SOME ASPECTS OF CONSTRUCTING LONG EPHEMERIDES OF THE SUN, MAJOR PLANETS AND THE MOON: EPHEMERIS AE95

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1. Introduction

The construction of long-term numerical ephemerides of the Sun, major planets and the Moon is based essentially on the high-precision numerical solution of the problem of the motion of these bodies and polynomial representation of the data. The basis of each ephemeris is a mathematical model describing all the main features of the motions of the Sun, major planets, and Moon. Such mathematical model was first formulated for the ephemerides DE/LE and was widely applied with some variations for several national ephemeris construction. The model of the AE95 ephemeris is based on the DE200/LE200 ephemeris mathematical model. Being an ephemeris of a specific character, the AE95 ephemeris is a basis for a special edition "Supplement to the Astronomical Yearbook for 1996–2000", issued by the Institute of the Theoretical Astronomy (ITA) (Glebova *et al.*, 1995). This ephemeris covering the years 1960–2010 is not a long ephemeris in itself but the main principles of its construction allow one to elaborate the long-term ephemeris on an IBM PC-compatible computer. A high-precision long-term numerical integration of the motion of major bodies of the Solar System demands a choice of convenient variables and a high-precision method of the numerical integration, taking into consideration the specific features of both the problem to be solved and the computer to be utilized.

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2. Numerical Solution of the Problem

2.1. EQUATIONS OF MOTION

The appropriate variables for the problem assume both the boundedness of their values and a simple algorithmic form of the differential equations for these variables. When describing the orbital motions of the bodies it is natural to deal with their barycentric rectangular coordinates. For the Moon's motion the choice of such coordinates depends on the compiler in use. If a compiler represents real numbers with 16 or 18 decimal figures then one has to use the geocentric rectangular coordinates. The compiler representing real numbers with 24 or more decimal figures permits one to perform the integration of the Moon's orbital motion equations in terms of the barycentric coordinates. When describing the rotational motions of celestial bodies it is preferable to use four Rodrigues-Hamilton parameters instead of three Euler angles, because Rodrigues-Hamilton parameters vary only periodically while one or two Euler angles grow linearly with time. It is quite possible to complete the mathematical model of the Solar System motion by the differential equations of the Earth's rotation. In this case another advantage of the Rodrigues-Hamilton parameters utilization is revealed. The differential equations of the Earth's rotation constructed for the Rodrigues-Hamilton parameters can be integrated numerically relatively either to the ecliptical or equatorial coordinate systems, whereas those in the Euler angles can be integrated only in the ecliptical coordinate system. When constructing the AE95 ephemeris the rectangular coordinates for the orbital motion (barycentric for the Sun and planets and geocentric for the Moon) and Rodrigues-Hamilton parameters for the Moon's rotational motion were used.

2.2. NUMERICAL INTEGRATOR

The numerical integration for the AE95 ephemeris was performed by means of a one-step predictor-corrector method of numerical integration based on the almost-uniform approximation of the right-hand sides of the differential equations by truncated Chebyshev polynomial series (Belikov, 1993). The essential advantage of this method consists in the polynomial representation both of the position and velocity components, providing almost-uniform approximation at any point of a step interval. For reducing round-off errors of the numerical integration the following ideas were realized:

a) Performing the integration with a constant step-size, since the differential equations of the Solar System motion represent a sufficiently smooth dynamical system;

b) Decreasing the number of steps by means of enlarging the step length, with a corresponding increasing the degree of the approximating polynomial;

c) Dividing the right-hand sides of all equations of the orbital motion by the heliocentric gravitational constant in order to improve the internal convergence control of the iterative process of the numerical integration performed on a IBM PC compatible computer.

A high-precision numerical integration of the equations of motion is carried out with a 8-day constant step-size and a 24-th degree of the approximating polynomial. The internal convergence control was determined by a permitted error (10^{-14}) of the convergence of the relative values of all the right-hand sides at each nodal point of the integration step (Eroshkin *et al.*, 1993). A comparison of the numerical integration results with the DE200/LE200 ephemeris data has revealed discrepancies not exceeding 80 *mm* in the radius vector values for the Sun, major planets and Moon over the interval from 1960–2010. Figure 1 shows the differences in the numerical integration of the AE95 equations with the initial conditions taken from the DE200/LE200 minus DE200/LE200 (for the geocentric distances for the Moon and the barycentric distances for the Mercury, Earth, Jupiter, Saturn, and Sun over the interval 1969–2019).

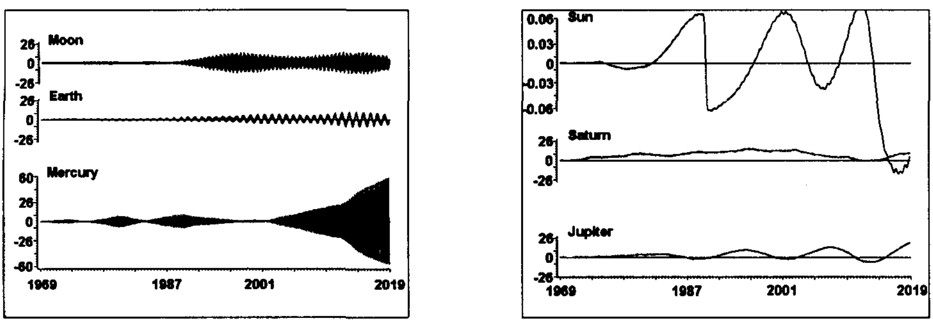

Figure 1. Comparison of the numerical integration with DE200/LE200 in the geocentric distance of the Moon, in the barycentric distances of the Earth, Mercury, Sun, Saturn, and Jupiter. All differences are in millimeters.

For every body the residual behavior is characterized by the presence of the harmonic with a period determined by the mean motion of the body and, in general, an increasing amplitude. One can see a tolerable accumulation of the round-off errors of the numerical integration for the interval 1969–2019.

3. Polynomial Representation

The polynomial approximation procedure uses as basic data the coefficient sets of the Lagrange-Chebyshev interpolation polynomials for the accelerations at each integration step in the INCH procedure. As a result of the numerical integration, a large volume of information for the polynomial coefficients is obtained at each step of integration. It is expedient to store the results in such a form. Proceeding from practical requirements one may choose an approximation interval $T \gg h$. The required accuracy can be achieved by selecting a proper degree of an approximating polynomial. For the polynomial representation of AE95 the truncated Fourier-Chebyshev series approximations were used, because they provided more accurate uniform approximation in comparison with the interpolation polynomial. The original software is developed for creating the ephemeris file simultaneously with the process of numerical integration (Trubitsina, 1995). In Table 1 the summary of the numerical tests of the problem is shown. The accuracy criterion for AE ephemerides is that an approximation error should be less than 10 millimeters at each point in the approximation interval for the planets and 1 millimeter for the Moon.

TABLE 1. Characteristics of the polynomial representation of AE95.

Object	Maximum approx. errors(*mm*)	Approx.interval (days) AE(LE/DE)	Polyn.degree AE(LE/DE)
Mercury	3.8	8(8)	12(11)
Venus	1.6	32(32)	11(11)
E-M barycenter	0.52	16(16)	12(14)
Mars	2.2	32(32)	8(9)
Jupiter	0.53	32(32)	8(8)
Saturn	0.79	32(32)	7(7)
Uranus	0.9	32(32)	7(7)
Neptune	0.43	32(32)	5(5)
Pluto	0.52	32(32)	5(5)
Geocentr.Moon	0.98	8(4)	14(11)
Sun	3.5	32(32)	12(14)
Earth	2.2	16	12

4. Observational Basis

On the basis of the model of numerical integration of the motion of major bodies, the AE94 ephemeris was employed for the reduction of optical observational data of the Sun and major planets and the radar ranging data for the inner planets.

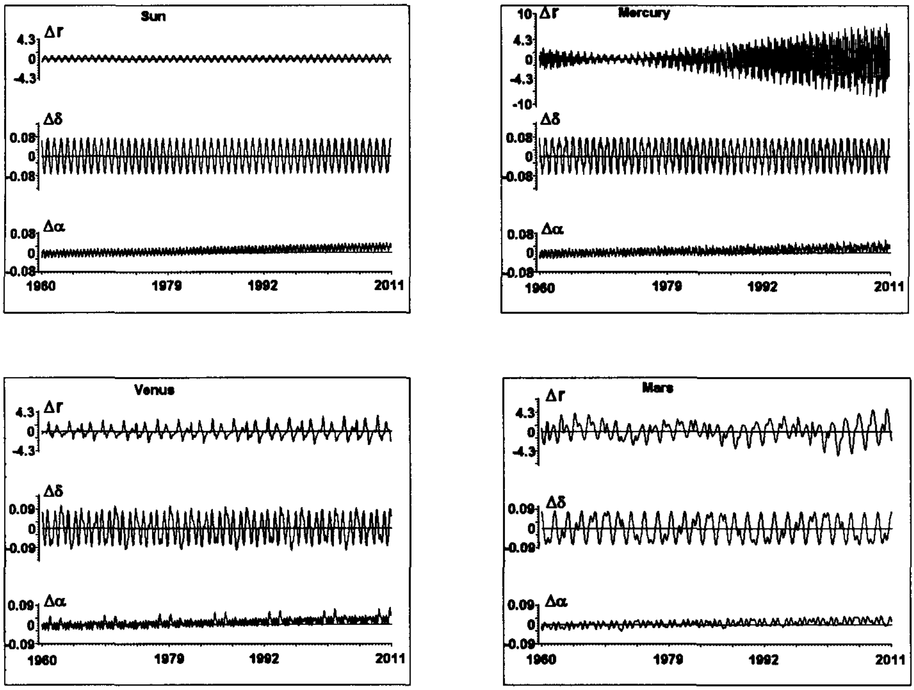

Figure 2. The geocentric differences AE95-DE200 in distance (kilometers), declination, and right ascension (arcseconds) for the Sun, Mercury, Venus, and Mars.

Some of these data covering the interval 1960–1991 were taken from the optical and radar observations data bank created in ITA by Dr. M.L. Sveshnikov. The observations were carried out at the observatories in Washington, Herstmonceux, Tokyo, Pulkovo and Kislovodsk (Russia), Nikolaev and Goloseevo (Ukraine), and Tashkent (Uzbekistan). The optical observations were done on the following instruments: the meridian circle, vertical circle, and transit instrument. The photographic observations were carried out by: the normal astrograph, zonal astrograph, 26″ refractor, and AKD astrograph. All the observations were reduced to the FK4 coordinate system, except the observations performed in the FK5 coordinate system. The reduction of the optical observations includes the corrections to a precession motion and an equinox drift. The preliminary processing was performed in order to determine the accuracy and the weight of every group of the observations. The weights of the groups of the radar ranging were used for the solution. For the optical observations of Mercury, Venus and the Sun an accuracy of 1″ was adopted; and for those of Mars an accuracy of 0″.5 was employed. Some variants of the solution of the normal systems containing

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different groups of observational data were investigated. The corrections to the orbital elements of some planets were determined and afterwards the corrections to the initial conditions of the planets' motion were calculated. A numerical integration of the equations of motion with improved initial conditions was carried out. As a result of the reduction of 37730 optical observation data for the Sun, Mercury, Venus and Mars and 8370 radar ranging data for the inner planets, spanning the interval 1960–1991, the ephemeris AE95 has been constructed in the form of Chebyshev polynomials for the time interval 1960–2010 . The reference frame of AE95 is the FK5 reference system. The AE95 initial epoch is JD 2440400.5. The AE95 ephemeris provides the barycentric positions, velocities, and accelerations of the Sun, the major planets, and the Earth-Moon barycenter, the geocentric position, velocity, and acceleration of the Moon, and the orientation of the lunar axes of the principal moments of inertia by means of four Rodrigues-Hamilton parameters values and their first and second derivatives with respect to time. The independent time variable of the AE95 ephemeris is the Dynamical Barycentric Time (TDB). Figure 2 shows differences between AE95 and DE200/LE200 for Mercury, Venus, Mars, and the Sun in the geocentric distances, declinations, and right ascensions $(\Delta r, \Delta\delta, \Delta\alpha)$.

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