CORRESPONDENCE.

ON THE DETERMINATION OF AVERAGE AGES BY METHODS OF WEIGHTING.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In the discussion which arose on Mr. Ackland's paper on an approximate method of Valuation of Whole-life Assurances, with allowance for selection, the fact was commented on by more than one speaker that, in determining the mean age or centre of gravity of a series, the terms of which can be represented by the expression $A+Br^x$, an increase in the value of r will cause an increase in the mean age. As this method of finding an average age appears likely to become of general use, it may be of interest to readers of the *Journal* to have a mathematical proof of the following general proposition:

Given
$$r^{\bar{x}}=\frac{S_1r^{x_1}+S_2r^{x_2}+\ldots+S_nr^{x_n}}{S_1+S_2+\ldots+S_n},$$

where $x_1, x_2 \ldots x_n$ and $S_1, S_2 \ldots S_n$ are any positive quantities independent of r, to prove that for all positive values of r an increase in the value of r will cause an increase in the value of $\bar{x}$.

Taking logarithms of both sides of the above equation and dividing by log r, we have—

$$\bar{x} = \frac{\log(S_1 r^{x_1} + S_2 r^{x_2} + \ldots + S_n r^{x_n}) - \log(S_1 + S_2 + \ldots + S_n)}{\log r}.$$

An increase in r in the above expression will cause an increase in $\bar{x}$ if

$$\frac{d}{dr}\left[\frac{\log(S_1 r^{x_1} + S_2 r^{x_2} + \ldots + S_n r^{x_n}) - \log(S_1 + S_2 + \ldots + S_n)}{\log r}\right]$$

is positive; *i.e.* if

$$\frac{(S_1 x_1 r^{x_1 - 1} + S_2 x_2 r^{x_2 - 1} + \ldots + S_n x_n r^{x_n - 1})}{(S_1 r^{x_1} + S_2 r^{x_2} + \ldots + S_n r^{x_n})} \frac{\log r}{(\log r)^2} - \frac{\bar{x} \log r}{r(\log r)^2}$$

is positive; *i.e.* if

$$\left[\frac{S_1 x_1 r^{x_1} + S_2 x_2 r^{x_2} + \ldots + S_n x_n r^{x_n}}{S_1 r^{x_1} + S_2 r^{x_2} + \ldots + S_n r^{x_n}} \log r - \bar{x} \log r\right]$$

is positive; *i.e.* if

$$r^{\frac{S_1 x_1 r^{x_1} + S_2 x_2 r^{x_2} + \ldots + S_n x_n r^{x_n}}{S_1 r^{x_1} + S_2 r^{x_2} + \ldots + S_n r^{x_n}}} > r^{\bar{x}},$$

if

$$r^{S_1 x_1 r^{x_1} + S_2 x_2 r^{x_2} + \ldots + S_n x_n r^{x_n}} > \left(\frac{S_1 r^{x_1} + S_2 r^{x_2} + \ldots + S_n r^{x_n}}{S_1 + S_2 + \ldots + S_n}\right)^{S_1 r^{x_1} + S_2 r^{x_2} + \ldots + S_n r^{x_n}}$$

if

$$(r^{x_1})^{S_1 r^{x_1}} (r^{x_2})^{S_2 r^{x_2}} (r^{x_3})^{S_3 r^{x_3}} \ldots (r^{x_n})^{S_n r^{x_n}} > \left(\frac{S_1 r^{x_1} + S_2 r^{x_2} + \ldots + S_n r^{x_n}}{S_1 + S_2 + \ldots + S_n}\right)^{S_1 r^{x_1} + S_2 r^{x_2} + \ldots + S_n r^{x_n}}$$

if

$$A^{S_1 A}\ B^{S_2 B}\ C^{S_3 C} \ldots N^{S_n N} > \left(\frac{S_1 A + S_2 B + \ldots + S_n N}{S_1 + S_2 + \ldots + S_n}\right)^{S_1 A + S_2 B + \ldots + S_n N}$$

where $A = r^{x_1}$ $B = r^{x_2}$. . . $N = r^{x_n}$.

Now, it is a well-known theorem in algebra that if we have any number, say n, of positive quantities, A, B, C . . . N, which are not all equal, then

$$A^A B^B C^C \ldots N^N > \left(\frac{A + B + C + \ldots + N}{n}\right)^{A+B+C+\ldots+N}$$

This inequality is perfectly general and will hold for any number of factors.

Take S_1 quantities each $= A$

„ S_2 „ „ $= B$

. . . .

. . . . .

„ S_n „ „ $= N$.

Therefore, we get:

$$\left\{\begin{array}{l} (A^{A})(A^{A})\ldots \text{to } S_1 \text{ factors} \\ \times (B^{B})(B^{B})\ldots \text{to } S_2 \quad ,, \\ \times \ldots\ldots\ldots\ldots \\ \times \ldots\ldots\ldots\ldots \\ \times\ (N^{N})(N^{N})\ldots \text{to } S_n \text{ factors} \end{array}\right\} > \left\{\frac{\begin{array}{l}(A+A+\ldots \text{to } S_1 \text{ terms}) \\ +(B+B+\ldots \text{to } S_2 \quad ,, \quad) \\ +\ \ldots\ldots\ldots \\ +(N+N+\ldots \text{to } S_n \text{ terms})\end{array}}{S_1+S_2+\ldots+S_n}\right\}^{\begin{array}{l}(A+A+\text{to } S_1 \text{ terms}) \\ +(B+B+\text{ to } S_2 \text{ terms}) \\ +\ldots+(N+N+\text{to } S_n \text{ terms})\end{array}}$$

$$\therefore\ A^{S_1A}\ B^{S_2B}\ C^{S_3C}\ldots N^{S_nN} > \left(\frac{S_1A+S_2B+\ldots+S_nN}{S_1+S_2+\ldots+S_n}\right)^{S_1A+S_2B+\ldots+S_nN}$$

I am, Sir,

Yours faithfully,

STEUART E. MACNAGHTEN.

18, *Lincoln's Inn Fields, W.C.*
4 *December* 1905.

ON THE USE OF $O^{[M]}$ SELECT PREMIUMS FOR VALUATION PURPOSES.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—Mr. King remarks, in his recent paper on the Valuation in groups of Whole-life Policies by Select Mortality Tables (§ 4), that the reserves by the $O^{[M]}$ Select Tables are greater than by any table or combination of tables hitherto used. It is interesting to notice that the great stringency of the $O^{[M]}$ Tables is due mainly to the net premiums employed. This is made clear if the reserves by the Select and Aggregate Tables for Model Office, No. 1, at the end of 50 years, are analyzed by the formula which is given by Mr. King.

Basis of Valuation	Value of Sums Assured	Value of Net Premiums	Actual Reserves
$O^{[M]}$, 3 %	1,450,683	750,263	700,420
O^{M}, 3 %	1,449,006	760,007	688,999
Difference	1,677	9,744	11,421

Of the whole difference in the reserves, 85 per-cent is due to the difference in the value of the net premiums.

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