THE DEFORMATION OF A GLACIER BELOW AN ICE FALL

Cambridge Austerdalsbre Expedition, Paper No. 5

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Abstract. Measurements have been made on Austerdalsbreen, Norway, to test the theory that wave ogives at the foot of an ice fall are formed by pressure. The pattern of deformation over the first three waves from the Odinsbre ice fall was studied by measuring the absolute and relative motions of 35 stakes during August 1956. There are large variations of longitudinal compression through the wave pattern, but no simple correlation exists between the compression and the crests; nor are the crests consistently rising with respect to the average surface. We conclude that the waves are not forming by pressure in this area. A detailed quantitative explanation of the observed pattern of deformation is found by improving the existing theory of glacier flow. The deformations taking place are explained as essentially independent of the presence of the waves; they are primarily due (*a*) to the compression that must occur in the ice as its slope decreases and it becomes thicker, and (*b*) to the bending and unbending to which the ice is subjected as it passes over a bed of changing curvature. Additional contributions to the deformation arise from the widening of the glacier channel, which is important at the immediate foot of the ice fall, and from the annual ablation.

The theory explains quantitatively the rotation and bending of the tunnel excavated in 1955; it was also used to make a prediction, which was successfully verified in a further experiment on the glacier in July and August 1957.

A pressure mechanism of wave formation can be reconciled with the observations if it is supposed that the compression in the lower two-thirds of the ice fall, which was measured photogrammetrically in August 1956, varies significantly with the seasons. However, the positions and amplitudes of the observed waves are fully accounted for by the combined plastic deformation and ablation mechanism recently described in another paper. There is therefore no longer any need for the pressure hypothesis as a primary cause of the waves.

Zusammenfassung. Es wurden am Austerdalsbreen, Norwegen, Messungen gemacht, um die Theorie, dass sich Wellen-Ogiven am Fusse eines Gletscherbruches durch Druck bilden, zu prüfen. Das Deformationsmuster über die ersten drei Wellen vom Odinsbre Gletscherbruch wurde dadurch studiert, dass während des Monats August 1956 der absolute und relative Gang von 35 Pfählen gemessen wurde. Im Wellenmuster sind grosse Abweichungen der Längsverdichtung zu erkennen, aber es besteht keine einfache Beziehung zwischen der Verdichtung und den Kämmen; noch erheben sich die Kämme im Hinblick auf die durchschnittliche Oberfläche regelmässig. Wir ziehen daraus die Folgerung, dass die Wellen sich in diesem Gebiet nicht durch Druck bilden. Eine in Einzelheiten zerlegte quantitative Erklärung des beobachteten Deformationsmusters wird durch Verbesserung der existierenden Theorie vom Gletscherfluss gefunden. Die vorkommenden Deformationen werden als im Grunde genommen unabhängig vom Vorhandensein der Wellen erläutert; sie beruhen in erster Linie auf (*a*) der Verdichtung, die im Eis mit abnehmendem Gefälle vor sich gehen muss, und das Eis wird dicker, und (*b*) dem Biegen und sich wieder gerade richten, dem das Eis unterworfen ist, wie es über ein Bett mit wechselden Krümmungen läuft. Weitere Mitwirkungen an der Deformation ergeben sich aus der Erweiterung des Gletscherkanals, was unmittelbar am Fuss des Gletscherbruches von Bedeutung ist, und aus der jährlichen Ablation.

Durch diese Theorie ergibt sich eine quantitative Erklärung für die Drehung und Biegung des Tunnels, der im Jahre 1955 ausgegraben wurde; sie wurde ausserdem zu einer Voraussage benutzt, die in einem weiteren Experiment am Gletscher im Juli und August 1957 erfolgreich bestätigt wurde.

Ein Druckmechanismus der Wellenbildung kann mit den Beobachtungen in Einklang gebracht werden, wenn angenommen wird, dass die Verdichtung im unteren Zweidrittel des Gletscherbruches, die im August 1956 nach dem Messbildverfahren gemessen wurde, mit den Jahreszeiten wesentlich wechselt. Für Position und Umfang der beobachteten Wellen kann jedoch durch plastische Deformation und Ablationsmechanismus zusammengenommen Rechnung getragen werden, wie es vor kurzem in einem andern Artikel beschrieben worden ist. Es besteht daher für die Druckhypothese als Grundursache der Wellen keine Notwendigkeit mehr.

Fig. 13. View of the stake system from Stake 1. The bottoms of stakes visible in the original are marked with dots. The top of the pipe is seen to the left of Stake 2 and in line with Stake B. To the left is the avalanche fan from Thorsbreen. Note crevasses to the right of the stake line, and figures near Stake BW and to the left of Stake C. (*Photographs by Judith Thomas, 23 August 1956*)

Fig. 14. Looking up the Odinsbre ice fall and showing the upper part of the stake system. Note pipe rig to right of Stake 3. (*Photographs by Judith Thomas, 23 August 1956*)

Fig. 15. View up-glacier from Stake E. Odinsbreen to left, Thorsbreen to right. Note the wave crest through Stake D. (*Photograph by Judith Thomas, 20 August 1956*)

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1. Introduction

It is commonly observed that when a glacier passes down an ice fall there are at the bottom a series of undulations curving transversely across its surface. These wave ogives, as they are sometimes called, were first noticed in 1843 by J. D. Forbes [1] on the upper part of the Mer de Glace, at the foot of the ice fall of the Glacier du Géant. Further downstream the undulations give place to a parallel series of dark bands which are often very conspicuous; they are convex downstream, like the undulations, and give the observer a vivid impression of the differential flow in the ice. The dark bands were seen by Forbes on the Mer de Glace and elsewhere in 1842 and are usually known as Forbes bands, although the nomenclature of the subject is in some confusion. The causes of both sets of features and the nature of the connexion between them have been much debated. Leighton has given a review [2] of the observational evidence and of the theories put forward to account for it, and some later work is described in references 3, 4 and 4a.

The purpose of the Cambridge Austerdalsbre Expedition was to learn as much as possible, by quantitative study, about the motion of a glacier at the foot of an ice fall. The immediate aims were to understand the formation of wave ogives and Forbes bands, and it was also hoped that the investigation would throw light on the problem of erosion at the foot of an ice fall. It had been proposed, originally by Forbes,[1] and later by Streiff-Becker [5] and by Haefeli,[6] and it was widely believed, that wave ogives were essentially pressure waves, caused by the pressure of the ice in the ice fall acting on the ice at the bottom. We were therefore particularly concerned to test this theory by measuring the plastic distortion of the ice which it implies.

Austerdalsbreen is one of the distributary glaciers from Jostedalsbreen, the ice cap in Western Norway (Fig. 1). Two separate ice falls, Odinsbreen and Thorsbreen, drop some 700 m. from the ice cap to a low valley where they join together to form the single glacier known as Austerdalsbreen. This glacier, which is 4 km. long and 0·7 to 1·0 km. wide, exhibits both wave ogives and Forbes bands very well. A good description of the glacier, and probably the earliest, was given by de Seue [16] in 1870 in a book which contains many measurements of glacier movement and which deserves to be better known; Slingsby [17] explored the glacier in 1894 and Evers [18] gives an account of its appearance in 1934. The velocity of the lower part of the glacier was measured photogrammetrically in 1937 by Finsterwalder, Evers and Pillewizer.[19, 20]

The present paper describes one aspect of the work of the Cambridge expedition, but it may be useful to take the opportunity of summarizing the whole venture. In 1954 a party of the Brathay Exploration Group led by A. B. Ware set up permanent painted reference marks and mapped the main features of the glacier. The Cambridge expedition started work in 1955 led by W. V. Lewis. C. Bull and J. R. Hardy carried out a gravity survey,[7] and from it calculated the thickness of the ice—this is believed to be the first recorded use of the method on a valley glacier. The main effort in man-power, however, in 1955 was devoted to the excavation of a tunnel in the ice at the foot of the Odinsbre ice fall, by parties of Cambridge undergraduates and others, led by G. de Boer. The subsequent deformation of the tunnel was measured and interpreted by J. W. Glen.[8] Further survey work and some preliminary velocity measurements were done under the direction of J. E. Jackson, and the writer mapped the wave ogives. There were also parties of the Brathay Exploration Group and from Cambridge at work above the ice fall and below the snout.

In 1956 there were two main projects: (*a*) the insertion of a pipe, by W. H. Ward, to measure glacier flow down to the bed of the glacier; (*b*) a detailed study by myself of the motion and deformation of the wave ogives, which is the subject of the present paper. This last was coupled with mapping of the Forbes bands by parties from the Brathay Exploration Group and from the Perse School, Cambridge. Dr. Glen also measured the rate of slip of the ice past the rock walls.[9] In January 1957 Mr. Ward led a small winter

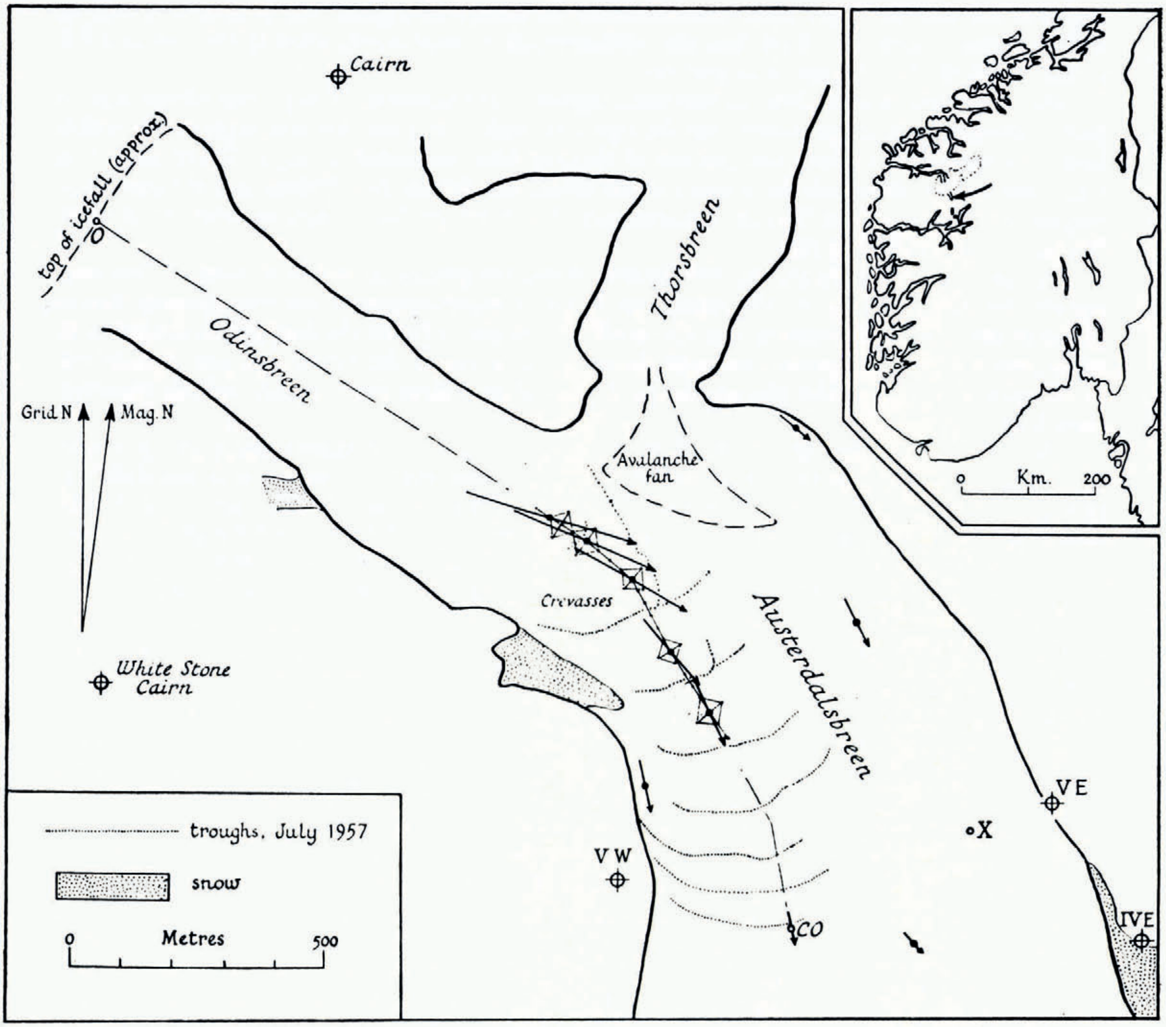

Fig. 1. Map of the upper part of Austerdalsbreen, showing the wave ogives from the foot of the Odinsbre ice fall and the stake pattern of August 1956. The arrows show the velocity at the marked points (August 1956). The velocity is plotted to the same scale as the map. The inset shows the position of Austerdalsbreen on a map of southern Norway

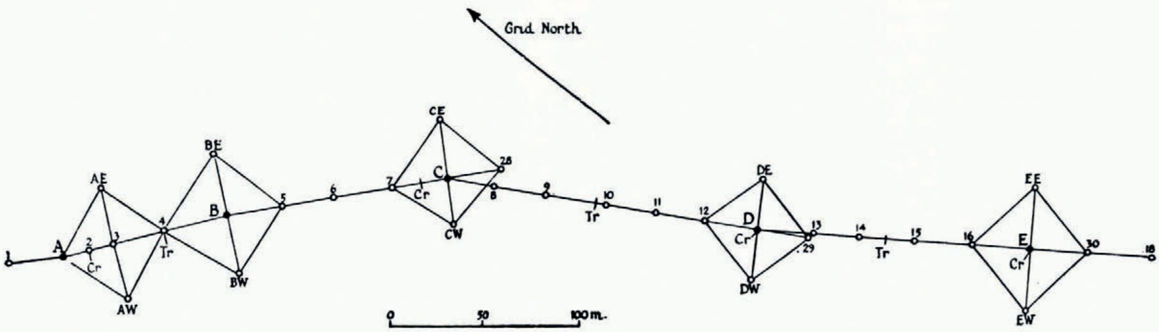

Fig. 2. Plan of the stake pattern of August 1956. The points where the line crossed the crests (Cr) and the troughs (Tr) of the wave ogives are shown

party to the glacier. They were not successful in finding the pipe, which was, and still is, buried under avalanche debris, but they did succeed in resurveying some of the markers left in the waves during the previous summer.

We continued observations on the wave ogives in the summer of 1957; the object was to survey the progress of the waves and the markers, and to test the working hypothesis which had been set up to explain the pattern of deformation measured in 1956. There were two visits: the first in early July by a party of the expedition led by Mr. Ward, and the second in August by a party of the Brathay Exploration Group led by M. F. Robins, who resurveyed the stakes we had left in July.

In 1958 a party again led by Mr. Ward visited the glacier at the beginning of July and resurveyed the wave ogives. We intend to continue observations of the year-to-year movements in the area of the waves, but the work we set out to do at the foot of the ice fall is in other respects now completed. In 1958 the main attention of the expedition was transferred to the middle section of the glacier and to the snout, and these new projects will be reported elsewhere.

This paper contains the results from the study of the wave ogives made in 1956 and 1957, namely, the geometrical features of the waves, their size and shape, and the distribution pattern of velocity, rate of deformation and rate of ablation in the wave area. Other observations on the glacier made in 1956 and 1957, including detailed analysis of the annual movements and observations on the Forbes bands, will be published separately. The leading results of the 1956 work, but not those of 1957, have already been briefly reported.[10]

The work on Austerdalsbreen in 1956 prompted G. R. Elliston, who took part in it, to undertake an identical series of measurements in 1957 on Svínafellsjökull with the Cambridge University South-East Iceland Expedition (leader: R. E. Lawrence). The results (private communication) are in many respects strikingly similar to those found on Austerdalsbreen.

The outcome of the investigation of the wave ogives on Austerdalsbreen was unexpected. We set out to find plastic distortions which should represent the process of wave formation by pressure, since the pressure theory seemed at that time to be the most plausible explanation of the waves. Instead we found that the pattern of deformation occurring over the wave area had no obvious connexion with the waves themselves. It can, in fact, be explained as an effect which is independent of the presence of the waves—being due, as it appears, to the distortion which must necessarily take place in the ice as it meets slight changes in the gradient and curvature of its bed. The theory of this effect, which is discussed in Section 5, is of general application to any valley glacier and gives essentially a correction term to the existing flow equations—a correction which becomes important in regions of rapidly changing bed curvature such as we are concerned with here.

At this point we were faced with the following situation: the plastic deformation in the wave system had been measured and appeared to show that the waves were not forming in that area by pressure in August 1956 or between July and August 1957. The present paper leaves the argument there. For completeness, however, we may mention the next development, which is given in full in reference 11.

In view of the lack of connexion between the measured deformations in the wave area and the waves themselves, it must be concluded that, if a pressure mechanism of wave production does operate on this glacier, it must do so at some other season of the year—or higher up the ice fall—for we could not rule out these possibilities directly from our observations. At the same time, the evidence that a pressure mechanism was not operating over the wave area in summer led to a search for some other process which might be producing the waves. Such a process was found and is very simple. All elements of ice are stretched out longitudinally as they pass down the ice fall, owing to the high local velocity, and they therefore present greater surface area. Those passing through the ice fall in the summer therefore lose more ice by ablation than those which spend the summer in regions of lower velocity. Waves are thus produced by a combination of ablation and plastic deformation—by the increased volume of ablation made possible by the plastic stretching in the ice fall. When the process is examined analytically,[11] it is found that the theory predicts correctly both the positions and amplitudes of the observed waves on Austerdalsbreen. That is not necessarily to say that a pressure mechanism does not operate on other glaciers. It does show, however, that the effect on Austerdalsbreen is fully accounted for, within the uncertainty of present data, without invoking the additional hypothesis of a pressure mechanism. The formation of waves by ablation in this way is something that must occur, it would seem, in any ice fall below the firn line, and there may be a related process in an ice fall above the firn line, although this case is more difficult to analyse.

2. The Stake System and the Measurements

(*i*) *The Stake System.* Fig. 1 shows the Odinsbre and Thorsbre ice falls. Wave ogives emanate from the bottom of both ice falls, but the situation on the Thorsbre side of the glacier is complicated by the presence of the avalanche fan shown in the figure. We therefore chose the comparatively straight Odinsbre ice fall and its associated waves for our study. Some seven or eight crests and troughs can be distinguished before the waves die away; the positions of the troughs on 3 July 1957 are shown in the figure. The maximum amplitude (one half of the total swing) is 6 m. and the wave length decreases from 240 m. to 75 m. as one goes down the glacier.* To a good approximation the waves are spaced at annual distances apart (the spacing may differ from an annual movement by 5 or 10 per cent, but discussion of this possible difference is being reserved for the time being).

To measure the plastic distortion occurring in the wave system, we set up, at the end of July 1956, the pattern of stakes shown in Figs. 1 and 2, starting in the lower part of the ice fall and extending through the first three waves, a total distance of 623 m. Figs. 13, 14 and 15 (p. 386) give some idea of the terrain covered. The pattern consisted of five key stakes lettered A, B, C, D, E, which were to be theodolite stations, and a series of intermediate numbered stakes arranged to form a single line which changed direction at A, B, C and D. This will be referred to as the "main line". The intervals between successive stakes were about 30 m. Near the upper end the main line was arranged to pass accurately (± 1 m.) through the position occupied in 1955 by the mouth of the tunnel. The course of the line was dictated to some extent by the need to avoid a very heavily crevassed area on its western side. In addition to the main line it was desired to have four stakes at the corners of a square centred on each of the key stakes, and the other stakes shown in Fig. 2 were inserted for this purpose (for practical reasons the topmost square was centred on Stake 3 rather than Stake A). There was a further single stake, lettered CO, at the point where the waves died out.

We used square stakes in round holes—the stakes, which were of 2·5 cm. (1 inch) square section, 3·05 m. (10 feet) long and made of hardwood, being set in the holes made by 3·2 cm. (1¼ inch) diameter drills. This technique, developed by Ward,[12] ensured that the stakes were firmly gripped by the ice and did not float.

(*ii*) *Theodolite Resections.* Fixed marks on the rock walls were surveyed by a combination of theodolite triangulation and resection. The positions of Stakes A, B, C, D, E, horizontally and vertically, were measured by theodolite resection to the marks on three occasions: between 28 and 30 July, between 11 and 14 August, and between 21 and 24 August.

(*iii*) *Taping.* Each of the stake intervals shown in Fig. 2 by a full line was measured by steel tape on four occasions: 30 July, 12, 21 and 27 August (except for five of the transverse intervals which were measured on three occasions only). Normally the measurement was

* It was unfortunately necessary to make length measurements in feet and inches. The calculations have been carried through in decimals of a foot and the final figures have been converted to metres.

made between corresponding points of the tops of the two stakes (e.g. the top N.E. corners of both stakes), with the tape freely suspended between the measuring points at a tension judged by the observer to be constant; sometimes, however, the ice surface was such that the tape had to touch it. If the top of a stake could not be reached easily, the measurement was made to a mark 30·5 cm. (1 foot) or 61 cm. (2 feet) from the top. Each measurement was normally made three times.

(*iv*) *Levelling.* The difference of height between the tops of each successive pair of stakes was measured by level and staff on three occasions: between 31 July and 4 August, between 13 and 17 August, and between 22 and 23 August.

(*v*) *Tilt of Stakes.* The stakes were bored in as near the vertical as possible. The angle to the vertical and the direction of tilt were measured on 30 July, 12 August and 21 August by using a simple plumb-line and a foot rule.

(*vi*) *Ablation and Reboring.* The height of the top of each stake above the ice surface was measured three or more times over the period 24 July to 27 August. The melting of the ice made it necessary to rebore the holes of all the stakes at least once during this period, and some were rebored twice. The height of the top of the stake was measured before and after each reboring.

(*vii*) *Offset Measurements.* The extent to which the stakes of any one leg of the system were out of line was measured three times over the whole period; the measurements are available but have not been used.

3. Results

The next step is to find out from the above measurements as much as possible about the absolute and relative motion of the ice at each stake. For the sake of precision the calculation of movement at each stake has been made to refer to that element of ice which was situated at the bottom end of the stake when it was first inserted. We shall call this element of ice the "reference element".

(*i*) *Longitudinal and Transverse Strain-rates.* The fact that the stakes were not exactly vertical and that their holes had to be rebored means that the distances measured between the tops of the stakes are not quite the same as the distances between the reference elements. The necessary corrections, which could all be calculated with the data available, involve the following: the tilts of the stakes, the amount of the reboring, the angle between the line of measurement and the horizontal, and an allowance for the occasions when the measurement could not be made to the top of the stake. The corrections were typically about 0·15 m. (the maximum correction of 0·48 m. arose in a case where the holes of the two stakes had been rebored very unequally, and where the line of measurement ran at 26 degrees to the horizontal). The distances between reference elements having thus been found, the average rate of change of each distance was calculated over the period, and hence the strain-rate $\dot{\epsilon}$ from the formula

$$\dot{\epsilon} = \frac{1}{\bar{l}}\frac{\Delta l}{\Delta t}, \qquad (1)$$

where Δl is the increase in the distance between the two stakes in the time interval Δt, and $\bar{l}$ is the average distance over this time. The strains being small (1) was a sufficient approximation to the true equation for the average strain-rate over the period:

$$\dot{\epsilon} = \frac{1}{\Delta t} \ln \frac{l_2}{l_1},$$

where l_1 and l_2 are the initial and final distances.

For the stake intervals on the main line all observations were given equal weight in the calculations, except for a few rejected in the field. The measurements on the stake intervals transverse to the main line were slightly more erratic and were therefore treated graphically;

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it was noticed that when a down-glacier wind had been recorded, which blew the tape sideways, the readings were abnormally high.

Let x denote the distance down the glacier. For definiteness, x is measured along the line defined by the reference elements on 12 August, with the origin taken at the top of the ice fall, to be consistent with the notation of reference 11. Let z denote the transverse coordinate, Oz being horizontal and perpendicular to Ox. Oy is perpendicular to Ox and Oz and hence is roughly normal to the glacier surface. The strains measured along the main line then give the longitudinal component of strain-rate $\dot{\epsilon}_x$. In Fig. 3 $\dot{\epsilon}_x$ is plotted against x (open circles). The best guide to the accuracy of the results is the degree of internal consistency. At the right-hand end of the graph, where $\dot{\epsilon}_x$ is fairly uniform, the points lie within 0·01 yr.$^{-1}$ of a

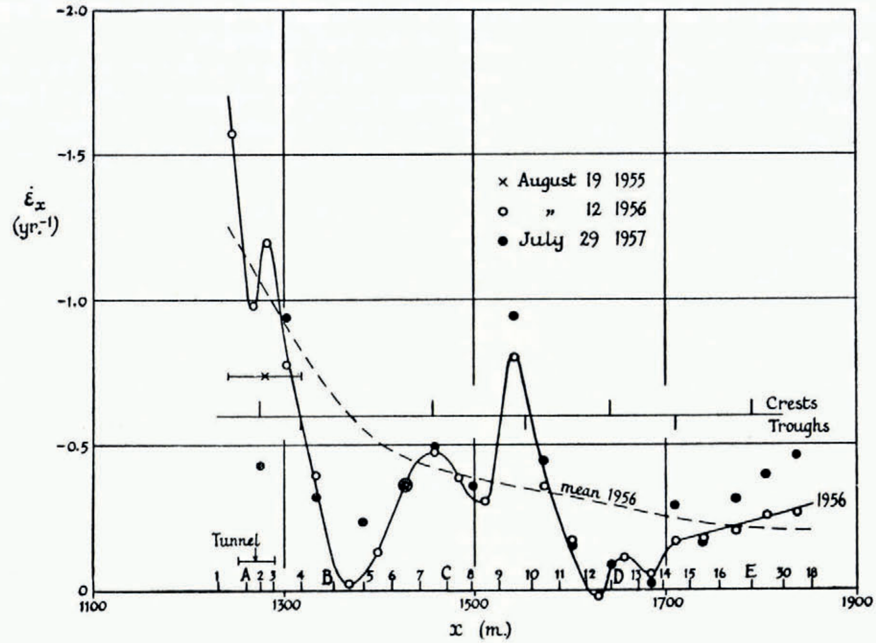

Fig. 3. The longitudinal strain-rate, $\dot{\epsilon}_x$, as a function of x, the distance down the glacier. The stake positions and crest-trough positions of 12 August 1956 are indicated. The position shown for the tunnel is that occupied on 27 August 1955, this being the mean date of the tunnel survey

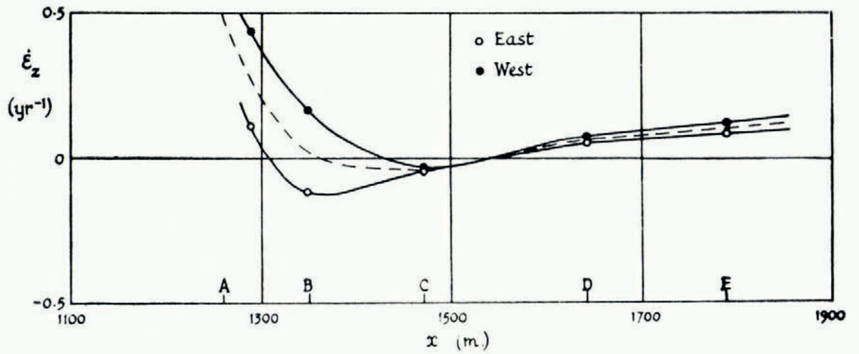

Fig. 4. The transverse strain-rate $\dot{\epsilon}_z$ as a function of x. The broken curve gives $\dot{\epsilon}_z$ along the main line

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smooth curve between the points. The most reliable checks on consistency come from the similar measurements made on the squares of stakes.[13] From such considerations it is concluded that the accuracy achieved in strain-rate is about $\pm 0 \cdot 01$ yr.$^{-1}$, and the curve labelled "1956" in Fig. 3 has been drawn on this basis.

The strains measured on the stake intervals perpendicular to the main line (3—AE, 3—AW, B—BE, B—BW, etc.) give the transverse component of strain-rate $\dot{\epsilon}_z$. In Fig. 4, where $\dot{\epsilon}_z$ is plotted against x, there are two sets of points, for the intervals on the "east" side of the line and the "west" side respectively. It will be seen that the strain-rates are consistently more tensile on the "west" side than on the "east" side, and it is no doubt significant that the stake line was close to the boundary of the severely crevassed area which lay on the "west" side. The broken line in Fig. 4 is the average of the two full lines and represents the strain-rate $\dot{\epsilon}_z$ along the main line. The points are widely spaced and there may, of course, be oscillations in the curve that we have missed (see the end of Section 5).

The strains measured on the stake intervals at 45 degrees to the main line, shown in Fig. 2, enable the shear strain-rate component $\dot{\epsilon}_{xz}$ to be calculated. The calculation of $\dot{\epsilon}_{xz}$, the interpretation of the results from the five squares of stakes, and the detailed connexion between the strains and the crevasses are reported in a separate paper,[13] since they are not immediately relevant to the present analysis.

(*ii*) *Vertical Velocities*. The theodolite resections of Stakes A, B, C, D, E, CO gave the vertical component of the movement of these stakes. The relative vertical velocity of each pair of neighbouring stakes was also known from the levelling surveys. The absolute vertical velocity V of each stake of the main line could therefore be calculated; the values are shown in Table I and are plotted in Fig. 5.

The errors in the theodolite observations appeared greater than those from the levelling. Consequently the figures quoted for V agree exactly with the relative velocities obtained by levelling, and the zero for velocity is fixed by using an averaging process on all the theodolite

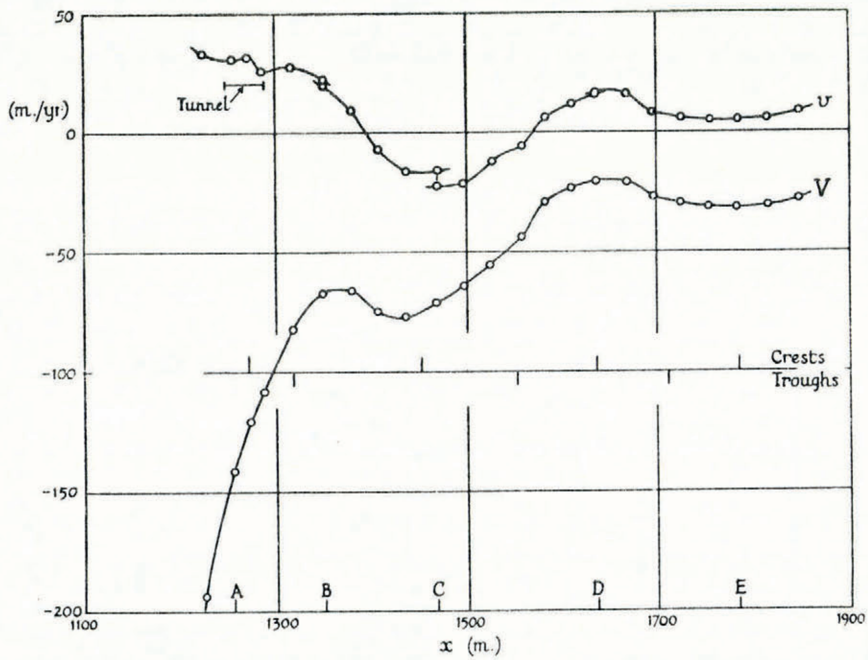

Fig. 5. The vertical velocity component, V, and the velocity component normal to the surface, v, as functions of x

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TABLE I. VELOCITIES OF THE MAIN LINE OF STAKES

Stake	x m.	θ degrees	θ_s degrees	V m./yr.	Horizontal velocity m./yr.	Grid bearing degrees	β degrees	U m./yr.	θ^* degrees
1	1228			−194·3				362·0	28·28
		27·3	27·1						
A	1259			−142·0	346·0	107·2	−15·5	334·0	23·05
		22·3	22·4						
2	1273			−121·2				327·2	20·37
		34·3	34·2						
3	1288			−108·8				314·8	19·08
		23·8	24·0						
4	1317			−82·6				301·5	15·32
		15·2	15·5						
B	1350			−67·3	296·0	113·9	−8·8	292·0	13·00
							−16·4	283·5	13·38
		7·8	7·9						
5	1381			−66·5				282·5	13·25
		13·6	13·0						
6	1410			−75·3				276·5	15·23
		11·9	12·2						
7	1440			−77·5				264·6	16·30
		12·5	12·3						
C	1470			−71·6	256·2	119·8	−10·6	251·5	15·90
							−30·2	221·4	17·95
		14·1	14·1						
8	1498			−64·6				212·4	16·92
		19·5	19·8						
9	1526			−55·8				206·4	15·13
		19·9	19·6						
10	1558			−44·2				182·5	13·60
		1·2	1·8						
11	1586			−29·6				173·3	9·68
		3·3	1·3						
12	1613			−23·5				168·8	7·92
		3·4	6·3						
D	1640			−20·7	173·8	138·9	−12·0	169·8	6·97
							−7·8	172·2	6·87
		16·1	15·1						
13	1671			−21·0				168·5	7·12
		25·9	25·5						
14	1696			−27·4				162·7	9·57
		12·8	13·1						
15	1725			−29·9				157·3	10·75
		7·8	7·7						
16	1754			−31·4				151·8	11·70
		5·6	5·9						
E	1787			−31·7	147·0	155·8	+9·0	145·1	12·33
		13·8	13·2						
30	1819			−30·5				136·9	12·55
		16·3	16·6						
18	1851			−28·0				129·0	12·27
CO	2252	—	—	−0·3	71·3	169·2	—	—	—

x = distance down the glacier from the top of the ice fall.
θ = dip of the line joining the reference elements.
θ_s = component along the stake line of the slope of the ice surface.
V = vertical component of velocity at each stake.
β = angle in the horizontal plane between the velocity of a stake and the line of the stakes.
U = horizontal component of velocity projected on to the vertical planes p defined by the legs of the stake system.
θ^* = dip of the streamline in the plane p.

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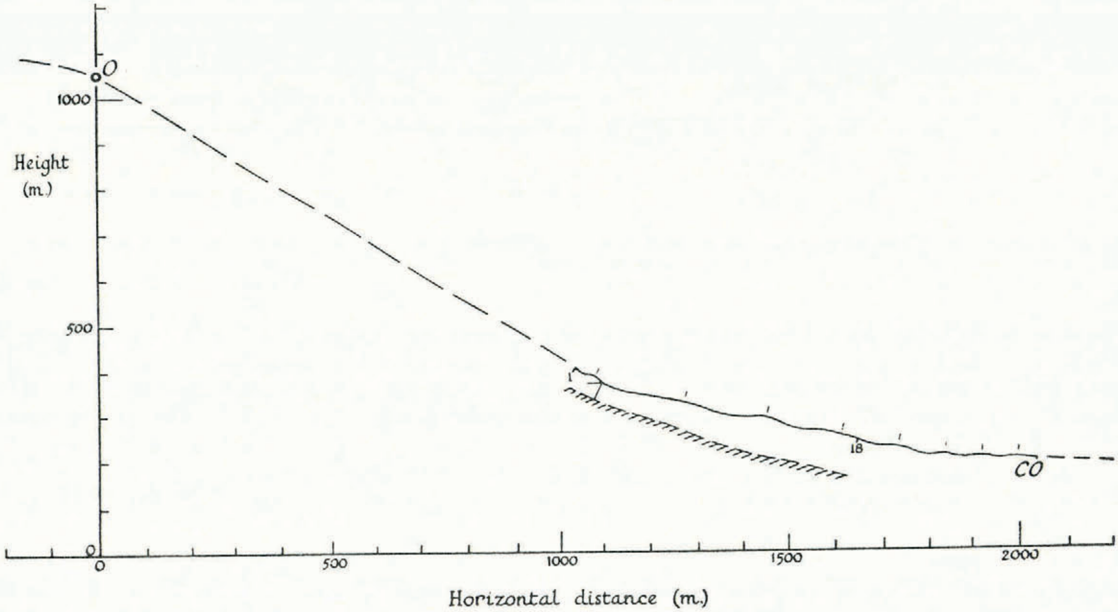

Fig. 6a. Profile (August 1956) down the line O-CO shown in Fig. 1. The bed is calculated from the surface velocities and the measured depth at the pipe (1956). The position of the tunnel on 27 August 1955 is also shown. The zero on the height scale is chosen arbitrarily

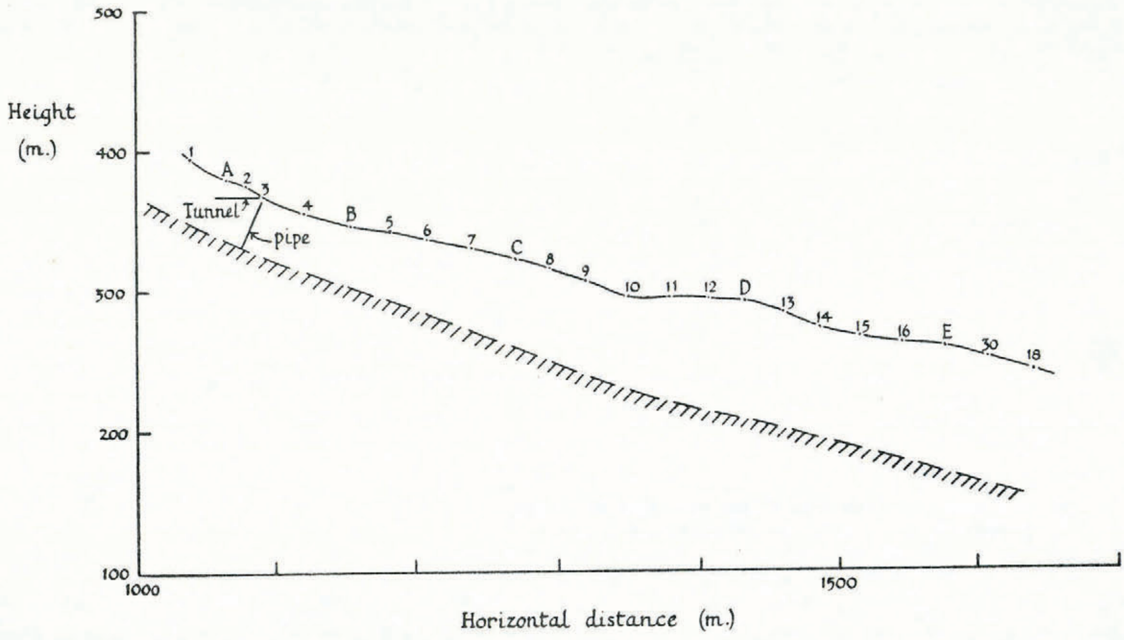

Fig. 6b. The part of the profile of Fig. 6a containing the main line of stakes, 1 to 18, on a larger scale

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measurements. The average adjustment necessary to the velocity V measured on any one stake by theodolite was 4·3 m./yr., and it is therefore thought that V has been measured to about ±2 m./yr. The relative values of V for neighbouring stakes, on the other hand, are thought to be correct, in most cases but not in all, to ±0·5 m./yr.

We may mention that the relative vertical velocity of successive stakes changed slightly over the period of measurement in a fairly consistent way: in 13 cases out of 17 the changes were in the directions that would be expected if the stakes were moving through a fixed velocity field.

(*iii*) *Long Profile.* Fig. 6a shows the longitudinal profile of the glacier from the approximate top of the ice fall O to CO. In the part from Stake 1 to CO, which was measured by levelling, the waves are clearly seen and the positions of eight crests are indicated. The section covered by the main line of stakes is also shown separately in Fig. 6b. Crests are seen near C, near D and near E and a less prominent crest appears just below A.

Table I gives the values of θ and θ_s calculated from the levelling measurements. θ for each stake interval is the angle between the horizontal and the line joining the reference elements on 13-17 August, and is positive when the slope is downhill in the downstream direction. θ_s is the same as θ except that it refers to the ice surface at each stake rather than to the reference element. θ_s is plotted against x in Fig. 7 (p. 399).

The exact positions of the crests and troughs can be defined in various ways. The positions shown in the figures are, in most cases, those which show the greatest departure from an

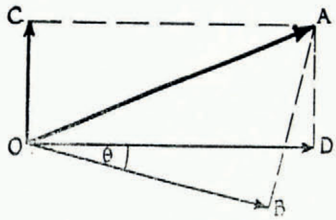

Fig. 8. For a given stake interval the diagram shows the velocity component OA, in the plane p, of one stake relative to the other. The vector OB is calculated from measurements by tape and the vector OC from measurements by level. Hence the relative horizontal and vertical components OD and OC are known

estimated average surface. A less subjective criterion, not entirely free from objection, is to take the crests and troughs as the points of inflexion of the curve of $\theta_s : x$—that is, the places of greatest curvature of the surface. It is then clear from the θ_s curve that the crest near Stake C seen on the profile has a fine structure, and is made up of two crests with a very small intervening trough. This fine structure is indicated in Figs. 7 and 9, where it seems relevant, but is omitted in the other figures.

(*iv*) *Horizontal Velocities.* The theodolite resections of Stakes A, B, C, D, E, CO also gave the horizontal components of the movements of these stakes (Fig. 1). The results from the taping of the main line do not, however, give directly the relative horizontal movements of all the stakes because (*a*) the taped distances are not horizontal, and (*b*) the movements, as Fig. 1 shows, are not exactly along the line of the stake system. We have adopted the following procedure. Each leg of the main line defines a vertical plane p. Let U denote the horizontal component of the velocity of a reference element projected on to this vertical plane. Then by combining the results from the taping and the levelling, as indicated in Fig. 8, we have calculated the relative values of U for the stakes along each leg of the main line. These relative values were used to check the theodolite observations. It again appeared that most of the error was in the theodolite observations, and so a correction to the absolute velocities was made on that basis. The values of U in Table I agree exactly with the relative values obtained from the taping and levelling. The average adjustment necessary to the

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value of U measured on any one stake by theodolite was 20 m./yr., and it is therefore thought that U has been measured to an accuracy of about ± 10 m./yr. The relative values of U for neighbouring stakes, on the other hand, are probably correct to $\pm 0 \cdot 5$ m./yr. or better.

Table I also shows the horizontal velocity (not projected on to the plane p), the angle β which this velocity makes with the plane p, and its grid bearing (the local grid north used was oriented roughly to geographic north). It will be noticed that where the line changes direction there are two values of U and β, one corresponding to each of the two possible planes p.

The values of θ^* shown in Table I are defined by

$$\tan \theta^* = -\frac{V}{U}.$$

θ^* thus gives the downward inclination, in the plane p, of the velocity vector at each reference element. It is double-valued where there are two possible planes p. θ^* is plotted against x in Fig. 7.

(v) *Ablation.* The mean rate of ablation over the period 30 July to 20 August is plotted against x in Fig. 9. Some results for other periods, notably 30 July to 7 August (faster ablation) and 7 August to 20 August (slower ablation) are also available; they are not shown but have been used to help in drawing the curve, maxima and minima only being inserted if they appear in at least two different periods of observation. It will be seen that there is a close correlation with the positions of the crests and troughs, the ablation rate at the crests being some 30 per cent higher than the rate at the troughs. One also notices that the fine structure in the crest near Stake C, small though it is, is faithfully reflected in the ablation curve.

4. Examination of the Pressure Wave Hypothesis

Fig. 3 shows that there is a longitudinal compression of between 0 and $1 \cdot 5$ yr.$^{-1}$ over practically the entire line of stakes, and that it oscillates considerably from point to point. It is perhaps especially remarkable that $\dot{\epsilon}_x$ changes from the very high value of $-1 \cdot 57$ yr.$^{-1}$ at the topmost stake interval to $-0 \cdot 03$ yr.$^{-1}$ between Stakes B and 5 only 122 m. lower down the glacier; there is a similar high gradient between Stakes 9 and D. One's first thought is to try to correlate the oscillations with the positions of the crests and troughs. If the waves are forming by plastic deformation one would expect to find high compression at the crests and less compression, or tension, at the troughs.

The oscillations in $\dot{\epsilon}_x$ have about the same frequency as the crests and troughs. The rate of compression is high at the composite crest near C and at the crest near E. On the other hand, the smallest compression of all, in fact a tensile strain, is measured close to the crest near D. One must conclude that if a correlation exists it is not an entirely simple one.

But there is another test of the hypothesis that the waves are forming by plastic deformation. If this were so, the crests would be rising and the troughs falling relative to their surroundings. Some test of this is given by the curve $V : x$ in Fig. 5, which came from the levelling results and is thus independent of the curve of $\dot{\epsilon}_x$ from the taping. Again, there are oscillations of the curve, this time at about one-half the frequency of the surface oscillations, but no simple correlation. In particular, the crest near D is rising, while those near C and E are relatively falling. Thus high compression at a crest (C and E) tends to be linked with a falling surface and low compression at a crest (D) is linked with a rising surface. This behaviour—seemingly at variance with the pressure hypothesis—and also the finer details of the curves, were very puzzling.

There is another way of examining the rate of rise and fall of the waves. Since the surface is inclined to the horizontal at angles up to 30 degrees one might think of trying a correlation with the velocity component, v, say, perpendicular to the average surface, rather than with the component V, which is vertical. There is, however, a serious difficulty—namely, to

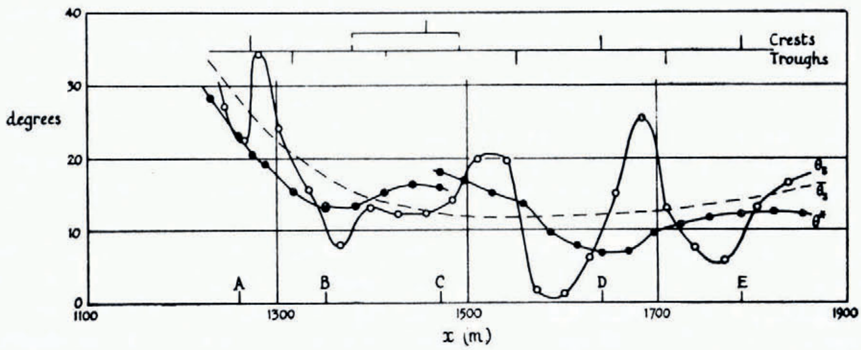

Fig. 7. The surface slope θ_s, the angle of dip θ^* of the streamline at the surface, and the "average" surface slope $\bar{\theta}_s$, as functions of x

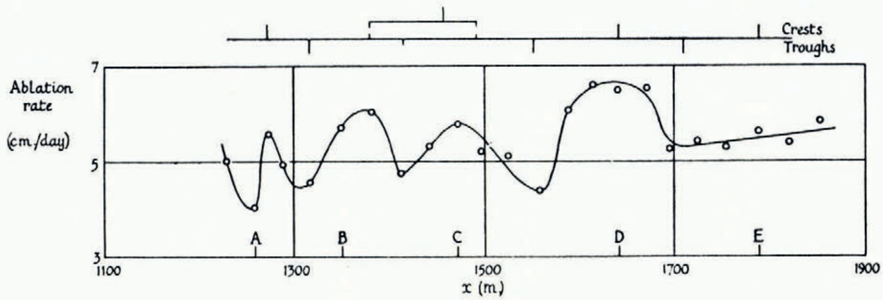

Fig. 9. The average rate of ablation between 30 July and 20 August 1956 as a function of x

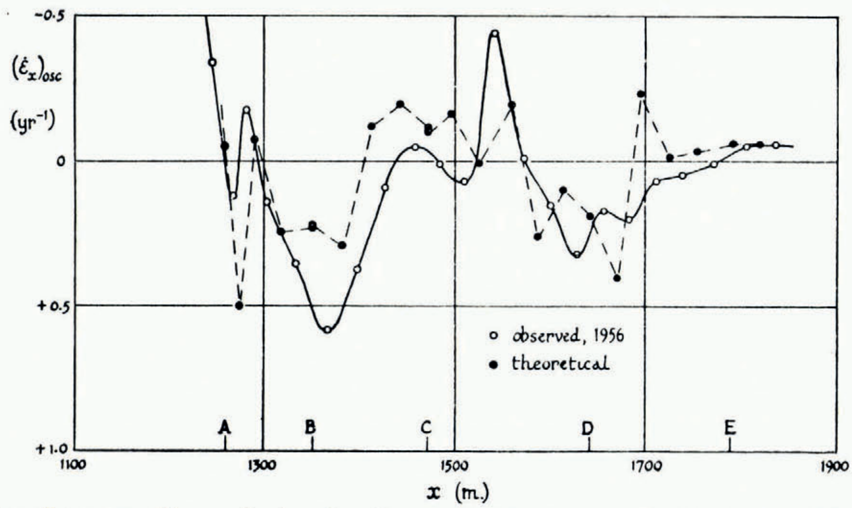

Fig. 12. The oscillatory part of $\dot{\epsilon}_x$ as a function of x. The measured values are compared with those deduced from a simple bending theory (see p. 403)

define the direction of the "average surface" without prejudicing the issue. The difficulty arises not merely because of the relief of the surface, but because of the very high value of the forward velocity of the ice, which is about 10 times the value of v. This means that the value of v obtained depends considerably on one's judgement of what is properly to be taken as the slope of the average surface. It must be emphasized, therefore, that conclusions about rise and fall relative to an average surface are reached with this important reservation.

By taking the curve shown in Fig. 7 as the slope $\overline{\theta_s}$ of the average surface the curve of $v : x$ in Fig. 5 results. v is defined as the velocity component in the plane p, and is therefore double-valued where there are two possible planes p. It will be seen that the curve leads to the same conclusions about the rise and fall of the crests near C, D and E as were reached from the curve of $V : x$. The same conclusions are also reached by comparing the curve of $\theta^* : x$ with the curve of $\overline{\theta_s} : x$.

We have now arrived at the following position. Three correlations were expected if the waves were forming by longitudinal pressure: (1) between the longitudinal strain-rate $\dot{\epsilon}_x$ and the wave pattern, (2) between the normal velocity component v and the wave pattern, (3) between $\dot{\epsilon}_x$ and v. None of these correlations was clearly shown in the measurements (although with (2) and (3) we must bear in mind the difficulty of defining v). We can only conclude, then, that the waves in the area under observation were not forming by plastic deformation while we were there in August 1956. (The possibility of wave formation in the area above the beginning of the stake line is discussed later, in Section 9.)

Nevertheless, the observed distribution of velocities, both vertical and horizontal, and especially the very considerable oscillations in $\dot{\epsilon}_x$ over the wave area demand an explanation. The explanation proposed is given in the next section.

5. Theory of the Strain-Rate Distribution

In previous papers the writer has formulated a theory of glacier flow which leads to the following equation for $\dot{\epsilon}_x$ in a parallel-sided glacier (reference 14 and equation (36) of reference 15):

$$\dot{\epsilon}_x = \frac{\phi'}{h} + u\kappa \cot \alpha. \qquad (2)$$

ϕ' is the rate of accumulation (negative if ablation), h the thickness (shortest distance from bed to surface), u the forward velocity of the ice averaged over the thickness, κ the curvature of the bed (positive when convex), and α the slope of the surface. $\dot{\epsilon}_x$ in this approximation is uniform with depth. Equation (2) is derived from a model in which the ice is supposed to behave as an ideally plastic material with a constant maximum shear stress. In this approximation the thickness is determined by the slope. On a given slope, if ice is being added at the upper surface, a longitudinal strain-rate ϕ'/h is necessary to preserve the existing thickness. On the other hand, when the slope changes the thickness has to change, and this is accomplished by a longitudinal strain-rate $u\kappa \cot \alpha$. The essential condition in the model which leads to equation (2) is that the shear stress on the bed of the glacier is constant. The fact that this condition is roughly fulfilled in real glaciers suggests that we are justified in using equation (2) as a rough guide to the value of $\dot{\epsilon}_x$. But there is an important proviso in the theoretical derivation: the slope and curvature must change relatively slowly with x—that is, $d\alpha/dx$ and $d\kappa/dx$ must be sufficiently small. At the foot of the Odinsbre ice fall this condition is not fulfilled, as we shall see, and it would be unreasonable to attempt to apply equation (2) as it stands. We must therefore reconsider the factors which determine $\dot{\epsilon}_x$.

Fig. 10 shows a longitudinal section through a parallel-sided glacier of thickness h. Suppose the bed at P has curvature κ and at Q, a distance dx from P, the curvature has changed to $\kappa + d\kappa$. The shaded element of glacier at P, if it is to remain conformable to the bed, must therefore bend about a horizontal axis as it moves from P to Q. For a first approximation we shall assume simple bending, but we do not know *a priori* which will be the neutral plane. If the origin is taken on the neutral plane the strain increment of the fibre y=constant will be

$$d\epsilon_x = y\, d\kappa. \qquad (3)$$

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Let the forward velocity of the glacier be u, so that $dx = u\,dt$, and divide both sides of (3) by dt. Then

$$\dot{\epsilon}_x = yu\frac{d\kappa}{dx}. \tag{4}$$

If the neutral plane is half-way through the thickness, the strain-rate at the upper surface ($y = \frac{1}{2}h$) is then

$$\dot{\epsilon}_x = \tfrac{1}{2}hu\frac{d\kappa}{dx}. \tag{5}$$

We note, first, that this bending strain-rate is proportional not to the curvature of the bed but to its rate of change $d\kappa/dx$, and, second, that it changes sign through the thickness of the glacier. In both these respects it differs from the strain-rate given by equation (2), which is proportional to κ and is uniform with depth. The bending strain does not appear in the theory which leads to equation (2) because it is assumed there that $|d\kappa/dx|$ is much less than $|\kappa/h|$, and therefore that the bending strain-rate is negligibly small compared with the strain-rate given by the term $u\kappa \cot \alpha$. A rough calculation shows that at the foot of the Odinsbre ice fall this is not the case, for $|d\kappa/dx|$ and $|\kappa/h|$ are of comparable magnitude.

In the absence of a comprehensive theory it seems plausible to take the following view. Equation (2) cannot take account of rapid fluctuations in κ, and is therefore only to be applied to deduce average values of $\dot{\epsilon}_x$ over large areas. On the other hand, equation (5) can be used as a first approximation to deduce the additional strains resulting from $d\kappa/dx$. On this understanding we therefore write for $\dot{\epsilon}_x$ at the upper surface the composite equation

$$\dot{\epsilon}_x = \frac{\phi'}{h} + u\kappa \cot \alpha + \tfrac{1}{2}hu\frac{d\kappa}{dx}. \tag{6}$$

In a full theory there would presumably be further terms involving $d^2\kappa/dx^2$ and higher derivatives. These terms will become important when $d\kappa/dx$ is so large that it is no longer permissible to assume simple bending—just as in the theory of elasticity simple bending theory is not accurate for a beam whose length is of the same order as its thickness. In fact, in the present case κ/h is about equal in order of magnitude to $d\kappa/dx$, as already noted, both being $\sim 10^{-5}$ m.$^{-2}$—that is, the change in curvature in a distance equal to the thickness is of the same order as the curvature. By using the term (5), then, we are, in effect, applying simple bending theory to a beam whose length is of the same order of magnitude as its thickness. We can therefore only expect a rough agreement with the theory.

A final term must be added to equation (6) to take account of the transverse strain-rate in the glacier. Suppose that κ, $d\kappa/dx$ and ϕ' were all zero, and that the glacier valley, instead of being parallel-sided, widened out so as to produce a transverse strain-rate $\dot{\epsilon}_z$. Since the volume of an element of ice remains constant, the strain-rate $\dot{\epsilon}_z$ must be accompanied, in general, by strain-rates $\dot{\epsilon}_x$ and $\dot{\epsilon}_y$. However, if the glacier is to maintain a roughly constant shear stress on the bed, it must maintain its thickness, and this means that $\dot{\epsilon}_y$ will be zero. Hence, for this special case,

$$\dot{\epsilon}_x = -\dot{\epsilon}_z.$$

Since in the present application $\dot{\epsilon}_z$ is a slowly varying function of x, it is reasonable to assume that it is due essentially to changes in width of the channel, and a study of the map supports this view. We therefore add the term $-\dot{\epsilon}_z$ on the right-hand side of equation (6) and obtain finally

$$\dot{\epsilon}_x = \frac{\phi'}{h} - \dot{\epsilon}_z + u\kappa \cot \alpha + \tfrac{1}{2}hu\frac{d\kappa}{dx}. \tag{7}$$

(Dr. Glen has pointed out to me that the longitudinal bending effect has a transverse analogue, whereby fluctuations in transverse curvature produce fluctuations in transverse strain-rate. This would mean that the curves in Fig. 4 have fluctuations which have been missed, and that $\dot{\epsilon}_z$ in equation (7) should be interpreted as the average value of the transverse strain-rate over the thickness.)

6. Application of the Theory

(*i*) *The Bending Term in Equation* (7). Let us first consider how the values of $d\kappa/dx$ may be calculated from the observations. κ in Fig. 10 was introduced as the curvature of the bed. But, since $1/\kappa$, the radius of curvature, is about twenty times the thickness of the glacier, κ may equally well be interpreted, to this approximation, as the curvature of the upper surface, or, what is the same thing in Fig. 10, the curvature of the streamline traced out by a point on the surface of the glacier. The last interpretation is the best for our purpose, for suppose there are undulations superimposed on the surface, carried along by it and having no relation to the bed. It would then clearly be wrong to interpret κ as the curvature of the surface, but still right to interpret it as the curvature of the streamline near the surface.

A further word should be said about the streamlines in the model of Fig. 10. The strain-rates $\dot{\epsilon}_x$ that we have calculated will be accompanied by equal and opposite strain-rates $\dot{\epsilon}_y$,

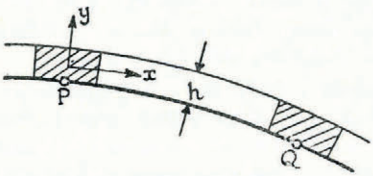

Fig. 10 (above). Illustrating the bending of a glacier as the curvature of the bed changes

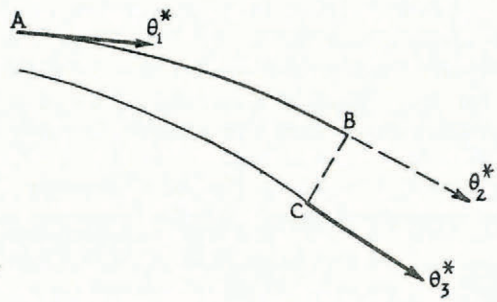

Fig. 11 (right). Illustrating the correction in θ^* to take account of the "splay" of the streamlines

since the volume of an element is constant and since, by hypothesis, $\dot{\epsilon}_z$ is zero. Thus the compression in the lower half of the glacier in the x direction must be accompanied by an expansion in the y direction, which will cause the streamline at $y=0$ to diverge slightly away from the bed. But in the upper half of the glacier the signs of $\dot{\epsilon}_x$ and $\dot{\epsilon}_y$ are reversed, and the streamline at the surface will therefore converge slightly towards the streamline at $y=0$. The final result is that, provided the neutral plane is half-way through the thickness, the streamlines at the upper and lower surfaces will be parallel.

When there is an overall longitudinal compression superimposed on this bending effect, as in the present case, the streamlines at the upper and lower surfaces will, of course, diverge from one another. One would expect, however, that the values of $d\kappa/dx$ would be much the same for the two streamlines. In view of these arguments we henceforth interpret κ in equation (7) as the curvature of the streamline at the upper surface.

Since we have not observed that the flow is steady with time, we must define more precisely what is to be meant by the "streamlines" in this context. It is reasonable to define them as the paths which the particles are following at the instant of observation: thus, if a time-exposure photograph of short duration were taken, the paths of the particles would appear as short lines; the family of curves which are everywhere tangential to these short lines are the "streamlines".

With this interpretation we can use the values of θ^* in Table I to calculate $d\kappa/dx$. If all the reference elements lay on one streamline, κ would simply be $d\theta^*/dx$. In fact they do not, since θ and θ^* are different, and a correction is necessary. Fig. 11 shows two neighbouring reference elements A and C and the streamlines through them. Let θ_1^*, θ_2^*, θ_3^* be the angles of dip of the streamlines at A, B, C respectively, where B is on the streamline through A and close to C. θ_1^*, θ_3^* are known and we wish to calculate θ_2^*. One finds that

$$\theta_2^* = \theta_3^* - \frac{\dot{\epsilon}_y l}{u}(\theta - \tfrac{1}{2}\theta_1^* - \tfrac{1}{2}\theta_3^*), \quad (8)$$

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to the first order in $\dot{\epsilon}_y l/u$, which is about 0·05, and to the first order of the term in the brackets, which is less than 0·1 for nearly all the intervals and has a maximum value of 0·31. In this formula l=AC, and θ is the angle of dip of the line AC. It will be observed that the correction term depends on $\dot{\epsilon}_y$, which measures the extent to which the streamlines diverge. Alternatively we can write

$$\kappa = \kappa_0 - \frac{\dot{\epsilon}_y}{u}(\theta - \overline{\theta^*}), \tag{9}$$

where $\kappa=(\theta_2^*-\theta_1^*)/l$, $\kappa_0=(\theta_3^*-\theta_1^*)/l$, $\overline{\theta^*}=\frac{1}{2}(\theta_1^*+\theta_3^*)$. $\dot{\epsilon}_y$ has been calculated from the equation

$$\dot{\epsilon}_y = -(\dot{\epsilon}_x+\dot{\epsilon}_z)$$

by using the graphs of $\dot{\epsilon}_x$ and $\dot{\epsilon}_z$ in Figs. 3 and 4. Then, by applying formula (8) to the values of θ^* in Table I, κ has been calculated. κ runs from $-3·21\times10^{-3}$ to $+1·71\times10^{-3}$ with a mean value of $-0·63\times10^{-3}$ m.$^{-1}$. This corresponds to a mean radius of curvature of −1590 m. The values of $d\kappa/dx$ at each reference element were then calculated by differences.

We have next calculated h, the shortest distance between the bed and the surface of the ice. The pipe inserted by Ward [10] was close to Stake 3 and is presumed to have reached bedrock at a depth of 39·3 m. on 28 August 1956. Allowing for ablation this corresponds to h=40·1 m. on 12 August. Starting with this value at Stake 3 we have then calculated the thickness of ice at the other stakes as follows. The increase of thickness Δh in a distance Δx is made up of two terms, one due to the divergence of the streamlines and one due to the difference between θ_s and θ^*. Thus

$$\Delta h = \frac{mh}{u}\Delta x+(\theta^*-\theta_s)\Delta x, \tag{10}$$

where m is the mean value of $\dot{\epsilon}_y$ through the thickness. The graph of $\dot{\epsilon}_y : x$ (not shown) is very similar in appearance to the graph of $-\dot{\epsilon}_x : x$ and shows oscillations. On the presumption that the oscillations are due to bending, and therefore change sign through the thickness, we have estimated the variation of m with x by drawing a curve on the $\dot{\epsilon}_y$ plot which leaves out the oscillations. Formula (10) was then used successively on the stake intervals, working out from Stake 3 in both directions (and taking $\theta_s=\theta$), and hence the values of the thickness h were obtained. They run from 39 m. at Stake 1 to 88 m. at Stake 18, and are plotted in Fig. 8 of reference 11. The bed of the glacier calculated from these values of h is shown in Figs. 6a and b of the present paper.

Knowing h, and $d\kappa/dx$, and taking $\sqrt{(U^2+V^2)}$ as a sufficient approximation for u, we then calculate the term $\frac{1}{2}hu\,d\kappa/dx$ in equation (7); Fig. 12, p. 399, shows the resulting theoretical curve. The oscillations in the curve are due to the term $d\kappa/dx$. What we have done in deriving these oscillations is essentially to make one differentiation of the observational data. To see this, first note that $d\kappa/dx=d^2\theta^*/dx^2$. Now the curve of θ^* is similar in form to the curve of V (since U does not fluctuate much). Thus we have, in effect, differentiated the curve of V twice. But the curve of V represents an integration of the levelling observations—for it was the *relative* vertical velocity of successive stakes that was observed. Hence $d\kappa/dx$ is obtained essentially by a single differentiation of the levelling observations. The fact that the four points at the extreme right of the theoretical curve in Fig. 12 are on a smooth curve gives confidence that the oscillations shown by the points are real and not simply scatter caused by the differentiation.

If our reasoning is correct, the oscillations shown by the observed curve of $\dot{\epsilon}_x$ in Fig. 3 should be similar to those now calculated. We have therefore drawn in Fig. 3 a mean curve of $\dot{\epsilon}_x$ which leaves out the oscillations, and have subtracted off this part to leave the observed oscillatory component of $\dot{\epsilon}_x$ plotted in Fig. 12. Comparing the two sets of points in Fig. 12 it is first to be remarked that the theory contains no adjustable parameters (except possibly the factor of $\frac{1}{2}$ in $\frac{1}{2}hu\ d\kappa/dx$ which it is unnecessary to change). It is then satisfactory that the predicted oscillations are of the correct order of magnitude. It will be noticed that the x values of the theoretical points are half-way between those of the observed points, and so the points cannot be directly compared, but the general trends shown by both sets of points are the same. In judging the extent of the agreement it must be remembered (*a*) that the thickness of the glacier is about 60 m. in the centre of the range, and that one would not expect a calculation based on simple bending to give correctly oscillations whose period is of this order or less, and (*b*) that the observed points in any case represent average strain-rates over 30 m. intervals.

In summary of this section let us first emphasize that the theoretical points come primarily, but not of course entirely, from the levelling observations. The similarity of the two sets of points in Fig. 12 thus means, roughly, that we have established an approximate connexion between (1) the levelling observations, and (2) the measurements by tape which gave $\dot{\epsilon}_x$. More precisely, the approximate connexion established is between the observed form of the surface streamline and the observed distribution of $\dot{\epsilon}_x$. The second point is that this approximate connexion, which exists in the experimental data independently of its interpretation, is what would be expected from the simple bending theory.

(*ii*) *The Slowly Varying Terms in Equation* (7). It remains to examine the other three, slowly varying, terms in equation (7). Taking ϕ' as $-6\cdot5$ m. of ice per year, which is the annual ablation measured from August 1956 to August 1957, and h from 39 to 88 m., we find values of ϕ'/h from $-0\cdot17$ to $-0\cdot07$ yr.$^{-1}$. For the second term we note that $\dot{\epsilon}_z$ is $0\cdot7$ yr.$^{-1}$ at the top of the stake system and $0\cdot12$ yr.$^{-1}$ at the bottom.

For the third term the mean κ, as already noted, is $-0\cdot63\times10^{-3}$ m.$^{-1}$ u runs from 409 m./yr. (with $\overline{\theta_s}=33\cdot0°$) at the top of the stake system to 133 m./yr. (with $\overline{\theta_s}=16\cdot0°$) at the bottom. We recall that $\overline{\theta_s}$ is the slope of an "average" surface which leaves out the waves (Fig. 7). Then, interpreting α as $\overline{\theta_s}$, we find values of $u\kappa\cot\alpha$ from $-0\cdot40$ to $-0\cdot29$ yr.$^{-1}$.

The values of the first three terms in (7) are thus respectively $-0\cdot17$ $-0\cdot7$ $-0\cdot40$ making a total of $-1\cdot27$ yr.$^{-1}$ at the upper end, and $-0\cdot07$ $-0\cdot12$ $-0\cdot29$ making a total of $-0\cdot48$ yr.$^{-1}$ at the lower end. These totals are to be compared with the slowly varying part of the observed $\dot{\epsilon}_x$, which was taken as $-1\cdot32$ yr.$^{-1}$ at the upper end and $-0\cdot20$ yr.$^{-1}$ at the lower. It will be noticed that the calculation has not taken into account the change in κ between the two ends of the line, which could increase the calculated range and improve the agreement. Considering the nature of the theory on which the slowly varying terms in (7) are based, it is perhaps surprising that the agreement with observation is so close.

(*iii*) *Re-examination of the v : x curve*. We should add a remark about the curve of $v : x$ (Fig. 5) in the light of the above considerations. Let us first note that the experimental data used to calculate v have now been included in the calculation of θ^*, and therefore that the v curve can give no essentially new information.

We have seen that v depends to a large extent on what is regarded as the slope of the average surface. If the above theory is right we must now conclude that the variations shown by the curve are largely spurious, being caused by the choice of surface. In fact we can now see that it would have been more logical to take for $\overline{\theta_s}$ in Fig. 7, not the curve shown, but one which followed the oscillations of the θ^* curve.

Assuming our theory to be correct, let us now see how a velocity component similar to v might be calculated and how it would behave. Since we know the form of the bed we could calculate at each reference element the velocity component normal to the bed. The result would simply be a curve of $mh : x$. m, it may be recalled, is the mean value of $\dot{\epsilon}_y$ through the thickness, and we have concluded that it varies only slowly with x. The curve of mh

would therefore be slowly varying, except for the slight ripples caused by the oscillations of h through the wave system. The crests would be rising a little faster than the troughs as the waves were accentuated by the overall plastic compression. This effect could not, of course, be regarded as a cause of the waves, because it could only operate to accentuate waves already there, and it is automatically included in the theory of wave formation given in reference 11. In the present context we remark that the effect is taken care of in the correction illustrated by Fig. 11.

(*iv*) *Conclusion.* We conclude that the observed longitudinal strain-rate is predominantly due to two terms: a relatively steady term of magnitude about $-0{\cdot}35$ yr.$^{-1}$ arising from the concave nature of the bed, and an oscillatory term of magnitude about $\pm 0{\cdot}2$ yr.$^{-1}$ caused by the bending and unbending of the glacier in response to changes in the curvature of the bed. In addition there is a relatively steady term due to ablation of about $-0{\cdot}1$ yr.$^{-1}$, and a slowly varying term due to the changing width of the valley running from $-0{\cdot}7$ to $+0{\cdot}04$ to $-0{\cdot}12$ yr.$^{-1}$.

Looking at Fig. 6b, and comparing it with the curves of $\dot{\epsilon}_x$ and of v, we can now see that the low compression measured at the surface just below B is caused by the fact that the glacier here is bending as it approaches a convex part of its bed—and the comparatively small slope of the bed here causes an apparently rising surface. Near D the same conditions are repeated. On the other hand, at C the slope of the bed is large, and so the surface appears to fall. Here, and a little lower down, the bed begins to be concave, and there is therefore a high compression in the surface as the glacier bends to conform to the new curvature.

The outstanding fact about this explanation of the observations is that it finds the reason for the oscillations of $\dot{\epsilon}_x$ in the form of the glacier bed—it does not connect them with the waves (except in so far as the waves cause slight changes of h, which in turn cause slight changes in the observed bending strains).

7. Prediction to Verify the Theory

If the glacier bed is the primary cause of the oscillations of $\dot{\epsilon}_x$ it may be predicted that, if $\dot{\epsilon}_x$ is remeasured at a later date, say a year later, *in the same position relative to the bed*, closely similar oscillations should be found. This should be so whatever the new configuration of the surface. These conclusions were reached in June 1957 one week before we were due to revisit the glacier. A party of ten people had already been organized for continuing the survey work and we therefore made it one of our tasks to test the above prediction.

The stakes of the 1956 main line were by that time 120 to 380 m. further down the glacier. We reconstructed the line in the position it had occupied the previous year, omitting the transverse stakes, and on 9 July we measured the distances between successive stakes by steel tape. The distances were remeasured on 18 August by a party of the Brathay Exploration Group under M. F. Robins. In this experiment, unlike that of the previous year, we avoided reboring the stake holes by inserting the stakes on 4-7 July to a depth of $5{\cdot}2$ m., their tops therefore being $2{\cdot}1$ m. below the surface. When the stakes were remeasured their tops were from $1{\cdot}0$ to $2{\cdot}9$ m. above the surface (and four of them had been lost by melting out). We also omitted the measurements of tilt made in 1956, in order to simplify the work, and being content with a somewhat lower accuracy. The mean date between the two surveys was 29 July. We took care therefore to put the stakes into the ice on 4-7 July 1957 in such a way that on 29 July 1957 the flow would have brought them into the positions occupied by the 1956 stakes on 12 August 1956 (the reference date for the 1956 measurements). This was done by theodolite resections, and a graphical system of trial and error, to an estimated accuracy of 3 m. Thus the values of $\dot{\epsilon}_x$ found in 1957 should be directly comparable with those of 1956. The 1957 values are plotted in Fig. 3. The variation with x is seen to be very similar to that measured in 1956—the low values of compression near B and D, the peak between C and D, and other smaller features, being common to both sets of points. There is no sign of a systematic shift in x, which confirms that the 1957 stakes were correctly positioned. Since four stakes melted out in 1957 some of the 1957 points refer to intervals of 60 m. instead of 30 m. The first of the 1957 points is the only one that shows any serious disagreement; the reason is not known, nor is it understood why the last three 1957 points are consistently high.

If the surface in 1957 had had a very different form from that of 1956 the experiment could be taken as direct confirmation that the bed rather than the surface topography is linked with the strain distribution. In fact, the surface was rather similar in the two years and so the experiment does not provide very strong evidence on this point. Nevertheless, the experiment verifies very well the prediction of the theory that the same oscillations would be found in 1957 as in 1956.

8. Explanation of the Motion of the 1955 Tunnel

A further test of the foregoing theory is given by the behaviour of the horizontal tunnel excavated in 1955.[8] The 1956 stake line was arranged to pass over the position previously occupied by the tunnel (Figs. 2, 6a, 6b) and a direct comparison between the two years is therefore possible. The horizontal component of velocity at the tunnel mouth in 1955 was measured as 320 m./yr. (mean date 18 August); this figure is of doubtful accuracy and does not differ significantly from the value at the same position (interpolated) in 1956, which was 327 m./yr. (mean date 12 August). The value of $\dot{\epsilon}_x$ measured [8] in 1955 is shown in Fig. 3. It was measured between two stakes separated by 82 m., as indicated, and is therefore an average value over this distance. It is rather lower than the 1956 values, and rather higher than the comparable value for 1957.

Glen [8] found in 1955 that the tunnel was (1) being carried down glacier, (2) being rotated, and (3) being bent. It is interesting to see to what extent this can be explained on the basis of the 1956 observations. As regards (1) it is sufficient to note the agreement of the velocity in the two years which has just been mentioned.

Four effects will contribute to the rotation of the tunnel. First, the change of slope, as mentioned by Glen. This gives a contribution to the angular velocity ω of $-u\kappa$, which at Stake A is $363\times 3\cdot 13\times 10^{-3}=1\cdot 13$ radians/yr. (We shall take ω as positive when the end of the tunnel falls relative to the entrance.) Second, there is a positive rotation due to the longitudinal compression, since the tunnel, being horizontal, makes an angle θ_s with the surface. This gives a contribution to ω of $-\frac{1}{2}\dot{\epsilon}_x \sin 2\theta_s$, which, with $\dot{\epsilon}_x=-1\cdot 10$ yr.$^{-1}$ and $\theta_s=26°$, is $+0\cdot 43$ radians/yr. Third, there is a similar effect from $\dot{\epsilon}_y$, which makes a contribution to ω of $\frac{1}{2}\dot{\epsilon}_y \sin 2\theta_s$; with $\dot{\epsilon}_y=0\cdot 70$ yr.$^{-1}$ this is $+0\cdot 28$ radians/yr. Fourth, the shear strain-rate component $\dot{\epsilon}_{xy}$ will tend to give a negative ω; this rotation, however, will vary with depth; it is zero at the surface and appears to make a negligible contribution. The result is a total positive angular velocity ω of $1\cdot 13+0\cdot 43+0\cdot 28=1\cdot 84$ radians/yr.$=105°$/yr. The observed angular velocity was $+1\cdot 70$ radians/yr.$=97°$/yr. (Fig. 5 of Glen's paper.[8]) Thus, we see that the observed rotation of the tunnel was mainly due to the lessening slope of the glacier bed, but that the longitudinal and normal strain-rates also make significant contributions.

In a similar way we may recognize several effects which will contribute to the bending of the tunnel. First, the effect of the change of curvature of the glacier bed, which, on a simple bending theory, gives for the rate of change of curvature κ' of the tunnel $d\kappa'/dt= u(\cos\theta_s+\sin 2\theta_s \sin\theta_s)\, d\kappa/dx$. $d\kappa/dx$ is changing very fast in this region, being $-0\cdot 71\times 10^{-5}$ m.$^{-2}$ at Stake A and $+6\cdot 73\times 10^{-5}$ m.$^{-2}$ at Stake 2. Using the mean of the two values gives $d\kappa'/dt=0\cdot 014$ m.$^{-1}$ yr.$^{-1}$, that is, the tunnel tends to become convex upwards from this cause. Second, the change of $\dot{\epsilon}_x$ and $\dot{\epsilon}_y$ with respect to x will cause a difference in the angular velocities of the two ends of the tunnel, and so will give a bending. The contribution to $d\kappa'/dt$ from this cause is $\sin\theta_s \cos^2\theta_s\, \partial(\dot{\epsilon}_x-\dot{\epsilon}_y)/\partial x$. Using the values of $\dot{\epsilon}_x$ and $\dot{\epsilon}_y$ appropriate to the two ends of the tunnel gives $\partial\dot{\epsilon}_x/\partial x=0\cdot 010$ m.$^{-1}$ yr.$^{-1}$ and $\partial\dot{\epsilon}_y/\partial x=-0\cdot 002$ m.$^{-1}$ yr.$^{-1}$,

https://doi.org/10.3189/S0022143000017081 Published online by Cambridge University Press

and hence $d\kappa'/dt = +0{\cdot}004$ m.$^{-1}$ yr.$^{-1}$. In this rough calculation we shall assume that it is permissible to neglect the contributions to $d\kappa'/dt$ from the derivatives $\partial\dot{\epsilon}_{xy}/\partial y$ and $\partial\dot{\epsilon}_{xy}/\partial x$. The total calculated rate of change of curvature of the tunnel is thus $+0{\cdot}018$ m.$^{-1}$ yr.$^{-1}$, which may be compared with the observed value of $+0{\cdot}036$ m.$^{-1}$ yr.$^{-1}$ (calculated from Fig. 5 of Glen's paper [8]). The rapid change of $d\kappa/dx$ makes the theoretical estimate rather uncertain, and we may also note that the observed bending is very non-uniformly distributed (and even becomes slightly negative). Within these uncertainties the conclusion is that the observed bending of the tunnel in 1955 was primarily caused by the local change of curvature of the glacier bed. The respective curvatures between 1 and A, A and 2, 2 and 3, are $-3{\cdot}04$, $-3{\cdot}21$, $-2{\cdot}23\times10^{-3}$ m.$^{-1}$. Thus the tunnel, initially straight, becomes convex upwards because the bed of the glacier, although concave, is tending to straighten out; the bending of the tunnel is an unbending of the glacier. (The curve of v in Fig. 5 confirms this analysis, but not quite rigorously, because the presence of the crest at Stake 2 would in any case tend to increase v.)

This explanation of the bending differs from that advanced by Glen,[8] who interpreted it as showing that the wave ogives were actually being formed in the ice surrounding the tunnel. If one confines attention to the limited region covered by the tunnel there is nothing to discriminate between the two explanations. But now that we have observations over a much larger area, and can see that similar bendings occurring elsewhere are not correlated with the wave ogives, the explanation put forward in this section is preferable; it shows that the tunnel area obeys the same principles as the rest of the wave area.

9. Final Remark on the Possibility of a Pressure Mechanism

There is one way in which a pressure mechanism of wave formation might be reconciled with the observations. It might be that the high compression measured at the extreme upper end of the 1956 stake line represents the lower edge of an extensive pressure wave which one year later would occupy the position of the crest near C. Now, first, there certainly is such a compression and photogrammetric measurements [11] show that it covers the whole of the lower two-thirds of the ice fall. But, second, whether this compression forms a pressure wave or not depends on whether it is greater at some seasons of the year than at others, and we have no information on this. The only reason for thinking that there is no very significant pressure wave formation of this sort is in the argument of reference 11—that the existing waves are fully accounted for by the ablation mechanism referred to in Section 1, which must in any case be present, and there is therefore no need to invoke a hypothetical pressure mechanism as the primary cause of the waves.

10. Acknowledgements

The Cambridge Austerdalsbre Expedition was the conception of W. V. Lewis, who organized and led the 1955 parties. In 1956, Mr. Lewis being prevented by illness from continuing, the organization at Cambridge was done by J. E. Jackson, and the parties on the glacier were led jointly by W. H. Ward, J. I. Davidson, J. W. Glen and myself. The 1957 and 1958 parties were organized and led by Mr. Ward.

The scientific work of 1956 and 1957 described in this paper was only made possible by much administrative work in England, and back-packing and similar tasks on the glacier, and I should like to express my thanks to all concerned. In 1956 there were at various times (not all at once) 40 to 50 people, mostly Cambridge undergraduates, working on the glacier —excluding the parties from the Perse School and the Brathay Exploration Group, who bring the total to about 80—and nearly all of them took part in one way or another in the work here described.

I should record my particular thanks to Mr. Ward, not only for the impetus he has personally given to the work, but also for his development of efficient drilling equipment and technique for inserting stakes in the ice, which has been a major factor in the success of our enterprise. I also record with gratitude the leading role which Dr. Glen has played in taking the measurements. In particular, in 1956, after the stake system had been set up and surveyed once, Dr. Glen took charge of the remaining surveys, which were meticulously performed. He is thus responsible for the greater part of the measurements used in this paper.

In the survey work we had the expert advice and assistance of Dr. Cuchlaine King, J. I. Sharp and Miss E. Peters. The reduction of the theodolite observations was done by Mr. Jackson and Mr. Ward. It is also a pleasure to mention the practical help given us on the glacier by Olav Sopp of Oslo University.

The expedition is indebted for financial help to the Royal Society, Cambridge University, the Mount Everest Foundation, Trinity College Cambridge, the Royal Geographical Society, the Tennant Fund of Cambridge University, and the University College of Swansea. I myself am indebted to Bristol University for a grant.

Dr. Glen has kindly read and commented on the manuscript of the paper.

MS. received 13 October 1958

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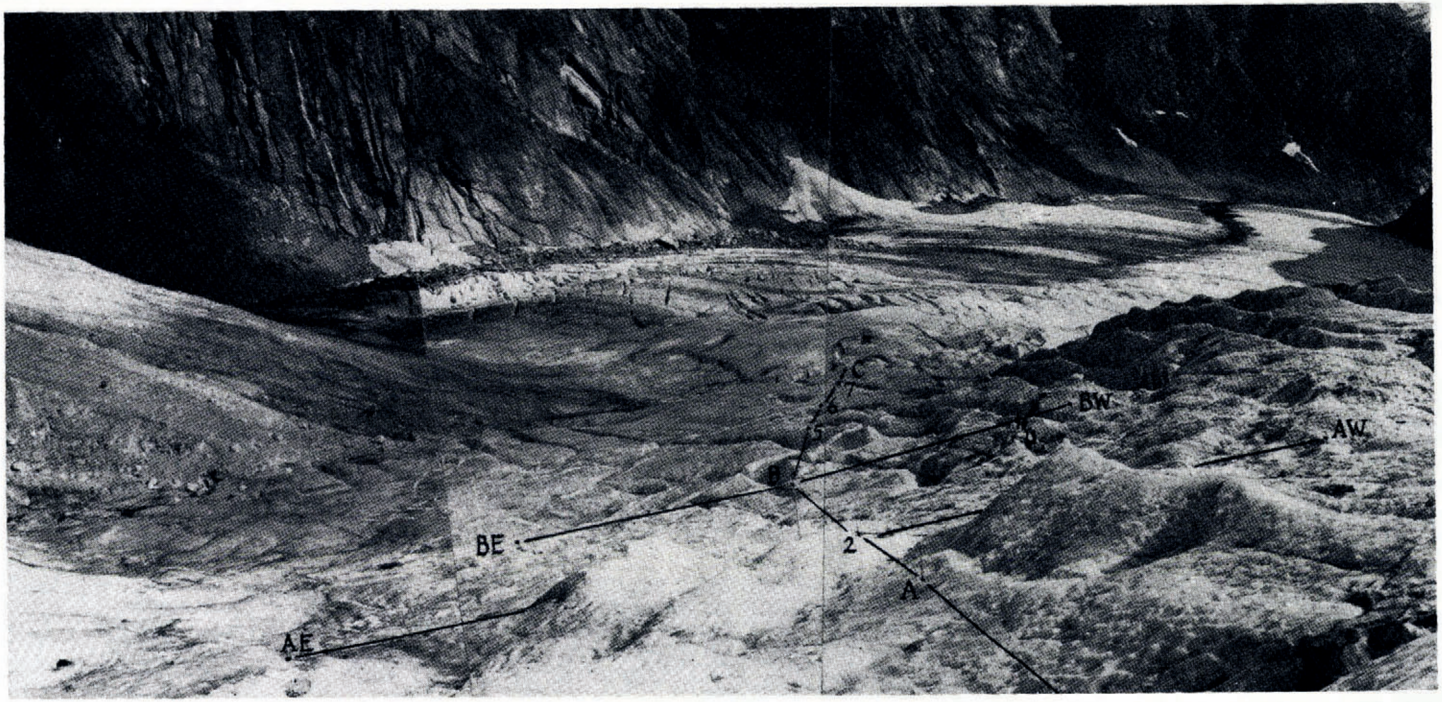
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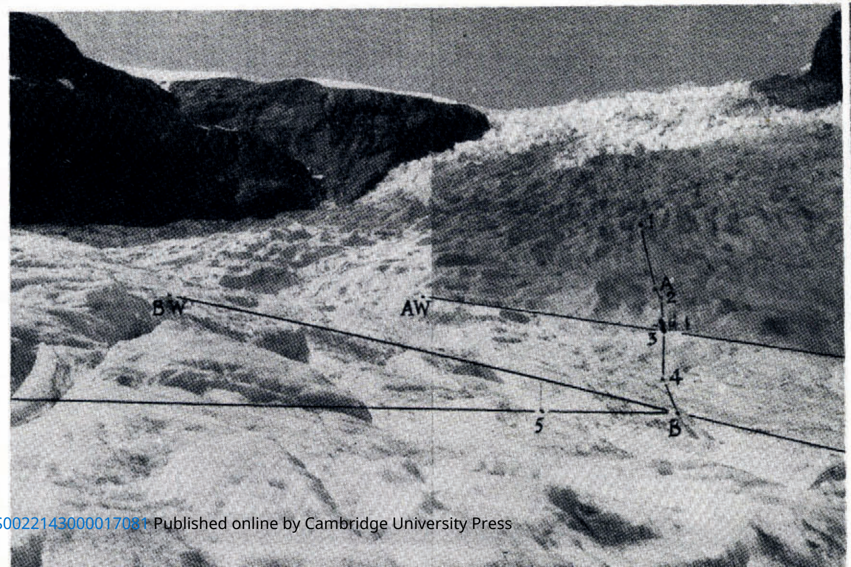
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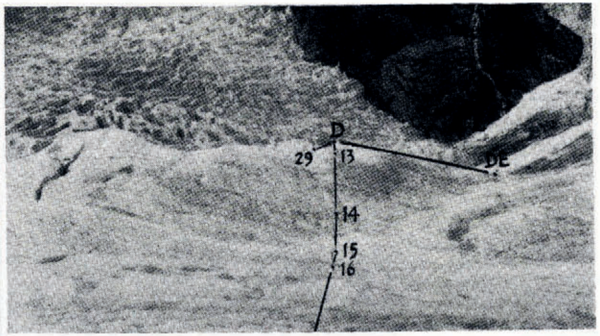
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