

DIFFUSION OF HEAVY IONS IN CONVECTIVE ENVELOPES,
WITH IMPLICATIONS FOR THE MASSES OF WHITE DWARFS

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Heavy ions rapidly sink out of the base of a white dwarf convective zone, and a balance is set up between accreting interstellar material and this sedimentation flux. The heavy element abundances are lower than they would be in the absence of convection. The accretion process is responsible for any gross departures from the cosmic abundance ratios among the heavy ions. A chemically pure white dwarf must have mass greater than $0.2 M_{\odot}$. The DB stars have masses lower than $0.4 M_{\odot}$.

The atmospheres of white dwarfs have long been known to be chemically very pure (Strittmatter and Wickramasinghe 1971; Weidemann 1975 and references therein). That sedimentation might reduce the heavy element abundances was realized by Schatzman (1958). Sedimentation in convectively unstable envelopes has been discussed recently by Fontaine and Michaud (1979), Alcock and Illarionov (1979a, henceforth AI) and Vauclair, Vauclair and Greenstein (1979). I will use the analysis of AI, because it has the advantage that it yields semi-analytic expressions that make the physics transparent, and will not discuss in detail the approximations made.

The sedimentation velocity of a heavy ion in a stellar envelope is approximately

$$w = \left(\frac{24 A^{1/2}}{Z^2(1+Z)} \right) \left(\frac{gT^{3/2}}{n\Lambda} \right) \quad (1)$$

where (Z,A) are the atomic (charge, mass) of the background plasma, g the gravity, T the temperature, n the number density in the plasma and Λ a coulomb logarithm. Note that this expression, which is probably good to a factor of 2, is independent of the properties of the sinking ion.

An approximate solution to the time-dependent diffusion equation yields the expected surface abundance

$$X(t) = X(0)e^{-t/\tau_0} + \frac{1}{n_0 w_0} \int_0^t \mathcal{F}(t') e^{-(t'-t)/\tau_0} \frac{dt'}{\tau_0} \quad (2)$$

where $X(0)$ is the initial abundance, n_0 and w_0 are evaluated at the base of the convection zone, \mathcal{F} is the accretion flux of the

element in question, t is time, and $\tau_0 = P_0 / (w_0 \rho_0 g)$, where P_0 and ρ_0 are the pressure and density at the base of the convection zone. Estimating τ_0 using the models of Fontaine and Van Horn (1976) gives very short times (0.1 to 10^8 yrs for $0.6 M_\odot$ DA) and leads us to CONCLUSION I: The primordial heavy elements sink out of sight rapidly, and a balance between accretion and sedimentation occurs. The heavy elements observed in DA spectra are accreted material.

The accretion flux is strongly time-dependent. Most of the volume of the galaxy is filled with a tenuous gas, and most white dwarfs we observe are in this tenuous region where the accretion flux is low. However, most of the mass is in clouds, and a typical time between encounters with clouds is $\sim 2 \times 10^6$ yr. If $\langle \mathcal{F} \rangle$ is the mean flux (including clouds) and \mathcal{F} the "present" flux then we can write

$$X = \frac{Z^2(1+Z)}{24 A^{1/2}} \frac{\Lambda}{g T_0^{3/2}} \left\{ \begin{array}{l} \mathcal{F} \text{ if } \tau_0 < 2 \times 10^6 \text{ yr} \\ \langle \mathcal{F} \rangle \text{ if } \tau_0 > 2 \times 10^6 \text{ yr} \end{array} \right. \quad (3)$$

CONCLUSION II: If $\tau_0 < 2 \times 10^6$ yr, the expected abundance is determined by accretion from the tenuous gas, while if $\tau_0 > 2 \times 10^6$ yr the expected abundance is determined by accretion from the clouds. CONCLUSION III: The surface abundances are lower than they would be in the absence of convection, because of the temperature dependence of X in equation (3).

If we examine τ_0 as a function of white dwarf mass, we find that for very low mass objects τ_0 becomes very large, up to 3×10^{10} yrs for an $0.22 M_\odot$ DA. This leads to CONCLUSION IV: White dwarfs with chemically pure photospheres must have mass $\geq 0.2 M_\odot$.

These considerations motivated a detailed study of the accretion of interstellar material onto stars (Alcock and Illarionov 1979b). There is not space to review this here, but the most interesting conclusions were (a) that accretion always occurs, but at rates quite different in general than the classic Bondi and Hoyle (1944) rate, and (b) that substantial chemical fractionation occurs in the accretion process. Equation (3) does not predict relative abundance anomalies amongst the heavy elements, which leads to CONCLUSION V: Non-cosmic relative abundances among the heavy elements in white dwarf photospheres are primarily the result of fractionated accretion.

We are left with the problem (Greenstein 1969) of hydrogen pollution of the DB stars. Michaud and Fontaine (1979) suggested that a helium star could electrostatically prevent protons from accreting. Alcock (1979a) showed that this could only occur if the star had a thermally driven wind. Preliminary work indicates that the DB stars will not have winds (Alcock 1979a). Vauclair, Vauclair and Greenstein (1979) suggested that the hydrogen might reside in a corona above the star; this corona would have an enormous ($\geq 10^{40}$ erg s^{-1}) x-ray luminosity and can be ruled out (Alcock 1979b). However, the mass in

the convection zone of a helium white dwarf increases strongly as the mass of the star decreases (Fontaine and Van Horn 1976) and becomes 10^5 times as massive as the accreted hydrogen if the mass of the star is below $0.4 M_{\odot}$. For a star of $0.3 M_{\odot}$, the sedimentation time is short enough that substantial underabundances of the heavy elements can occur, and the hydrogen "pollution time" is longer than the cooling time for a white dwarf. This leads us to CONCLUSION VI: The helium white dwarfs have mass lower than $0.4 M_{\odot}$. This conclusion has also been reached by Wesemael (1979).

This low mass appears to conflict with recent determinations of the radii of DB and DC stars (Liebert 1976, Shipman 1979). Shipman deduced a mean radius for the DB stars in his sample of $0.0111 R_{\odot}$, which corresponds to a mass slightly greater than $0.65 M_{\odot}$. However, Shipman estimated the error in the color-flux calibration he used to be about 20%, which yields a radius uncertainty $\Delta R/R \approx 0.1$. The corresponding mass uncertainty is $\Delta M/M \approx 0.3$. This means that the low mass for the helium stars required by the absence of hydrogen in their spectra is not grossly inconsistent with the atmospheric analyses. We note that Weidemann (1977) suggested that the DB stars have lower masses than the DA stars.

The gravitational redshifts of spectral lines in white dwarfs have been measured by Trimble and Greenstein (1972). The mean redshift of their sample (10 DB's) is $28 \pm 20 \text{ km s}^{-1}$. The gravitational redshift for a $0.4 M_{\odot}$ white dwarf is $15\text{-}20 \text{ km s}^{-1}$ (depending on the composition of the core). There is no apparent disagreement between the theory and the observations here.

The argument leading to the conclusion that the helium atmosphere degenerates have low mass has some interesting corollaries, with predictions that can be tested by observations. First, since the accretion rate is a strongly decreasing function of velocity, the fraction of degenerate stars that have helium atmospheres should be lower in samples with low space velocities. Second, since convection does not occur in high luminosity degenerates, there should not be any helium atmosphere stars with high luminosity. Third, since stars with DA/B spectra probably are in the process of changing from type DA to type DB or vice versa, the luminosities of stars of type DA/B should be either at the high end or the low end of the observed DB luminosity function.

The above considerations suggest that more work on a number of fronts would be very profitable. On the observational side, the statistics of helium versus hydrogen atmosphere white dwarfs need further examination. Accurate parallaxes for white dwarfs are necessary in the determination of their radii. On the theoretical side, a study of diffusion at high densities is needed. More white dwarf envelope models for helium stars of low mass are needed to improve the limits on the masses of the stars. Further atmospheric analyses of the kind described by Shipman (1979) are also needed.

There are two critical questions regarding the DB stars that remain unanswered. First, how in the evolution of a star is such a low

mass degenerate formed? Second, how does the star lose all of its hydrogen?

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