# 23

# Causal Sets

## 23.1 Introduction

In this chapter we apply the concepts presented in Chapter 22 to a topic that has been of much theoretical interest in recent decades: *causal set theory* (CST).

It is interesting and useful to review the status of CST relative to other foundational topics in mathematical physics that have been and remain of prime significance to empirical physics, namely, special relativity (SR), general relativity (GR), and quantum mechanics (QM). Those great frameworks are built on foundations that presuppose the continuity of space and time.

Despite this, it is an empirical fact that we are surrounded by discreteness in the form of atoms, so it is natural to see how that discreteness fits in with the continuity of space and time. We have attempted in this book to show that the application of discrete principles to QM can go far beyond the basic Planck quantum of action. We have gone as far as discretizing empirical time, in the form of the *stage* concept. However, as far as relativity in either of its manifestations is concerned, the case for any form of discreteness has not yet been established empirically, although great efforts are being made in that direction in the program known as *quantum gravity*. We have particularly in mind the discretization approaches to GR of Regge (Regge, 1961) and Penrose (Penrose, 1971).

Regge's approach is a classically based discretization of the Riemannian manifold concept, so has at best a generalized propositional classification (GPC) of two.[1] Penrose's spin network paradigm started as a quantum description of spacetime and has led to much work in the area of spin networks in the hope of unifying GR and QM. However, workers in spin networks and related branches of quantum gravity have focued their attention on the mathematical

[1] When Regge's approach is used to regularize Feynman path integrals in quantum gravity, its GPC drops to one, as it then becomes an exercise in mathematical physics.

complexities and hardly any on the role of observers and their apparatus, leading to a GPC classification of quantum gravity as one, that is, as mathematical physics and not physics. Claims made that such programs represent empirical physics have not been empirically substantiated to date; quantum gravity at this time remains empirically vacuous, despite the best efforts of many theorists over many decades.[2]

In various attempts to account for the existence of space, time, and matter in the Universe, physicists often adopt one of two opposing viewpoints. These may be labeled *bottom-up* and *top-down*, reflecting the basic difference between reductionist physics and emergent physics. QM makes an appearance in both approaches because while it really models the emergent side of observation, its mathematical structure has a strong reductionist flavor.

A number of bottom-up approaches to cosmology proceed from the assertion that at its most basic level, the Universe can be represented by a vast collection of discrete events embedded in some sort of mathematical space. For example, the *pregeometric* approach asserts that conventional classical time and space and the classical reality that we appear to experience all emerge on macroscopic scales due to the complex connections between certain fundamental microscopic, unobserved and unobservable, pregeometric entities. This approach was pioneered by Wheeler (Wheeler, 1980) and has received attention more recently by Stuckey (Stuckey, 1999).

A bottom-up approach to cosmology that we can relate to in quantized detector networks (QDN) is the *causal set hypothesis*, which asserts that spacetime itself is discrete at the most fundamental level. In CST models it is postulated that classical, discrete events are generated either randomly or through some agency, though neither the nature of these events nor the mechanism generating them is explained or discussed in detail. QDN comes in cost-free at this point. In this chapter we show how the particular mathematical properties and dynamical principles of QDN generates causal set structures naturally, without any extra assumptions.

Although it seems natural to generate causal sets by the discretization of a pseudo-Riemannian spacetime manifold of fixed dimension, as is done in lattice gauge theories and Regge's approach to GR (Regge, 1961), causal set events need not in principle be regarded as embedded in some background spacetime of fixed dimension $d$. One view taken by some theorists is that conventional (i.e., physical) spacetime emerges in some appropriate limit as a consequence of the causal set relations between discrete events. If so, then it is reasonable to expect that in the correct continuum limit, metric structure should emerge naturally (Brightwell and Gregory, 1991). An even more intriguing hypothesis

[2] This is not the same thing as QM over classical curved GR background spacetimes, for which there is some empirical evidence, such gravitational lensing and the need to correct GPS signals because of gravitational curvature and time dilation (Ashby, 2002).

is the suggestion that the dimension of this emergent spacetime might be scale dependent (Bombelli et al., 1987), making the model potentially compatible with GR in four spacetime dimensions, string theory and $p$-brane cosmology, and higher dimensional Kaluza–Klein theories. Such a hypothesis fits in with the general philosophy taken in this book that physics is contextual: different effective theories may explain different experiments.

**The Origin Problem**

One of the problems we have in accepting any pregeometric approach is what we may call the *origin problem*. This is the problem of explaining where any foundational concept comes from. It seems scientifically inconsistent to try to explain empirical physics on the basis of unverifiable hypotheses.

Mathematics came to terms with the origin problem in set theory by admitting that there are concepts, such as those of set, integer, infinity, and infinitesimal, that cannot be explained further and have to be taken as primitive concepts that cannot be derived from deeper principles and have to be treated as axiomatic. We have taken the same approach in our concept of *primary observer*. It is not inconsistent to base our foundations on such a concept, because we ourselves are primary observers: we can point to real, physical examples of the concept.

Our approach to causal set structure then is predicated on the concept of a primary observer. Any causal set structure being discussed will be contextual to the primary observer involved and their apparatus, so we do not conjecture any absolute pregeometric structure.

It turns out that the separation and entanglement properties of quantum register labstates discussed in the previous chapter have enough structure in them to provide the necessary causal set attributes of interest here. A novel feature of our approach is the occurrence of *two* distinct but interleaved causal set structures, in contrast to the one normally postulated in CST. One of these causal sets arises from the entanglement and separation properties of labstates themselves, while the other arises from the split and partition properties of the quantum registers and operators representing modules. In our view, these two distinct causal sets correspond to the two distinct classes of information transmission observed in physics. One of these involves nonlocal quantum correlations in signal labstates and does not respect Einstein locality. The other involves classical information transmission equivalent to local operations carried out by the observer on apparatus, and that does respect Einstein locality.

In the next section we review classical CST. Then we show how the separation and entanglement concepts discussed in the previous chapter lead to a natural definition of the concepts of families, parents, and siblings used in CST. Then we show how two sorts of causal set structure arise in QDN, one of which is associated with labstates and the other with modules.

## 23.2 Causal Sets

A number of authors (Bombelli et al., 1987; Brightwell and Gregory, 1991; Markopoulou, 2000; Requardt, 1999; Ridout and Sorkin, 2000) have discussed the idea that spacetime could be discussed in terms of causal sets. In the causal set paradigm, the universe is envisaged as a set $\mathcal{C} \equiv \{x, y, \ldots\}$ of objects (or *events*) that may have a particular binary relationship among themselves denoted by the symbol $\prec$, which may be taken to be a mathematical representation of a temporal ordering. For any two different elements $x, y$, if neither of the relations $x \prec y$ nor $y \prec x$ holds, then $x$ and $y$ are said to be *relatively spacelike*, *causally independent*, or *incomparable* (Howson, 1972). The objects in $\mathcal{C}$ are usually assumed to be the ultimate description of spacetime, which in the causal set hypothesis is often postulated to be discrete (Ridout and Sorkin, 2000). Minkowski spacetime is an example of a causal set with a continuum of elements (Brightwell and Gregory, 1991), with the possibility of extending the relationship $\prec$ to include the concept of *null* or *lightlike* relationships.

The causal set paradigm supposes that for given elements $x, y, z$ of the causal set $\mathcal{C}$, the following relations hold:

$$\begin{aligned} \forall\, x, y, z \in \mathcal{C}, &\quad x \prec y \text{ and } y \prec z \Rightarrow x \prec z && \text{(transitivity)} \\ \forall x, y \in \mathcal{C}, &\quad x \prec y \Rightarrow y \not\prec x && \text{(asymmetry)} \\ \forall x \in \mathcal{C}, &\quad x \not\prec x. && \text{(irreflexivity)} \end{aligned} \tag{23.1}$$

A causal set may be represented by a Hasse diagram such as shown in Figure 23.1 (Howson, 1972). In a typical Hasse diagram, the events are shown as labeled circles or spots and the relation $\prec$ as a solid line or link between the events, with the "temporal" ordering running from bottom to top. Stage diagrams in QDN are a form of Hasse diagram, with the added complication of including modules.

Hasse diagrams can be discussed in familial terms. For example, in Figure 23.1, event $1_0$ is the *ancestor* of all subsequent events; $2_2$ and $3_2$ are the *parents* of their *child* $2_3$; $3_2$ and $4_2$ are *siblings*; and so on.

One method of generating a causal set is via a process of "sequential growth" (Ridout and Sorkin, 2000). At each step of the growth process a new element is created at random, and the causal set is developed by considering the relations

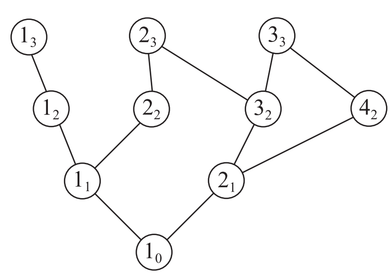


Figure 23.1. A typical Hasse diagram.

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between this new event and those already in existence. Specifically, either the new event $y$ may be related to another event $x$ as $x \prec y$, or $x$ and $y$ are said to be unrelated. Thus the ordering of the events in the causal set is as defined by the symbol $\prec$, and it is by a succession of these orderings, i.e., the growth of the causet, that constitutes the passage of time. The relation $x \prec y$ is hence interpreted as the statement: "$y$ is to the future of $x$." Further, the set of causal sets that may be constructed from a given number of events can be represented by a Hasse diagram of Hasse diagrams (Ridout and Sorkin, 2000).

The importance of CST is that in the large-scale limit of very many events, causal sets may yield all the properties of continuous spacetimes, such as metrics, manifold structure, and even dimensionality, all of which should be determined by the dynamics (Bombelli et al., 1987). For example, it should be possible to use the causal order of the set to determine the topology of the manifold into which the causet is embedded (Brightwell and Gregory, 1991). This is the converse of the usual procedure of using the properties of the manifold and metric to determine the lightcones of the spacetime, from which the causal order may in turn be inferred.

Distance may be introduced into the analysis of causal sets by considering the length of paths between events (Bombelli et al., 1987; Brightwell and Gregory, 1991). A *maximal chain* is a set $\{a_1, a_2, ..., a_n\}$ of elements in a causal set $\mathcal{C}$ such that, for $1 \leq i \leq n$, we have $a_i \prec a_{i+1}$ and there is no other element $b$ in $\mathcal{C}$ such that $a_i \prec b \prec a_{i+1}$. We may define the path length of such a chain as $n - 1$. The distance $d(x, y)$ between comparable (Howson, 1972) elements $x, y$ in $\mathcal{C}$ may then be defined as the maximum length of path between them, i.e., the "longest route" allowed by the topology of the causet to get from $x$ to $y$. This implements Riemann's notion that ultimately, distance is a counting process (Bombelli et al., 1987). For incomparable elements, it should be possible to use the binary relation $\prec$ to provide an analogous definition of distance, in much the same way that light signals may be used in special relativity to determine distances between space-like separated events.

In a similar way, "volume" and "area" in the spacetime may be defined in terms of numbers of events within a specified distance. Likewise, it should be possible to give estimates of dimension in terms of average lengths of path in a given volume. An attractive feature of causal sets is the possibility that different spatial dimensions might emerge on different physical scales (Bombelli et al., 1987), whereas in conventional theory, higher dimensions generally have to be put in by hand.

## 23.3 QDN and Causal Sets

Our approach to causal sets in QDN differs from the above in that we do not assume any structure to the information void, such as spacetime per se.

Our discrete set structure arises from the separation and entanglement properties of labstates and modules.

Furthermore, in the QDN approach, various relations assumed in the "sequential growth" mentioned above must be interpreted carefully. In quantum physics, past, present, and future can never have the status they have in Block Universe models. At best we can only talk about conditional probabilities, such as asking for the probability of a possible future stage *if* we assumed we were in a given present stage. This corresponds directly to the meaning of the Born interpretation of probability in QM, where all probabilities are conditional.

Another point is that the causal set relations discussed by Sorkin and others imply that the various elements $a, b, \ldots, z$ have an independent existence outside the relations themselves and that these relations merely reflect some existing attributes. This is a vacuous Block Universe perspective that QDN does not accept.

A further criticism of classical CST comes from dynamics. In some diagrams, relatively space-like events with no previous causal connection are permitted to be the parents of the same event. The question arises then, given two such unrelated events at a given "time" $n$, how any information from either of them could ever coincide, that is, be brought together to be used to create any mutual descendants. The only sensible answer is that there must be some external agency organizing the flow of that information. But the whole point of standard CST, however, is that there is no external space in which these events are embedded, or any external "memory," observer, or information store correlating such information. It is not clear then how such processes could be encoded into the dynamics. In our QDN approach, however, we have no such issue: the primary observer is such an agency, manipulating apparatus in order to produce such processes.

## 23.4 Quantum Causal Set Rules

Stage diagrams are greatly useful in understanding the architecture of apparatus. Similarly, causal set diagrams are greatly useful in understanding the pattern of causality as labstates "flow" through that apparatus. With reference to Figure 23.2, the conventions in causal set diagrams are as follows.

### Time

Stages go up the page, starting from stage $\Sigma_0$.

### Separations

These are represented by squares surrounding single integers identifying separations. As with detectors on a given stage in a stage diagram, labeling is arbitrary. Our convention then is to label atoms left to right.

### Entanglements

A rank-$r$ entanglement is represented by a left to right rectangle equivalent to $r$ adjacent squares, labeled by the atoms involved in the entanglement.

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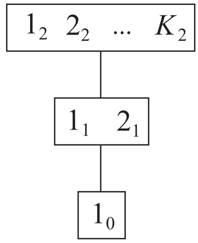


Figure 23.2. The causal set diagram for the DS experiment.

**Lines**
These connect elements of the causal set as required from the modules involved.

## 23.5 Case Study 1: The Double-Slit Experiment

We are now in a position to reexamine the QDN analysis of the double-slit (DS) experiment given in Chapter 10 and analyze its causal set structure.

Looking at Figure 10.2, we would get the impression that it is a Hasse diagram as it stands and so the causal set structure of the experiment would appear to be obvious. This is not what we are after, however, because the causal set structure we are interested in involves labstates and not detectors. These are two separate concepts.

Our point is this. The stage $\Sigma_1$ labstate given in Eq. (10.13) is a signality-one state with two identifiable components $\alpha^1 \widehat{\mathbb{A}}_1^1 \mathbf{0}_1$ and $\alpha^2 \widehat{\mathbb{A}}_1^2 \mathbf{0}_1$ added together, giving the impression that this is not an entangled state. If we return to a more explicit signal basis representation, the entanglement at stage $\Sigma_1$ becomes clear. If instead of (10.13) we write

$$\boldsymbol{\Psi}_1 = \alpha^1 \mathbf{1}_1^1 \mathbf{0}_1^2 + \alpha^2 \mathbf{0}_1^1 \mathbf{1}_1^2, \tag{23.2}$$

then it is clear that $\boldsymbol{\Psi}_1$ is actually an entangled state, an element of the entanglement $\mathcal{Q}_1^{\overline{12}}$, one of the partition elements of the quantum register $\mathcal{Q}_1^{[12]} \equiv Q_1^1 Q_1^2$.

Note that in the above, the superscripts refer to the two slits, labeled 1 and 2.

**Remark 23.1** In standard QM, the state at stage $\Sigma_1$ in the DS experiment would also usually be written in nonentangled form $|\Psi_1\rangle = \alpha^1|\text{slit } 1\rangle + \alpha^2|\text{slit } 2\rangle$, masking the fact that it is actually an entangled state (contextual on the observer's ability to look at both slits simultaneously). This is a far more subtle point than it appears and goes to the heart of the difference between classically conditioned thinking and the sort required to make sense of quantum processes.

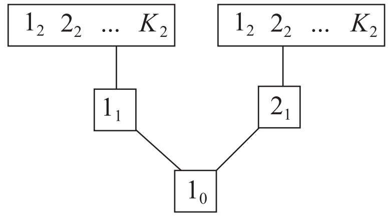


Figure 23.3. The causal set diagram for the monitored DS experiment.

The stage $\Sigma_2$ labstate requires some commentary also, because Figure 10.2 is misleading in the same respect as at stage $\Sigma_1$. The final stage labstate $\mathbf{\Psi}_2$ is an element of the entanglement $\mathcal{Q}_2^{\overline{123...K}}$, where $K$ is the number of detectors in the detecting screen $S$. The final labstate may be written as a sum of computational basis representation states:

$$\mathbf{\Psi}_2 = \sum_{j=1}^{K} \left\{ \sum_{a=1}^{2} \alpha^a U_{2,1}^{j,a} \right\} \underline{\mathbf{2^{j-1}}}_2. \tag{23.3}$$

With this analysis of the entanglement structure of the DS experiment, the appropriate QDN causal set diagram is given by Figure 23.2. Temporally successive stages run bottom to top. Separations are in square boxes, and entanglements are in rectangular boxes, labeled by the atoms involved.

## 23.6 Case Study 2: The Monitored Double-Slit Experiment

The interference pattern observed in the DS experiment disappears when which-path information is available to the observer, that is, when it is possible to identify which path a photon had taken from source to detector. In this situation, known as the monitored double-slit experiment, the labstate at stage $\Sigma_1$ can no longer be considered entangled. In consequence, there is doubling of final screen elements in the relevant causal set diagram, as shown in Figure 23.3. While the two slit elements at stage $\Sigma_1$ are not considered entangled once which-path information is obtained post each run, the labstate for stage $\Sigma_1$ may be considered entangled from the perspective of the observer at stage $\Sigma_0$, that is, *before* any which-path information is obtained at stage $\Sigma_2$. This underlines the fact that such a causal set diagram changes with context.

It should be noted that the two stage-$\Sigma_1$ separations $1_1$ and $2_1$ in the monitored DS experiment are physically disjoint, in that they start outcome branches that do not occur in the same run. They are classically distinct. The stage-$\Sigma_1$ labstate is not the tensor product of elements from these two separations; neither is it a superposition of those two separations.

A significant point to make here is that Figure 23.3 is not a Block Universe diagram.

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## 23.7 Case Study 3: Module Causal Set Structure

We mentioned above that QDN leads naturally to another causal set structure, one associated with the modules in a given experiment. For a given experiment, this causal set structure is associated with the links between real and virtual detector nodes, and on that account will be referred to as the *dual causal set*. It is at this point that our stage diagram convention of representing modules by boxes, rather than the circles associated with the detectors, becomes useful.

Given a conventional stage diagram, such as Figure 13.3, the two-photon interferometer, the first step is to fill in the missing module boxes. These occur when signals propagate through the information void directly. In Figure 23.4 we show how Figure 13.3 is amended: the new modules $V^1$ and $V^2$ represent information void propagation.

The next step is to remove all clear circles (that is, except final stage detectors), join up the module boxes according to how they interact with the real and virtual detectors, and then readjust the resulting diagram into a vertical Hasse form. Figure 23.4 gives Figure 23.5, the module Hasse diagram for the two

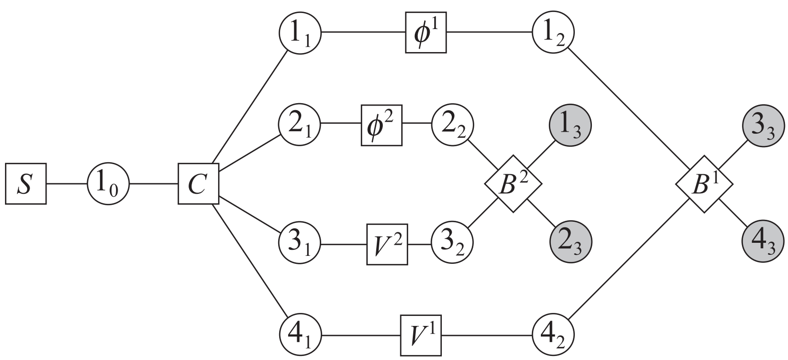


Figure 23.4. The amended stage diagram for the two photon interferometer.

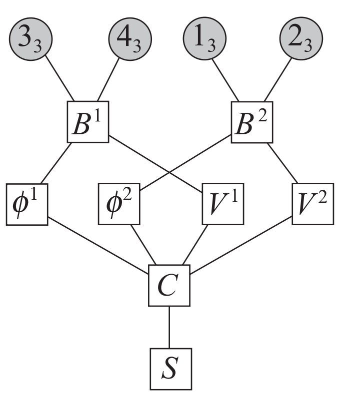


Figure 23.5. Two-photon interferometer module causal set diagram.

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photon interferometer. This casual set diagram is entirely classical: there are no entanglements.

The reader is invited to analyze the other experiments discussed in this book in terms of their labstate and apparatus causal set structures. It will become apparent that there are significant complexities in even relatively simple experiments.