would need to look elsewhere for full proofs. My only real criticism here is that Breuer's focus is rather strongly centred on $\mathcal{H}^1(X)$, the subject matter of the later chapters: for the general reader, it would enhance the usefulness of the book to mention (if only briefly) a few other important G-modules associated with X, such as the first homology and cohomology modules, and their decompositions in terms of holomorphic and antiholomorphic differentials. Examples of such applications, which might be mentioned here, include Macbeath's investigation of the action of G on homology, Sah's use of modular representations to construct abelian coverings, and the recent use by Streit and Wolfart of the action on differentials to study fields of definition.

Chapter 4 concerns the question of whether (and, if so, in how many ways) a finite group can be generated by elements with specific properties, usually as an epimorphic image of some Fuchsian group. Breuer brings together a number of methods which have recently been developed for studying surface coverings, the inverse Galois problem, and the symmetric genus problem. Although much of this material is readily available in research papers, it is very useful to have it collected, unified and further developed here.

In Chapters 5–7, with the preliminaries disposed of, Breuer makes a detailed attack on the two problems stated earlier. Much of this is his own recent research, using algorithms specifically developed for the purpose; I found it rather impressive, though significantly harder going than the more familiar material in the first part of the book. Some of the results consist of long and detailed tables, obtained by careful case-by-case analysis, frequently assisted by GAP. A typical example is the classification of the groups G which act irreducibly on $\mathcal{H}^1(X)$: there are 36 of these, the largest group (and genus) being $L_2(13)$ for $g = 14$; in each case Breuer gives the signature of the corresponding Fuchsian group and the matrices representing a set of generators of G on $\mathcal{H}^1(X)$.

The text is well organized, and (given the technical nature of some of the subject matter) not too hard to read; a well-motivated postgraduate student should have no great difficulty in making progress through it, especially in the more expository first half. The presentation of the material is excellent, with attractive layout and very few misprints. There is a very useful and up-to-date bibliography, and the index is also good. Taken together, the clear expository Chapters 1–4 and the interesting new algorithms and results in Chapters 5–7 make this book a valuable addition to the literature, which should be essential reading for anyone interested in the connections between Riemann surfaces and finite groups.

G. JONES

McLean, W. *Strongly elliptic systems and boundary integral equations* (Cambridge University Press, 2000), xiv+357 pp., 0 521 66375 X (paperback), £20.95 (US$32.95).

The book offers a concise and self-contained study of distributional solutions of linear, strongly elliptic three-dimensional systems of partial differential equations, which covers the existence of such solutions and their integral representations in terms of single- and double-layer potentials.

After a brief review of some fundamental elements of functional analysis on normed spaces, distributions and their Fourier transforms are discussed succinctly and the full range of Sobolev spaces is introduced and described in some detail, with particular reference to Lipschitz domains. This is followed by a study of the Dirichlet, Neumann and mixed boundary value problems for strongly elliptic systems, in which variational formulations are derived and the existence, uniqueness, regularity and continuous dependence of their solutions on the data are established. The main tool in this analysis is the Fredholm Alternative.

Next, single-layer and double-layer potentials are defined, and the mapping properties of the boundary operators generated by these potentials are investigated. By having their solutions sought in the form of potentials, the above boundary value problems in both interior and exterior domains are subsequently reduced to Fredholm integral equations of the first kind on the domain surface, which are then solved by means of the Fredholm Alternative. The regularity of the solutions of these integral equations is also discussed.

In the last part of the book, the theory developed for general strongly elliptic systems is illustrated in application to the Laplace equation, the Helmholtz equation and the equilibrium equations of linear elasticity in the homogeneous and isotropic case. Some technical analytic details are gathered together in three appendices.

The mathematical treatment is rigorous and the presentation is clear and easy to follow. The book is a good source of information for scientists and engineers interested in learning about boundary integral equation techniques, and in the development of boundary element methods for the approximate solution of a wide class of linear boundary value problems.

C. CONSTANDA

KUSRAEV, A. G. *Dominated operators* (Mathematics and its Applications, vol. 519, Kluwer, 2000), xiii+446 pp., 0 792 36485 6 (hardback), £120.

In spite of the widening contacts between mathematicians in the former Soviet Union (fSU) and the western world, there remain many legacies of the decades of separate development of mathematics in the two regions. One of these is that research in some disciplines has developed in rather different directions. One such field is that of vector and Banach lattices and operators on them. As far back as 1935–1936 Kantorovich introduced the notion of a *lattice-normed space*, which generalizes the notion of a real normed space to consider vector spaces endowed with a 'norm' taking values in a vector lattice rather than the reals. If we take $E = \mathbb{R}$, then we obtain the classical (real) normed spaces, while taking $X = E$, with the lattice modulus taking the role of the norm, gives another class of examples. The role of bounded operators in this setting is played by the so-called *dominated operators*, T, defined by the requirement that the 'norm' of Tx is dominated by the result of applying S to the 'norm' of x for all $x \in X$, where S is a positive linear operator on E. In the case of normed spaces this notion coincides with that of the traditionally bounded operators, while, when $X = E$, this is precisely the class of regular operators (those which can be written as the difference of two positive operators). Professor Kusraev has researched into these and related notions for around 20 years and to a large extent the work under review may be seen as a summary of his research over that period, while at the same time setting it into context. This work provides a convenient opportunity to find out what has been happening in the fSU in this field, one which has been almost completely neglected outside the fSU.

The work starts with a chapter on *Boolean Algebras and Vector Lattices*, which is explicitly presented as a summary of needed results and to establish notation which, in a field where even western literature often disagrees as to terminology and where the fSU usually employs completely different terminology, is vital. After a chapter on *Lattice-Normed Spaces*, a chapter on *Positive Operators* sets the scene for the basic chapter on *Dominated Operators*. There follow chapters on two particular concrete classes of operators, *Disjointness Preserving Operators* and *Integral Operators*, and their relationship with the class of dominated operators. If the vector lattice E is actually a normed lattice, then one can norm (in the traditional sense) a lattice-normed space by composing the norm on E with the lattice norm on X. This allows a generalized abstract setting in which to study *Operators in Spaces with Mixed Norm*, which is the topic of the seventh chapter. Topics included here include various kinds of summing operators and Kaplansky–Hilbert modules. The final chapter is on *Applications of Boolean-Valued Analysis* in this area (an appendix provides an introduction to Boolean-valued models for those unfamiliar with this field).

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