

# THE SHORT PERIOD LATITUDE VARIATIONS DERIVED FROM RECALCULATED PAST ILS OBSERVATIONS

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## ABSTRACT

Ten frequency components of short period latitude variations and corrections for the declination at an epoch and the proper motion in declination are estimated simultaneously. The short period variations are nearly diurnal and semi-diurnal and are supposed to be due to nutations and Earth tides. Analysis is by the least-squares chain method with weights for individual observations, recently developed by Manabe et al. (1977). Weights are assigned on the assumption that the observational errors follow a normal distribution, whose dispersion changes night by night. Among all possible 1024 combinations of the short period variations, the best is chosen on the basis of the minimum AIC procedure. The data used are the results of the homogeneous recalculation of the ILS observations, 1899-1978. This is considered to be the optimal astrometric data set for investigating nutations, because of its long time-span, homogeneity and size. Declinations are estimated for each of the seven observational periods with mean errors smaller than  $0''.01$ . The mean errors of the proper motions are of the order  $0''.002/y$ . Among the short period variations, significant estimates are obtained for the terms corresponding to the lunar fortnightly, solar semi-annual and lunar nodal nutations, and the  $M_2$  tide. The mean errors are shown to be greatly reduced and the entire data set is analyzed. An effort is made to distinguish the preferred theoretical model.

## INTRODUCTION

It is a challenging problem for astrometry to decide which theoretical model of nutations is most consistent with observations. For example an accuracy of  $0''.001$  is required to distinguish between Molodensky's (1961) and Wahr's (1979) models. Manabe et al. (1979) estimated the nutations and Earth tides from the recalculated ILS data (Yumi and Yokoyama 1980) in each of seven periods. They obtained nutation parameters which are generally in agreement with theoretical predictions. However, the mean errors of their estimates are not small enough to distinguish the preferred model. Therefore, it is useful to

estimate the nutations using the whole 80-year series of ILS data. Furthermore, it is very important to combine the data in different periods and analyze them simultaneously in order to realize a uniform system of declinations and to provide a homogeneous series of polar motion data. The most serious obstacle to doing this is an irregularity in the scale values of micrometer screws. In the present analysis, after eliminating long period latitude variations and daily variations of scale values from observational equations by using the least-squares chain method (Manabe et al. 1977), we estimate constant and secular parts of declination corrections and the short period variations of latitude. The estimates thus obtained are free from systematic errors due to irregular variations of scale values. The frequency components of the short period variations that are taken into account are the fortnightly, semi-annual, annual and principal nutations and the  $M_2$  and  $S_2$  tides. In the following notations are the same as those found in Manabe et al. (1977).

#### METHOD OF ANALYSIS

The VZT observable is a difference  $D$  of readings of a micrometer. Apart from accidental errors,  $D$  depends systematically on latitude variations, declination errors and errors in the adopted scale values. These parameters are so small that their products can be neglected. Then, after making various corrections such as differential refractions by using the adopted scale values  $m_i$  at  $t_i$  for a star  $s_i$ , the observational equations for the  $r$ -th night are given by

$$E(\Delta D_r) = X_r \beta + L_r (P_r \Delta m_r)^T \quad (1)$$

where  $\beta = (\Delta \delta^T \Delta u'^T A^T)^T$ ,  $\Delta m_r$  is a daily correction of the adopted scale values,  $\Delta D_r = D_r - \text{diag}[m_1 \dots m_{n_r}] \zeta_r$  is an O-C vector and  $\zeta_r$  is a vector of zenith distances calculated on the basis of catalogue places and the IAU nutation table. Matrices  $X_r$  and  $L_r$  are  $X_r = \text{diag}[m_1 \dots m_{n_r}] (R_r T_r K_r)$  and  $L_r = (-e_{n_r} \zeta_r)$  with  $n_r$ -vector  $e_{n_r} = (1 \dots 1)^T$ . According to the least-squares chain method, the last term in equation (1) can be eliminated by using a matrix  $W_r = (W_{ij}^r)$  with

$$W_{ij}^r = w_i \delta_{ij} - w_i w_j \left\{ m_i m_j [w \zeta \zeta] - (m_i \zeta_j + m_j \zeta_i) [w m \zeta] + \zeta_i \zeta_j [w m m] \right\} \\ / \left\{ [w m m] [w \zeta \zeta] - [w m \zeta]^2 \right\}, \quad (2)$$

where  $w_i$  is a weight and  $[ ]$  is Gauss' summation symbol. Let us denote the resultant normal equation by  $S \hat{\beta} = Q$ , where  $S = \sum_r X_r^T W_r X_r$

and  $Q = \sum_p^T X_p W_p \Delta D_p$ . If we neglect irregular and temperature dependent variations of  $m_j$  which are of relative order  $10^{-4}$ , and small periodic variations of  $\zeta_j$  due to the nutations,  $S$  satisfies a relation

$$S H = 0 \quad , \quad \text{with } H = \begin{bmatrix} e_n & 0 & a \\ 0 & e_n & b \\ 0 & 0 & 0 \end{bmatrix} \quad ,$$

where  $n$ -vectors  $a$  and  $b$  are constant and secular parts of  $\zeta$  such that  $\zeta_j = a_{sj} + (\bar{t}_r - t_0)b_{sj}$ . Therefore, the rank of  $S$  is deficient by three and three additional conditions are required on  $\beta$ . We adopt conditions

$$H^T \beta = 0 \text{ or } e_n^T \Delta \delta = 0 \quad , \quad e_n^T \Delta u' = 0 \text{ and } a^T \Delta \delta + b^T \Delta u' = 0 \quad (3)$$

at an epoch of correction. It can be shown that these conditions are equivalent to setting  $|\beta| = \min$ .

The equation  $S\hat{\beta}=Q$  has been solved with conditions (3) for all possible 1024 combinations of the short period variations. The best combination of the short period variations has been chosen with the aid of the minimum AIC procedure (Akaike 1973).

## RESULTS

The table shows the minimum AIC estimates of the principal terms of the short period variations. The last three columns show the ratio  $\gamma$  of the estimated and the theoretical nutation amplitudes to those of a rigid Earth (Kinoshita 1977) for the diurnal components, and the tidal factor  $\Lambda = 1 + k - \ell$  for the semi-diurnal components.

It is clear that the  $2L-\alpha$  term agrees well with the theoretical values. But the mean error is too large to distinguish the better theoretical value. The agreement of the  $2L'-\alpha$  term with the theoretical values appears poor. However, if the contribution of the  $O_1$  tide to this component ( $-0.0016 \sin(2L'-\alpha)$  at  $39^\circ 8' N$  with  $\Lambda = 1.2$ ) is subtracted from the estimate, we obtain much better agreement. The most serious discrepancies between the present estimates and the theoretical values are in the  $\pm \Omega-\alpha$  terms. The differences are much larger than the mean errors, which are small enough to distinguish between the two models. This may be due to some unknown systematic errors with long time-scale. In fact if we use the data for 1899-1955 only, we obtain  $\gamma = 0.99687 \pm 17$  for  $\Omega-\alpha$  and  $\gamma = 1.00380 \pm 119$  for  $-\Omega-\alpha$ . The differences between these values and those in the table far exceed the mean errors. The  $\gamma$ 's of other components also change, but the differences can be regarded as caused by statistical fluctuations.

Table. Minimum AIC estimates of the short period variations at 1950.0.

	Amp.	Phase	$\gamma$ or $\Lambda$	Wahr	M-II
$2L' - \alpha$	0 <sup>h</sup> :0923 ± 10	0 <sup>o</sup> :87 59	1.0081 105	1.0283	1.022
$-2L' - \alpha$	0.0036 8	32.36 13.16	1.0439 2398		
$2L - \alpha$	0.5476 21	0.04 22	1.0332 389	1.0344	1.0315
$-2L - \alpha$	0.0253 20	-19.87 4.39	1.1201 8728		
$l - \alpha$	0.0430 50	170.18 40.41	1.7196 2004		
$-l - \alpha$	0.0496 39	-3.35 4.47	1.9856 1576	1.2246	1.2247
$\Omega - \alpha$	8.0282 10	-0.01 1	0.9973 1	0.99640	0.99667
$-\Omega - \alpha$	1.1791 9	0.33 5	1.0015 8	1.00314	1.00290
$2L' - 2\alpha$	0.0069 5	78.89 4.17	0.8961 649		
$2L - 2\alpha$	0.0067 40	77.78 73.65	1.8611 1.1111		
Periods 1 - 5					
$\Omega - \alpha$	8.0247 14	-0.02 1	0.9969 2		
$-\Omega - \alpha$	1.1818 14	-0.21 7	1.0038 12		

As for the declination corrections, the mean errors at 1950 range from 0<sup>h</sup>:007 for the stars observed throughout the whole period to 0<sup>h</sup>:2 for the stars observed during 1899-1906. The range of the mean errors of the proper motion corrections is from 0<sup>h</sup>:00017 /y to 0<sup>h</sup>:0054/y. It is remarkable that the mean errors for stars observed only for six years are much reduced in the present calculation from those in the past calculation (Manabe et al. 1979) which also uses only six years of data.

## REFERENCES

- Akaike, H. 1973, 2nd International Symposium on Information Theory, eds B. N. Petrov and F. Csáki, 257.
- Kinoshita, H. 1977, *Celest. Mech.*, 19, 215.
- Manabe, S., Sasao, T. and Sakai, S. 1977, *Publ. Int. Latit. Obs. Mizusawa*, 11, 23.
- Manabe, S., Sakai, S. and Sasao, T. 1979, *Publ. Int. Latit. Obs. Mizusawa*, 13, 53.
- Molodensky, M. S. 1961, *Commun. Obs. Roy. Belgique*, 3<sup>e</sup> Série, 10, Fasc. 2.
- Wahr, J. M. 1979, Ph.D. Thesis, Univ. Colorado.
- Yumi, S. and Yokoyama, K. 1980, Results of the International Latitude Service in a Homogeneous System 1899.9 – 1979.0, Mizusawa, Japan.