomic polynomial. Since $I \cap J = 0$, we have the fibered product diagram

$$\begin{array}{ccc} Z[Z_{p^n}] & \longrightarrow & Z[Z_{p^n}]/J \\ \downarrow & & \downarrow \\ Z[Z_{p^n}]/I & \longrightarrow & Z[Z_{p^n}]/I + J. \end{array} \tag{4.3}$$

Identifying the rings in (4.3) we have the diagram

$$\begin{array}{ccc} Z[Z_{p^n}] & \longrightarrow & Z[\zeta_{p^n}] \\ \downarrow & & \downarrow \\ Z[Z_{p^{n-1}}] & \longrightarrow & Z_p[Z_{p^{n-1}}] \end{array} \tag{4.4}$$

where ζ_{p^n} is a primitive p^n-th root of unity and $Z[\zeta_{p^n}]$ is the p^n-th cyclotomic integers.

Note that since $Z[\zeta_{p^n}]$ is a Dedekind domain, every ideal is projective, and every f.g. Z-torsion free $Z[\zeta_{p^n}]$ module is isomorphic to a direct sum of ideals.

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If $M = \mathscr{A}_1 \oplus \ldots \oplus \mathscr{A}_k$, we let

$$\mathrm{cl}\,(M) = \prod_{i=1}^{k} \mathscr{A}_i \in C(Z[\zeta_{p^n}]),$$

the ideal class group of $Z[\zeta_{p^n}]$. Since

$$\mathscr{A}_1 \oplus \ldots \mathscr{A}_k \simeq Z[\zeta_{p^n}]^{k-1} \oplus \prod_{i=1}^{k} \mathscr{A}_i,$$

$\mathrm{cl}(M)$ is trivial if and only if M is a free $Z[\zeta_{p^n}]$ module. The following lemma is an application of Milnor's construction:

LEMMA (4.5). *Let M be a f.g. Z-torsion free $Z[\zeta_{p^n}]$ module. There is an exact sequence $0 \to Z[Z_{p^{n-1}}]^k \to P \to M \to 0$ where P is a f.g. projective $Z[Z_{p^n}]$ module, and P is free if* $\mathrm{cl}\,(M) = 1$.

Proof. Let $M = \mathscr{A}_1 \oplus \ldots \oplus \mathscr{A}_k$ where $\mathscr{A}_i$ is an ideal in $Z[\zeta_{p^n}]$ $i = 1, \ldots k$.

$$Z_p[Z_{p^{n-1}}] \underset{Z[\zeta_{pn}]}{\otimes} M = \mathscr{A}_1/\bar{I}\cdot\mathscr{A}_1 \oplus \ldots \oplus \mathscr{A}_k/\bar{I}\cdot\mathscr{A}_k$$

where $\bar{I}$ is the ideal in $Z[\zeta_{p^n}]$ corresponding to I. Given an ideal $\mathscr{A}$ in $Z[\zeta_{p^n}]$, $\mathscr{A} \simeq \mathscr{B}$ where $\mathscr{B}$ is relatively prime to $\bar{I}$, and

$$\mathscr{A}/\bar{I}\cdot\mathscr{A} \simeq \mathscr{B}/\bar{I}\cdot\mathscr{B} = \mathscr{B}/\bar{I} \cap \mathscr{B} \simeq (\mathscr{B} + \bar{I})/\bar{I} = Z[\zeta_{p^n}]/\bar{I} \simeq Z_p[Z_{p^{n-1}}].$$

Therefore

$$Z_p[Z_{p^{n-1}}] \underset{Z[\zeta_{pn}]}{\otimes} M \sim Z_p[Z_{p^{n-1}}]^k \sim Z_p[Z_{p^{n-1}}] \underset{Z[Z_{pn-1}]}{\otimes} Z[Z_{p^{n-1}}]^k,$$

and we apply Milnor's construction to obtain the pullback diagram:

$$\begin{array}{ccc} P & \rightarrow & M \\ \downarrow & & \downarrow \\ Z[Z_{p^{n-1}}]^k & \xrightarrow{\psi} & Z_p[Z_{p^{n-1}}]^k. \end{array}$$

P is a f.g. projective $Z[Z_{p^n}]$ module, and if $\mathrm{cl}\,(M) = 1$, P is free.

Now consider the exact sequence

$$0 \to \mathrm{Kern}\,(\psi) \xrightarrow{i_1} P \xrightarrow{\pi_2} M \to 0$$

where $i_1(x) = (x, 0)$ and $\pi_2(x, y) = y$.

$$\mathrm{Kern}(\psi) = \phi_{p^n}(t)\cdot Z[Z_{p^{n-1}}]^k = \phi_p(t^{p^{n-1}})\cdot Z[Z_{p^{n-1}}]^k = p\cdot Z[Z_{p^{n-1}}]^k \simeq Z[Z_{p^{n-1}}]^k.$$

Therefore we have the sequence

$$0 \to Z[Z_{p^{n-1}}]^k \to P \xrightarrow{\pi_2} M \to 0.$$

We now prove Theorem (3.1).

THEOREM (3.1). *Let A be a f.g. Z-torsion free $Z[Z_{p^n}]$ module. There is an exact sequence of $Z[Z_{p^n}]$ modules $0 \to A \oplus P \to F \oplus Z[Z_{p^{n-1}}]^k \to B \to 0$ with the following properties:*

a) *P is a f.g. projective $Z[Z_{p^n}]$ module, and F is f.g. free;*

b) *B is a $Z[Z_{p^{n-1}}]$ module; and*

c) *if* $\mathrm{Sw}(A) = 0$, *P is free and* $\mathrm{Sw}(B) = 0$ *in* $G_0(Z[Z_{p^{n-1}}])/S$.

Proof. Given A, let $0 \to A \to F_1 \to C \to 0$ be an exact sequence with F_1 f.g. free and C Z-torsion free. Note that $\mathrm{Sw}(A) = 0 \Leftrightarrow \mathrm{Sw}(C) = 0$. Let $B = \{x \in C | (t^{p^{n-1}} - 1) \cdot x = 0\}$. B is a $Z[Z_{p^{n-1}}]$ module, and $\mathrm{Sw}(A) = \mathrm{Sw}(C) = \mathrm{Sw}(B) + \mathrm{Sw}(C/B)$ since $0 \to B \to C \to C/B \to 0$ is exact. C/B is Z-torsion free and is annihilated by J since $\phi_{p^n}(t) \cdot C \subset B$. Therefore C/B is a projective $Z[\zeta_{p^n}]$ module, and is free if and only if $\mathrm{cl}\ (C/B) = 1$. From [**4**, § 13] it follows that

$$\varphi: G_0(Z[Z_{p^n}])/S \to (G_0(Z[Z_{p^{n-1}}])/S \oplus C(Z[\zeta_{p^n}])$$

by $\mathrm{Sw}(C) \to (\mathrm{Sw}(B), \mathrm{cl}(C/B))$ defines an isomorphism. Thus $\mathrm{Sw}(A) = 0 \Leftrightarrow \mathrm{Sw}(C) = 0 \Leftrightarrow \mathrm{Sw}(B) = 0$ and $\mathrm{cl}\ (C/B) = 1$. Let $0 \to Z[Z_{p^{n-1}}]^k \to P' \to C/B \to 0$ be the exact sequence of Lemma (4.5). P' is f.g. projective and is free if $\mathrm{Sw}(A) = 0$. Since P' is projective, there is a commutative diagram

$$(4.6)\qquad \begin{array}{ccccccccc} 0 & \longrightarrow & Z[Z_{p^{n-1}}]^k & \longrightarrow & P' & \longrightarrow & C/B & \longrightarrow & 0 \\ & & \big\downarrow \hat{f} & & \big\downarrow f & & \big\downarrow 1 & & \\ 0 & \longrightarrow & B & \longrightarrow & C & \longrightarrow & C/B & \longrightarrow & 0 \end{array}.$$

Now choose a surjection $g: F_2 \to B$ where F_2 is a free $Z[Z_{p^n}]$ module. The sequence

$$(4.7)\qquad 0 \to K \to F_2 \oplus P' \xrightarrow{\gamma} C \to 0$$

is exact where $\gamma(x, y) = g(x) - f(y)$, and $K = \{(x, y) | g(x) = f(y)\}$. Since the image of g is B, and $f(x) \in B \Leftrightarrow x \in Z[Z_{p^{n-1}}]^k$ (by 4.6),

$$K = \{(x, y) \in F_2 \oplus Z[Z_{p^{n-1}}]^k | g(x) = \hat{f}(y)\}.$$

Therefore the sequence

$$(4.8)\qquad 0 \to K \to Z[Z_{p^{n-1}}]^k \oplus F_2 \xrightarrow{\epsilon} B \to 0$$

is exact where $\epsilon(x, y) = g(x) - \hat{f}(y)$. Now since $0 \to A \to F_1 \to C \to 0$ and $0 \to K \to P' \oplus F_2 \to C \to 0$ are exact, $A \oplus P' \oplus F_2 \simeq F_1 \oplus K$ by Schanuel's Lemma. Adding F_1 to (4.8), and using this isomorphism, we obtain the exact sequence

$$0 \to A \oplus P \to F \oplus Z[Z_{p^{n-1}}]^k \to B \to 0$$

where $P = P' \oplus F_2$ and $F = F_2 \oplus F_1$.

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University of Wisconsin,
Milwaukee, Wisconsin

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