Seventh Meeting, 14*th May* 1897.

Professor GIBSON in the Chair.

The Bessel Functions and their Zeros.

By Dr PEDDIE.

A Geometrical Theorem with application to the Proof of the Collinearity of the mid-points of the Diagonals of the Complete Quadrilateral.

By R. F. MUIRHEAD, M.A., B.Sc.

Geometrical Note.

By R. TUCKER, M.A.

On the sides BC, CA, AB of the triangle ABC are described two sets of equilateral triangles,
the set Ba'C, Cb'A, Ac'B externally, and
the set BaC, CbA, AcB internally.
The lines Aa', Bb', Cc' cointersect in Q, the centre of Perspective of the triangles ABC, $a'b'c'$,
and the lines Aa, Bb, Cc in P, the centre of Perspective of ABC, abc.
Since a, a', b, b', c, c' are on the perpendicular bisectors of BC, CA, AB, their joins cointersect in the circumcentre, O, which is the centre of Perspective of abc, $a'b'c'$.

Now $$\mathrm{O}a' = 2\mathrm{R}\cos(60° - \mathrm{A}),$$
$$\mathrm{O}a = -2\mathrm{R}\cos(60° + \mathrm{A}),$$

hence $$aa' = a\sqrt{3}, \quad bb' = b\sqrt{3}, \quad cc' = c\sqrt{3}:$$

and also $$\Sigma(aa')^2 = 3 \quad \Sigma(a^2) = 3k.$$

Using trilinear coordinates,

Q is $$\alpha\sin(60° + \mathrm{A}) = \beta\sin(60° + \mathrm{B}) = \gamma\sin(60° + \mathrm{C});$$

P is $$\alpha\sin(60° - \mathrm{A}) = \beta\sin(60° - \mathrm{B}) = \gamma\sin(60° - \mathrm{C}).$$

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Hence we have the equations to

PQ, $\Sigma a\sin(60^\circ + A)\sin(60^\circ - A)\sin(B - C) = 0,$

OQ, $\Sigma a\sin(60^\circ + A)\cos(60^\circ - A)\sin(B - C) = 0,$

OP, $\Sigma a\sin(60^\circ - A)\cos(60^\circ + A)\sin(B - C) = 0,$

which evidently pass through the symmedian point.

From the Δ Oab we get

$$c'^2 = (ab)^2 = a^2 + b^2 + ab\cos C - \sqrt{3}\,ab\sin C\,;$$

$$a'^2 = b^2 + c^2 + bc\cos A - \sqrt{3}\,bc\sin A\,;$$

$$b'^2 = c^2 + a^2 + ca\cos B - \sqrt{3}\,ca\sin B.$$

Hence $$\Sigma a'^2 = \tfrac{5}{2}\Sigma a^2 - b\sqrt{3}\,\Delta.$$

From the Δ O$a'b'$ we get

$$c''^2 = (a'b')^2 = a^2 + b^2 + ab\cos c + \sqrt{3}\,(2\Delta)\,;\ \text{and so on.}$$

Hence $$\Sigma a''^2 = \tfrac{5}{2}\Sigma a^2 + b\sqrt{3}\,\Delta.$$

also $$a'^2 + a''^2 = 3(b^2 + c^2) - .a^2,\ \text{etc.},$$

and $$a''^2 - a'^2 = 4\Delta\sqrt{3} = b''^2 - b'^2 = c''^2 - c'^2.$$

Let Δ', Δ'' be the areas respectively of Oab, O$a'b'$,

then $$2\Delta' = 5\Delta - \sqrt{3}k/4,$$

and $$2\Delta'' = 5\Delta + \sqrt{3}k/4.$$

Hence $$\Delta' + \Delta'' = 5\Delta.$$

If ω', ω'' be the Brocard angles of the triangles

$$\cot\omega' = \frac{5\cot\omega - 3\sqrt{3}}{5 - \sqrt{3}\cot\omega}, \qquad \cot\omega'' = \frac{5\cot\omega + 3\sqrt{3}}{5 + \sqrt{3}\cot\omega}.$$

Again $$(\mathrm{A}a')^2 = c^2 + a^2 - 2ca\cos(60^\circ + B)$$

$$= \frac{K}{2} + 2\Delta\sqrt{3} = 2\Delta(\cot\omega + \sqrt{3})\,;$$

$$(\mathrm{A}a\,)^2 = c^2 + a^2 - 2ca\cos(60^\circ - B) = 2\Delta(\cot\omega - \sqrt{3}):$$

hence $$(\mathrm{A}a')^2 + (\mathrm{A}a)^2 = K = (\mathrm{B}b')^2 + (\mathrm{B}b)^2 = (\mathrm{C}c')^2 + (\mathrm{C}c)^2.$$

The points a, a' are given by

$$\sin 60^\circ,\quad \sin(C - 60^\circ),\quad -\sin(60^\circ - B),$$

$$-\sin 60^\circ,\quad \sin(C + 60^\circ),\quad \sin(60^\circ + B),$$

hence the triangles abc, $a'b'c'$ are concentroidal with ABC.

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