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RESEARCH ARTICLE

The Repayment Structure of Agricultural Loans under a Full Repayment Constraint

Marcos Escobar-Anel[1], Gaurav Khemka[2] and Zheng Xu[1]

[1]Western University, London, ON, Canada and [2]Australian National University, Canberra, Australia
Corresponding author: Zheng Xu; Email: zheng.xu1@anu.edu.au

Abstract

We investigate the optimal asset allocation and repayment strategy of an agricultural loan under a guaranteed repayment condition in a continuous-time setting. We propose two forms of the problem: an analytically solvable "separable" problem and a more realistic "nominal" problem that is investigated numerically. In the numerical study, we calibrate our model to publicly available farm data and explore various forms of repayment structures. While the widely used constant repayment structure has a surprisingly outstanding performance, we also design two repayment structures for the nominal problem that perform quite well.

Keywords: Contingent repayment; farm leverage; farm household; optimal debt management; stochastic dynamic programming

JEL codes: C61; G51; Q14

1. Introduction

Debt financing is an integral part of farming operations as it increases the capital level for production and allows farmers to expand their business, thereby leading to a higher expected return from farming. However, debt financing is accompanied by repayment obligations, which are usually independent of the income from farming operations, such as a bank loan. Farming operations, in reality, can be highly variable and are affected by variables largely outside the farmers' control, such as weather and the prices of their output. Consequently, the repayment structure should play an important role in the farm leverage decision.

Since time-contingent loans (for example, standard bank loans) are the most common form of financing instruments in the real world, the repayment rate has usually been modeled as an exogenously given constant in the economics literature. However, since income contingent loans (ICLs) to fund higher education have been successfully implemented in many countries (e.g., Australia, New Zealand, and England), various studies have considered the possibility of developing a similar product in the context of agricultural financing. Botterill and Chapman (2004) and Botterill and Chapman (2009) introduce an income stabilization instrument for agricultural credit named revenue contingent loan (RCL). By its name, the RCLs link the loan repayments to the generated revenue to avoid repayment hardship, especially when the revenue is low. Botterill et al. (2017) propose RCLs as an alternative to the drought assistance subsidy payments by the Australian government. They argue that RCLs improve equity and provide sufficient protection against default risk.

Against this background, we analyze the effects of various repayment structures, including those seen in practice (for example, bank loans) and new proposals (various types of contingent loans, including RCL and ICL), on the lifetime welfare of farm households in a continuous-time setting. The model also derives the farm households' optimal inter-temporal consumption and investment strategies. As this is an important first step in understanding this problem, we prioritize solving the model analytically; for this, we make a simplifying assumption that the loan is guaranteed to be repaid by a pre-specified terminal date; that is, we ignore the possibility of credit risk (Collins and Karp, 1993), which will be studied in a separate work.

In setting up the problem with the repayment guarantee, we make use of the mathematical framework of portfolio insurance strategies from the field of finance, thus building a link between agriculture economics and traditional finance fields. Portfolio insurance strategies can be regarded as the optimal solution to a modified utility maximization problem with certain exogenous guarantee constraints (Balder and Mahayni, 2010). The original constraint of simply a minimum level of terminal wealth (for example, in the case of avoiding defaults on the outstanding loan) has been extended to many other forms, such as value at risk (VaR) in the literature (see Basak and Shapiro, 2001; Kraft and Steffensen, 2013; Chen et al., 2018). Two of the most prominent insurance strategies in the finance literature are constant proportion portfolio insurance (CPPI) and option-based portfolio insurance (OBPI), both based on the seminal works of Merton (1969, 1975), where a functional form of hyperbolic absolute risk aversion (HARA) utility is maximized.

Our main objective consists of integrating the literature on agricultural debt financing with the literature on income-contingent loans to assess how repayment structures affect the farmer's lifetime utility in the presence of revenue variations and to provide analytical solutions to such problems. To the best of our knowledge, a framework that encompasses the repayment structure, consumption, and investment strategies is absent from the literature. As explained before, we impose a full repayment guarantee to gain better insight into this type of low-credit-risk investors while making our model analytically tractable. Further, we find that a problem that does not explicitly take account of the utility from repayments has a trivial solution, implying that the farmer would be indifferent to different repayment structures. Thus, we construct a framework that assigns utility to repayments. In this spirit, we propose two reward functions to be maximized under a general form of the net revenue process and a full repayment guarantee constraint. A closed-form optimal solution is found for one of the reward functions, while the other reward function and the corresponding problem are studied numerically.

We also perform an empirical test to compare several types of repayment structures (deterministic, stochastic, and hybrids). The results highlight the advantages of deterministic repayment rates, particularly the constant repayment rate (like those seen in standard bank loans), due to its straightforward structure. This finding sheds new light on the optimality of deterministic repayments and the impact of parameters while opening the door to further innovations and studies on optimal debt management.

Our first contribution to the literature is to establish a theoretical farm household decision-making framework for analyzing the effects of different repayment structures. In the literature, the farm leverage decision is largely studied through an investigation of the optimal debt ratio, while little attention is given to the repayment structure of the debt. From the farmer's perspective, the repayment rate implicitly reflects the speed at which the loan is repaid and the farmer's credit access. Thus, it should be an important factor in farmers' leverage decisions, from which our research stems. Most of the recent literature focuses on assessing the theoretical models empirically such as de Mey et al. (2014), Uzea et al. (2014), Ifft et al. (2015), de Mey et al. (2016), Bampasidou et al. (2017), Aderajew et al. (2019), and Key (2020), while few innovations are made in the theoretical framework of farm capital structure. Notable exceptions include Wu et al. (2014), who explain the farm capital structure choice in the presence of various credit access scenarios, and Wauters et al. (2015), who consider the trade-off between total-farm-risk and off-farm-risk.

An important branch of literature related to our framework involves formulating the debt decision problem alongside simultaneous consumption and production decisions in a farming household setting. Studies such as Phimister (1995a), Benjamin and Phimister (1997), Ramirez

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et al. (1997), Cheng and Gloy (2008), and Briggeman et al. (2009) explore this dynamic trade-off between consumption and investment. Interestingly, Ramirez et al. (1997) follows the Merton model (Merton, 1969) and applies stochastic optimal control to solve for the optimal leverage and consumption strategies in a continuous-time setting. However, the optimal leverage found by Ramirez et al. (1997) has the same structural implications as the static Collins-Barry model, and shortfalls in the operating return can be offset by additional borrowings implicitly. As identified by Wu et al. (2014), such an assumption that the farmers have full access to credit and their risk profiles are homogeneous differs from actual capital markets. In contrast, our framework fixes the initial loan amount and prohibits further borrowing over the loan horizon. Alternatives in the literature, such as Phimister (1995a) and Phimister (1995b), explicitly consider borrowing constraints in life-cycle models, while Benjamin and Phimister (1997) introduce transaction costs and adjustment costs for new borrowings.

As a second contribution, we provide the analytical solution to the farm leverage decision in the *separable* problem[1] along with the numerical illustrations for the *nominal* problem,[2] while studies in this field, almost exclusively, consider the effect of repayment rates from a numerical perspective in a discrete-time framework (such as Leatham and Baker, 1988, who model the farmer's choice between a fixed-rate and an adjustable-rate loan by using a discrete sequential stochastic programming approach). In doing so, we also provide an analytical investigation of the effect of RCLs on the lifetime welfare of the farmer to the agricultural financing literature. We note that such an analytical formulation needs to be improved even in the significantly more studied and developed field of ICLs, from which RCL is derived. In the higher education sector, the central strand of literature focuses on the public finance outcomes and the design of ICLs at the macroeconomic level (Garcia-Penalosa and Wälde, 2000; Barr and Crawford, 2004; Chapman, 2006a, 2006b, a; Del Rey and Racionero, 2010; Eckwert and Zilcha, 2012). The closest studies are those by Van Long (2014, 2019), which propose a general theoretical framework for a utility maximization problem in discrete time for analyzing a piecewise-linear repayment schedule but do not provide a method of solving the framework.

Finally, we contribute to the broader economic literature by noting that our model can be applied to any industry with productive capital and incurred debt as long as the form of the income or net revenue process aligns with our model. In continuous-time stochastic Ramsey growth models with Cobb-Douglas production function, the optimal pathwise consumption strategy generally lacks a closed-form solution; see Baten and Miah (2007) and Feicht and Stummer (2010). A feedback form solution is provided by Morimoto and Zhou (2009), while Feicht and Stummer (2010) and Menoncin and Nembrini (2018) recognize a closed-form solution under a strict limitation on the risk-aversion parameter within a CRRA utility framework. To achieve a closed-form solution, our model instead simplifies the Cobb-Douglas production function into a linear form by making the parameters state-dependent. The solution is then found using a change of control approach.

The remainder of the paper is structured as follows. In Section 2, we introduce the problem and set up two reward functions for analyzing the effects of different repayment structures – the separable problem and the nominal problem. We provide an analytical solution to the separable problem in Section 3 and to a corresponding "net" problem. In Section 4, we use publicly available Australian farm data to parameterize our model and provide an illustration of the separable problem. More importantly, we solve the nominal problem numerically and present a speculated optimal solution to the nominal problem that is assessed against other repayment structures and found to exhibit the most favorable performance. Section 5 concludes.

[1]The *separable* problem is defined by (PS), where the consumption, repayment, terminal wealth and terminal outstanding loan amount contribute to the lifetime utility through separated utility functions.

[2]The *nominal* problem is defined by (PN), in which both consumption and repayment contribute to lifetime utility through the same utility function, as do terminal wealth and the terminal outstanding loan amount.

2. Problem formulation

Consider a farmer whose sole external financing tool is a loan that was taken out at time $t = 0$ with a maximum repayment period of $t = T$. In a continuous-time setting, at time $t \in [0,T]$, the farmer's net wealth W_t is defined as the summation of the amounts invested in the risky asset and the amount of cash in the bank (B) less the outstanding loan amount D_t. For simplicity of presentation, we will assume two risky assets,[3] namely land (L), and farm capital excluding land (K).

However, we note that our approach can be extended to multiple asset classes without the loss of generality. In general, our methodology follows through if working with more generic sources of risk, like technology, productivity shocks, or commodity prices, i.e., $K = f_K(X_1, \ldots, X_m)$, $L = f_L(X_1, \ldots, X_m)$.

The representation for wealth would be $W_t = L_t p_t^L + K_t p_t^K + B_t p_t^B - D_t$, where L_t, K_t and B_t denote the units invested in each asset class while p_t^L, p_t^K, and p_t^B are the corresponding unit price at time t. Then we define the nominal wealth $\tilde{W}_t$ as the total available amount of investment as

$$\tilde{W}_t = W_t + D_t = L_t p_t^L + K_t p_t^K + B_t p_t^B. \tag{1}$$

The evolution of p_t^B reflects the risk-free rate provided by the cash account, which is assumed to be exogenously given as a constant r^B over the whole period $[0, T]$. Thus, the process of p_t^B is $\mathrm{d}p_t^B / p_t^B = r^B \mathrm{d}t$.

Similarly, the evolution of p_t^L and p_t^K reflects the appreciation of land value and the depreciation of farm capital, respectively. Their processes are modeled as geometric Brownian motions (GBMs), and the sign of the drift parameter μ captures appreciation:[4]

$$\begin{aligned} \mathrm{d}p_t^L / p_t^L &= \mu^L \mathrm{d}t + \sigma^L \mathrm{d}z_t^L, \\ \mathrm{d}p_t^K / p_t^K &= \mu^K \mathrm{d}t + \sigma^K \mathrm{d}z_t^K, \end{aligned} \tag{2}$$

where z_t^L and z_t^K define standard Brownian motions with σ representing the respective diffusion coefficient.

The overall effects of these price changes result in an instantaneous capital gain amount of $L_t \mathrm{d}p_t^L + K_t \mathrm{d}p_t^K + B_t \mathrm{d}p_t^B$ for the farmer.

2.1. Net revenue from farm production

In addition to the overall capital gain resulting from the instantaneous changes in unit prices, the farmer also generates revenue from farming. Let y_t be the cumulative net revenue (i.e., revenue less cost) until time t, then the instantaneous net revenue is

$$\mathrm{d}y_t = \mu_t^y (L_t, K_t) \mathrm{d}t + \sigma_t^y (L_t, K_t) \mathrm{d}z_t^y,$$

where μ^y and σ^y denote the drift and volatility of the net revenue process, respectively, while z_t^y defines a standard Brownian motion.

We note that both μ^y and σ^y are functions of L_t and K_t because the net revenue is generated from these two resources. Other sources of risk could have been considered for more generic drift and diffusion terms e.g., $\mu_t^y(X_{1,t}, \ldots, X_{m,t})$, $\sigma_t^y(X_{1,t}, \ldots, X_{m,t})$. Consistent with reality, this revenue-asset relationship incentivizes the farmer to allocate wealth to these two asset classes. We write μ^y and σ^y in terms of land value $p_t^L L_t$ and capital value $p_t^K K_t$ as follows:

[3]Other assets, such as machinery or livestock, as long as their price dynamics also follow geometric Brownian motions, can also be considered in our model without loss of analytical tractability.

[4]A negative μ means depreciation effectively.

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$$\mathrm{d}y_t = \left(\alpha^L p_t^L L_t + \alpha^K p_t^K K_t\right)\mathrm{d}t + \left(\beta^L p_t^L L_t + \beta^K p_t^K K_t\right)\mathrm{d}z_t^y. \tag{3}$$

Although the writing above appears linear in L_t, K_t, our model allows for α and β to be state dependent, i.e., $\alpha_t = \alpha(L_t,K_t)$, $\beta_t = \beta(L_t,K_t)$. Thus, the well-known diminishing marginal effect could be modeled by (3) by assuming that α_t would decrease while land value and capital value increase. For simplicity of presentation, we implement the case of constant α and β. Note that Grigorieva and Khailov (2005) considers a similar optimal loan repayment schedule problem in the case of linear production function but without the Brownian motion component.

Finally, we note that the three stochastic processes of $p_t{}^L$, $p_t{}^K$ and y_t can be correlated, and such correlation is captured by a correlation matrix $\boldsymbol{\rho}$ which satisfies

$$\mathrm{d}\boldsymbol{z}_t = \left(\mathrm{d}z_t^L, \mathrm{d}z_t^K, \mathrm{d}z_t^y\right)^\top = \boldsymbol{\rho}\left(\mathrm{d}\bar{z}_t^L, \mathrm{d}\bar{z}_t^K, \mathrm{d}\bar{z}_t^y\right)^\top = \boldsymbol{\rho}\mathrm{d}\bar{\boldsymbol{z}}_t,$$

where $\mathrm{d}\bar{\boldsymbol{z}}_t$ are uncorrelated Brownian motions. The correlation matrix $\boldsymbol{\rho}$ should be non-singular to avoid perfect correlation amongst the three random processes.

2.2. Loan and wealth dynamics

We assume the farmer has an initial loan amount D_0 at time $t = 0$, and the existing loan must be repaid during the period $[0, T]$. Given the repayment guarantee, the outstanding loan amount D_t at time t should satisfy

$$D_t = \int_t^T e^{-r^B(s-t)}\mathrm{d}R_s + D_T e^{-r^B(T-t)}, \tag{4}$$

where $\mathrm{d}R_s$ is the instantaneous repayment amount, and the equation above should hold for any realized wealth path.

In our study, the functional form of $\mathrm{d}R_t$ is very flexible; the only condition is zero quadratic variation (i.e., R_t is a differentiable function of time),[5] therefore $\mathrm{d}R_t$ can be expressed as $\mathrm{d}R_t = a_t\mathrm{d}t$, where a_t could be a general function of any of the underlying variables, parameters, or processes described previously, for example, t, y_t, W_t, and D_t. Thus, the dynamics of D_t, according to equation (4), satisfies

$$\begin{aligned}\mathrm{d}D_t &= r^B D_t\mathrm{d}t - \mathrm{d}R_t \\ &= \left(r^B D_t - a_t\right)\mathrm{d}t.\end{aligned} \tag{5}$$

At any time $t \in [0,T]$, the farmer needs to determine the instantaneous consumption rate c_t and reallocate the assets after receiving the instantaneous overall capital gain and the net revenue dy_t, as well as settling the instantaneous repayment $\mathrm{d}R_t$. With the definition of wealth in equation (1), the budget constraint is derived as:

$$\mathrm{d}\tilde{W}_t = \mathrm{d}y_t - c_t\mathrm{d}t - \mathrm{d}R_t + L_t\mathrm{d}p_t^L + K_t\mathrm{d}p_t^K + B_t\mathrm{d}p_t^B$$

Let $\tilde{\pi}_t^L = L_t\mathrm{d}p_t^L/\tilde{W}_t, \tilde{\pi}_t^K = K_t\mathrm{d}p_t^K/\tilde{W}_t$ and $\tilde{\pi}_t^B = \tilde{\pi}_t^K = B_t\mathrm{d}p_t^B/\tilde{W}_t$ denote the proportion of nominal wealth invested in each asset class. Then, from equation (1), we receive $\tilde{\pi}_t^L + \tilde{\pi}_t^K + \tilde{\pi}_t^B = 1$.

Rewriting the dynamics of nominal wealth in terms of the π's, we get in matrix form

$$\mathrm{d}\tilde{W}_t = \left(\tilde{W}_t\left[r^B + \tilde{\boldsymbol{\pi}}_t^\top\tilde{\boldsymbol{\mu}}\right] - (c_t + a_t)\right)\mathrm{d}t + \tilde{W}_t\tilde{\boldsymbol{\pi}}_t^\top\boldsymbol{\sigma}\mathrm{d}\boldsymbol{z}_t \tag{6}$$

with $\tilde{\boldsymbol{\pi}}_t = (\tilde{\pi}_t^L, \tilde{\pi}_t^K)^\top$; $\tilde{\boldsymbol{\mu}} = (\alpha^L + \mu^L - r^B, \alpha^K + \mu^K - r^B)^\top$; $\tilde{c}_t = c_t + a_t$ is the nominal consumption and

[5]Richer repayment structures, such as those driven by Brownian motions, can be considered as a qualitative different class worth exploring in future analyses.

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma^L & 0 & \beta^L \\ 0 & \sigma^K & \beta^K \end{bmatrix}.$$

Similarly, π, the proportion of *net* wealth invested in each asset class could also be defined. Then we have $\pi_t^L = L_t p_t^L / W_t$, $\pi_t^K = K_t p_t^K / W_t$ and $\pi_t^B = B_t p_t^B / W_t$, as well as $\boldsymbol{\pi}_t = (\pi_t^L, \pi_t^K)^\top$, which lead to $\tilde{W}_t \tilde{\boldsymbol{\pi}}_t^\top = W_t \boldsymbol{\pi}_t^\top$.

With the dynamics of nominal wealth and equation (1), the dynamics of net wealth could be derived as

$$\begin{aligned} \mathrm{d}W_t &= \mathrm{d}\tilde{W}_t - \mathrm{d}D_t \\ &= \left(W_t\left[r^B + \boldsymbol{\pi}_t^\top \tilde{\boldsymbol{\mu}}\right] - c_t\right)\mathrm{d}t + W_t \boldsymbol{\pi}_t^\top \boldsymbol{\sigma} \boldsymbol{\rho} \mathrm{d}\bar{\boldsymbol{z}}_t. \end{aligned} \tag{7}$$

Since the last expression in the right-hand side of (7) is independent of D_t, the dynamics of net wealth are unrelated to D_t, indicating that net wealth has no connection to debt or repayments whatsoever.

2.3. Objective functions

Our main interest is to compare different repayment structures in terms of farmer satisfaction with a fixed amount of initial loan under the constraint of full repayment, which could be expressed as a constrained utility maximization problem. In this section, we propose a utility framework in which not only net consumption and terminal wealth but also repayments contribute to lifetime utility. We then present two reward functions: a separable problem in (PS) and a nominal problem in (PN) within this framework. Of the two, the nominal problem is of greater interest as it can accommodate a broad range of repayment schemes, including those where either or both of $a_t = 0$ and $D_T = 0$ (for example, due to early repayment, or only paying off the loan at the very end). The separable problem is limited as it can only evaluate repayment schemes where $a_t \neq 0$ and $D_T \neq 0$. Some preliminary results and motivation can be found in Appendix A.

To capture the important input variables and controls, we adopt the following notation $J(\cdot)$ for the optimization problem and $V(\cdot)$ for the reward function. It should be noted that the functions are written in nominal terms to keep repayments as part of the objective.

Let us define $\mathcal{A}_t$ as the set of unconstrained feasible consumption, repayment, and investment strategies to distinguish it from $\mathcal{B}_t$, the set of all the feasible consumption, repayment, and investment strategies at time t induced by the repayment constraint (4). These are:

$$\mathcal{A}_t = \left\{\left(c_s, a_s, \pi_s^L, \pi_s^K\right)_{s\in[t,T]}\right\},$$

and

$$\mathcal{B}_t = \left\{\left(\tilde{c}_s, a_s, \tilde{\pi}_s^L, \tilde{\pi}_s^K\right)_{s\in[t,T]} \middle|\ \tilde{c}_s \geq a_s \geq 0, \tilde{W}_T \geq D_T \geq 0\ \mathcal{P} - a.e.\right\}.$$

The aim of $\mathcal{B}_t$ is to guarantee a positive net consumption rate c_s and a positive net terminal wealth W_T when the loan has been incorporated.

We now create the utility framework where the farmer does not only gain utility from the inter-temporal consumption c_s but also from the inter-temporal repayment a_s. This repayment rate a_s could be regarded as a pre-defined obligation by the loan provider, and the farmer could gain utility when meeting those quantitative obligations. Following this idea, the terminal repayment amounts to D_T due to the full repayment constraint should also contribute to the lifetime utility, while one additional explanation might be that it provides the farmer access to a funding source without any concern of repaying it before the end of the period. Since the initial loan amount is fixed, the difference in the lifetime utility results from the repayment scheme a_s and thus D_T.

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Within such a utility framework, we propose two reward functions under CRRA utility:

$$\begin{aligned}\hat{V}\big(W_t, t, D_t; (c_s, a_s, \boldsymbol{\pi}_s)_{s\in[t,T]}\big) &= E_t\left[\int_t^T e^{-\delta(s-t)}(u(c_s) + u(a_s))\mathrm{d}s + \varepsilon e^{-\delta T}(v(W_T) + v(D_T))\right] \\ &= E_t\left[\int_t^T e^{-\delta(s-t)}\frac{c_s^{1-\gamma}}{1-\gamma}\mathrm{d}s + \varepsilon e^{-\delta T}\frac{W_T^{1-\gamma}}{1-\gamma}\right] \\ &\quad + E_t\left[\int_t^T e^{-\delta(s-t)}\frac{a_s^{1-\gamma}}{1-\gamma}\mathrm{d}s + \varepsilon e^{-\delta T}\frac{D_T^{1-\gamma}}{1-\gamma}\right] \\ &= V\big(W_t, t, 0; (c_s, 0, \boldsymbol{\pi}_s)_{s\in[t,T]}\big) + V\big(D_t, t, 0; (a_s, 0, (0,0))_{s\in[t,T]}\big),\end{aligned} \tag{8}$$

where the second term in last equation is due to $\boldsymbol{\pi}_s = (0{,}0)$ implied by the dynamics of D_t; and

$$\begin{aligned}\bar{V}\big(\tilde{W}_t, t, D_t; (\tilde{c}_s, a_s, \tilde{\boldsymbol{\pi}}_s)_{s\in[t,T]}\big) &= E_t\left[\int_t^T e^{-\delta(s-t)}u(\tilde{c}_s)\mathrm{d}s + \varepsilon e^{-\delta T}v(\tilde{W}_T)\right] \\ &= E_t\left[\int_t^T e^{-\delta(s-t)}\frac{\tilde{c}_s^{1-\gamma}}{1-\gamma}\mathrm{d}s + \varepsilon e^{-\delta T}\frac{\tilde{W}_T^{1-\gamma}}{1-\gamma}\right],\end{aligned} \tag{9}$$

where ε is a positive constant that denotes the relative weight the farmer places on terminal utility, and a high value of ε reflects that the farmer regards terminal wealth as important relative to the inter-temporal consumption. δ stands for the utility discounting factor, while γ is the coefficient of relative risk aversion.

The main difference between these two reward functions is how the repayment rate a_s and D_T are allocated, i.e., $\hat{V}$ assigns individual utility to a_s and D_T term, while $\bar{V}$ treats c_s and a_s as a whole and assigns utility to the whole of the nominal consumption $\tilde{c}_s$ and the nominal terminal wealth $\tilde{W}_T$.

By maximizing these two reward functions at time t, we have two objective function candidates:

$$\hat{J}\Big(W_t, t, D_t\Big) = \sup_{\left(c_s, a_s, \pi_s^L, \pi_s^K\right)\in\mathcal{A}_t} \hat{V}\big(W_t, t, D_t; (c_s, a_s, \boldsymbol{\pi}_s)_{s\in[t,T]}\big), \tag{PS}$$

and

$$\bar{J}\big(\tilde{W}_t, t, D_t\big) = \sup_{\left(\tilde{c}_s, a_s, \tilde{\pi}_s^L, \tilde{\pi}_s^K\right)\in\mathcal{B}_t} \bar{V}\big(\tilde{W}_t, t, D_t; (\tilde{c}_s, a_s, \tilde{\boldsymbol{\pi}}_s)_{s\in[t,T]}\big). \tag{PN}$$

We define the problem (PS) as the *separable* problem and the problem (PN) as *nominal* problem. The nominal problem captures the fact that both net consumption and repayment are drawn from the same pool of nominal wealth and are allowed to influence each other. Furthermore, in the *nominal* problem, net consumption and repayment contribute to the lifetime utility together. This implies that the utility remains unchanged if a certain amount of wealth is reallocated between net consumption and repayment. However, since such re-allocation affects terminal nominal wealth, the farmer faces a trade-off between consumption and repayment, as well as a trade-off between present and future utility when deciding how to allocate them.

In contrast, in the *separable* problem, net consumption and repayment contribute to the lifetime utility through separate utility functions: net consumption is financed from net wealth, while repayment is made from the outstanding loan balance. Consequently, there is no trade-off between consumption and repayment at each point in time. As a result, net consumption and repayment can be treated independently in the *separable* problem, which justifies its decomposition into two independent sub-problems, as shown below.

We observe that $\hat{V}$ is composed of two individual reward functions in the form of V in equation (8), where one is solely affected by c_s and W_T, while the other is affected by a_s and D_T. More

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importantly, the dynamics of W_t and D_t are independent of each other, which means $\hat{J}$ can be further written as

$$\begin{aligned}\hat{J}(W_t,t,D_t) &= \sup_{a_s\in\mathcal{A}_t}\left\{\sup_{\left(c_s,\pi_s^L,\pi_s^K\right)\in\mathcal{A}_t} V\big(W_t,t,0;(c_s,0,\boldsymbol{\pi}_s)_{s\in[t,T]}\big)+V\big(D_t,t,0;(a_s,0,(0,0))_{s\in[t,T]}\big)\right\}\\ &= \sup_{a_s\in\mathcal{A}_t} V\big(D_t,t,0;(a_s,0,(0,0))_{s\in[t,T]}\big)+\sup_{\left(c_s,\pi_s^L,\pi_s^K\right)\in\mathcal{A}_t} V\big(W_t,t,0;(c_s,0,\boldsymbol{\pi}_s)_{s\in[t,T]}\big).\end{aligned} \tag{10}$$

Thus, this optimization problem $\hat{J}$ is solvable if each sub-problem is solvable, which we investigate in Section 3.

However, the nominal problem in (PN) resembles an OBPI problem in the presence of an additional constraint on consumption. To the best of our knowledge and ability, this problem does not have a closed-form solution. This means we will approximate (PN) via (PNA), i.e., by fixing the allocations and consumption while optimizing the repayment, this is:

$$\bar{J}\big(\tilde{W}_t,t,D_t\big) \geq \tilde{J}\big(\tilde{W}_t,t,D_t\big) = \sup_{a_s\in\mathcal{B}_t} \bar{V}\big(\tilde{W}_t,t,D_t;(\tilde{c}_s^*,a_s,\tilde{\boldsymbol{\pi}}_s^*)_{s\in[t,T]}\big), \tag{PNA}$$

where $\tilde{c}_s{}^\star$ and $\tilde{\boldsymbol{\pi}}_t^*$ are the optimal solutions to the net problem (A.2).

3. Analytical solutions

While in this paper, we work with the constraint of certain repayments; we would like to ascertain the impact of repayments on the farm household's welfare. This leads us to the separable and nominal problem described in (PS) and (PN). Although we cannot solve the nominal problem (PN) analytically, we can solve the separable problem (PS) based on equation (10) and proposition Appendix A.1.

Corollary 3.1. *The solution to problem (PS) with wealth dynamics (6) is closed-form, in particular, the life-time utility, the optimal net consumption, repayment rate, asset allocation, and nominal wealth dynamics are, respectively:*

$$\hat{J}(W_t,t,D_t) = \frac{g(t)^\gamma}{1-\gamma}W_t^{1-\gamma}+\frac{h(t)^\gamma}{1-\gamma}D_t^{1-\gamma}.$$

$$c^*(W_t,t) = \frac{W_t}{g(t)}.$$

$$a^*(D_t,t) = \frac{D_t}{h(t)}. \tag{11}$$

$$\boldsymbol{\pi}^* = \frac{1}{\gamma}(\boldsymbol{\sigma\rho\rho}^\top\boldsymbol{\sigma}^\top)^{-1}\tilde{\boldsymbol{\mu}}.$$

$$\begin{aligned}\mathrm{d}\tilde{W}_t^* = &\left\{\tilde{W}_t^*\left[r^B+\frac{1}{\gamma}\Big(1-\frac{D_t}{\tilde{W}_t^*}\Big)\tilde{\boldsymbol{\mu}}^\top(\boldsymbol{\sigma\rho\rho}^\top\boldsymbol{\sigma}^\top)^{-1}\tilde{\boldsymbol{\mu}}\right]-\frac{\tilde{W}_t^*-D_t}{g(t)}-\frac{D_t}{h(t)}\right\}\mathrm{d}t\\ &+\frac{1}{\gamma}(\tilde{W}_t^*-D_t)\tilde{\boldsymbol{\mu}}^\top(\boldsymbol{\sigma\rho\rho}^\top\boldsymbol{\sigma}^\top)^{-1}\boldsymbol{\sigma}\mathrm{d}\boldsymbol{z}_t,\end{aligned}$$

where

$$A = \frac{\delta + r^B(\gamma - 1)}{\gamma} + \frac{1}{2}\frac{\gamma - 1}{\gamma^2}\tilde{\boldsymbol{\mu}}^\top(\boldsymbol{\sigma\rho\rho}^\top\boldsymbol{\sigma}^\top)^{-1}\tilde{\boldsymbol{\mu}},$$
$$B = \frac{\delta + r^B(\gamma - 1)}{\gamma};$$

and

$$g(t) = \frac{1}{A}\left(1 + \left[\varepsilon^{1/\gamma}A - 1\right]e^{-A(T-t)}\right),$$
$$h(t) = \frac{1}{B}\left(1 + \left[\varepsilon^{1/\gamma}B - 1\right]e^{-B(T-t)}\right).$$

Proof. Given equation (10), the separable problem (PS) has been decomposed to two sub-problems where proposition Appendix A.1 could be applied because these sub-problems are in the same form of the net problem (A.2). ■

Corollary 3.1 implies that while the farmer still invests the loan in a risk-free manner, the repayment rate is no longer irrelevant because now it enters into the reward function and the optimal repayment rate is found to be a deterministic function of time t.

4. Empirical analysis

The aim of this empirical analysis is to provide an illustrative application of our model. Here, we first estimate the parameter of the model using publicly available data Australian farm data (Section 4.1). We note that the model is general enough that it can be easily adopted in any jurisdiction. Next, we outline the repayment structures considered and explain the metrics used to compare performances (Section 4.2). Following this, we explore repayment structures for the various problems at hand in Section 4.3. First, given its analytical properties, we provide results for the separable problem, (PS). In Section 4.3.2, we study the nominal problem in (PN) numerically using the approximation in (PNA) for various repayment structures. Moreover, given the lack of a closed-form solution, we also explore and motivate two ansatz for the optimal allocation, consumption, and repayment structure of (PN). This allows us to compare, in Section 4.3.2, of all candidate solutions.

4.1. Parameter calibration

To estimate the parameters in equations (2) and (3), we apply the Maximum Likelihood Method (see, Aldrich, 1997) to the performance data of large-size cropping farms obtained from the Australian Bureau of Agricultural and Resource Economics and Sciences (ABARES).[6] The data set from ABARES contains the average farm business data (including, e.g., land value, capital value, area operated, and depreciation) from 1990 to 2022, with large farms classified as those with total cash receipts (in 2022–23 dollars) greater than \$1,000,000. The estimates are shown in Table 1, and the details are provided in Appendix B).

Since μ^K represents the mean appreciation rate of capital, it is worth noting that our estimate, $\hat{\mu}^K = -0.0971$, is negative and thus captures the depreciation. However, this negative rate should not be misconstrued as a deterrent for investing in capital. In fact, in the net revenue process, the

[6]We attempted MLE on data of small, medium and large size farms across various industries. However, the estimates for the large-size cropping farm are the most reasonable, as both $\hat{\alpha}^L$ and $\hat{\alpha}^K$ are positive. This aligns with the belief that land and capital both have positive effects on the mean of net revenue.

Table 1. The parameter estimates based on ABARES large-size cropping farm performance data

Estimates	Estimated Value	Standard Deviation
Parameters of land price dynamics		
$\hat{\mu}^L$	0.0693	0.0483
$\hat{\sigma}^L$	0.2603***	0.0336
Parameters of capital price dynamics		
$\hat{\mu}^K$	– 0.0971***	0.0259
$\hat{\sigma}^K$	0.0258***	0.0033
Parameters of net revenue dynamics		
$\hat{\alpha}^L$	0.0192	0.0193
$\hat{\alpha}^K$	0.1800***	0.0950
$\hat{\beta}^L$	0.0162***	0.0050
$\hat{\beta}^K$	– 0.1955***	0.0250

Notes: * p<0.10, ** p<0.05, *** p<0.01.

contribution of capital to the drift term, as represented by $\hat{\alpha}^K$, far exceeds that of $\hat{\alpha}^L$. Within the context of the net revenue process's diffusion term, dy_t, the negative value of $\hat{\beta}^K$ implies that capital, when considered as an asset class, can help mitigate the overall risk associated with net revenue. This characteristic makes capital investment quite appealing. On the other hand, the positive value of $\hat{\beta}^L$ signifies the risk associated with land ownership. This positive correlation is consistent with the understanding that larger land holdings generally entail higher risk.

Given the parameter estimates presented in Table 1, we proceed to estimate the correlation matrix by centering the data. The estimates are:

$$\hat{\rho} = \begin{bmatrix} 1 & -0.1118 & -0.0271 \\ -0.1118 & 1 & -0.0757 \\ -0.0271 & -0.0757 & 1 \end{bmatrix}.$$

We see that the stochastic processes described by equations (2) and (3) are negatively correlated. Although most of the literature supports a positive correlation between the land price and capital price (e.g., the land price and "buildings and structures" in Delbecq et al. (2014); the land price and building in Sardaro et al. (2020)), Just and Miranowski (1993) found with the United States data that the increase in land prices can be partially attributed to capital erosion; that is, when the opportunity cost of alternative investments declines, land becomes a comparatively more attractive investment, resulting in increased demand for land and potentially higher land prices. Baldoni and Ciaian (2023) also showed with European data that land value is negatively correlated with the livestock units (which can be regarded as a representative of capital) by coefficient estimates.

In terms of the negative correlations between farm net revenue and land (L) as well as capital (K), Beckman and Schimmelpfennig (2015) found that the land price and the prices paid by farmers (i.e., the farm input prices) have a negative impact on the farm net revenue. Deaton and Lawley (2022) also indicated a potential negative correlation between net revenue and the value of land and buildings in certain regions of Canada.

For the numerical analysis, the borrowing rate is calibrated to be $r^B = 0.04$ by Bayraktar and Young (2007), Livshits et al. (2007), and Nakajima (2017). The inter-temporal utility discount factor is $\delta = 0.02$ (see, for example, Shin et al., 2007; Zeng et al., 2016; Lichtenstern et al., 2021; Gong and Li, 2006), who use a range between 0.005 and 0.05). The relative risk aversion is set to

Table 2. The factors of optimal strategy based on the estimates in Table 1

Factors of the Optimal Strategy	Value
π^L	37.32%
π^K	55.06%
A	0.0404
B	0.0300

$\gamma = 2$ to reflect a moderate level of risk aversion by Lim et al. (2008), which is also close to the estimate $\gamma = 2.211$ from Pennings and Garcia (2001). The initial wealth of the farmer is $W_0 = 100,000$, with an initial debt amount of $D_0 = 50,000$, and the maximum borrowing period is set at $T = 20$ years, which means that the loan should be repaid fully within this time span. The relative weight on terminal utility, as denoted by ε, is set to be 1, implying equal weight by the farmer on inter-temporal consumption and terminal wealth.

As per Proposition Appendix A.1 and Corollary 3.1 and with the parameter setting discussed above, the optimal net investment strategy $\boldsymbol{\pi}$ (with regards to L and K) to net problem (A.2) and the separable problem (PS), along with the values of A and B associated with $g(t)$ and $h(t)$ respectively, are shown in Table 2. Corollary 3.1 demonstrates that the *separable* problem can be decomposed into two sub-problems, where $\boldsymbol{\pi}$ appears only in the reward function of the net problem, as shown in equation (8). Therefore, the optimal $\boldsymbol{\pi}$ for the *separable* problem is solely determined by, and exactly the same as, the optimal $\boldsymbol{\pi}$ for the net problem.

In the *separable* problem (PS), the farmer's optimal investment strategy is to allocate 37.32% and 55.06% of his net wealth to land and capital, respectively. Excluding these investments, the remainder of his nominal wealth is invested in the cash account. Since the sum of π^L and π^K is less than 1, this result implies that the loan is effectively invested in a risk-free manner, which is due to the full repayment requirement. As investment in land and capital is risky, it is possible that the farmer's terminal nominal wealth $\tilde{W}_T$ will fall below the terminal loan amount D_T if he invests more than his net wealth in risky assets.

4.2. Repayment structures and optimization problems

This section outlines the repayment structures that we explore in the subsequent sections. In addition to the widely used constant repayment structure, we investigate various contingent and hybrid repayment structures. We conclude this section with a description of our comparison metric, the equivalent initial net wealth, which we use in the numerical analysis in Section 4.3.

The contingent repayment structures are based on the works of Botterill and Chapman (2004); Botterill and Chapman (2009) and are designed according to the concept that the repayment rate is proportional to the mean of a specific type of earnings, such as capital gain or revenue. On the other hand, the hybrid structure combines both constant and contingent components, forming a more comprehensive class of repayment structures. The intuition is that by making (a portion of) the repayment proportional to earnings, the farmer can reduce their repayments during periods of lower earnings and increase them when earnings are higher, thereby achieving a consumption-smoothing effect.

In this study, we consider two types of earnings, capital gains, and net revenue, and create contingent repayment structures based on them.[7] We will provide the definitions and then derive the dynamics for these structures. The instantaneous capital gain before consumption, denoted as dG_t, is defined as:

[7] We also attempted the contingent structures where the repayment rate is proportional to earnings directly. However, the performance of those structures is inadequate due to the high volatility of repayment.

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Table 3. The repayment structures

Loan Scheme	Repayment Structure
Constant (C)	$a_t^C = b_1$
Capital Gain Contingent (CG)	$a_t^{CG} = b_2 \mathbb{1}_{\{t\leq\tau\}} \tilde{W}_t[r^B + \tilde{\boldsymbol{\pi}}_t^\top \tilde{\boldsymbol{\mu}}]$
Revenue Contingent (R)	$a_t^R = b_2 \mathbb{1}_{\{t\leq\tau\}} \tilde{W}_t \tilde{\boldsymbol{\pi}}_t^\top \boldsymbol{\alpha}$
Hybrid of Constant and Capital Gain Contingent (HC)	$a_t^{HC} = \mathbb{1}_{\{t\leq\tau\}}(b_1 + b_2 \tilde{W}_t[r^B + \tilde{\boldsymbol{\pi}}_t^\top \tilde{\boldsymbol{\mu}}])$
Hybrid of Constant and Revenue Contingent (HR)	$a_t^{HR} = \mathbb{1}_{\{t\leq\tau\}}(b_1 + b_2 \tilde{W}_t \tilde{\boldsymbol{\pi}}_t^\top \boldsymbol{\alpha})$

$$\mathrm{d}G_t = \tilde{W}_t\left[r^B + \tilde{\boldsymbol{\pi}}_t^\top \tilde{\boldsymbol{\mu}}\right]\mathrm{d}t + \tilde{W}_t \tilde{\boldsymbol{\pi}}_t^\top \boldsymbol{\sigma} \mathrm{d}\boldsymbol{z_t}, \tag{12}$$

Rewriting the net revenue process in terms of $\tilde{W}_t$ and $\tilde{\pi}_t$, we have:

$$\begin{aligned}\mathrm{d}y_t &= \left(\alpha^L p_t^L L_t + \alpha^K p_t^K K_t\right)\mathrm{d}t + \left(\beta^L p_t^L L_t + \beta^K p_t^K K_t\right)\mathrm{d}z_t^y \\ &= \tilde{W}_t\left[\left(\alpha^L \tilde{\pi}_t^L + \alpha^K \tilde{\pi}_t^K\right)\mathrm{d}t + \left(\beta^L \tilde{\pi}_t^L + \beta^K \tilde{\pi}_t^K\right)\mathrm{d}z_t^y\right] \\ &= \tilde{W}_t\left[\tilde{\boldsymbol{\pi}}_t^\top \boldsymbol{\alpha}\mathrm{d}t + \tilde{\boldsymbol{\pi}}_t^\top \boldsymbol{\beta}\mathrm{d}z_t^y\right],\end{aligned} \tag{13}$$

where $\boldsymbol{\alpha} = (\alpha^L, \ \alpha^K)^\top$ and $\boldsymbol{\beta} = (\beta^L, \ \beta^K)^\top$. We note that given the stochastic nature of the repayments, the exact time of full repayment is unknown. Thus, to formulate the expressions for each repayment type, it is necessary to introduce the concept of a repayment stopping time, τ:

$$\tau = \inf\{t : D_t = 0\}.$$

The repayment structures we consider are summarized in Table 3, where b_1 and b_2 serve as descriptive parameters. The Capital Gain Contingent and Revenue Contingent repayment structures in our study essentially link the repayment rate to the mean level of instantaneous capital gain and revenue, as shown in equations (12) and (13) respectively. Note, for each of the repayment structures, the descriptive parameters, b_1, b_2, are our control variables, and their values are determined numerically.

Based on the decomposition of the separable problem (PS), it is apparent that irrespective of the chosen repayment strategy, the optimal consumption and investment strategies remain unchanged. This happens since the repayment rate solely impacts the dynamics of D_t and not that of W_t. Therefore, for any repayment structure a^j, we can reformulate the optimization problem (PS) as follows:

$$\hat{J}\left(W_0, t=0, D_0; \left(a_s^j\right)_{s\in[0,T]}\right) = \sup_{\boldsymbol{b}} \hat{V}_t\left(W_0, t=0, D_0; \left(c_s^*, a_s^j, \boldsymbol{\pi_s^*}\right)_{s\in[0,T]}\right),$$

where $\boldsymbol{b}$ is the descriptive parameter vector of the structure a^j, while $\boldsymbol{\pi}_s{}^*$ and $c_s{}^*$ are the optimal solution from Corollary 3.1.

However, given that the nominal problem (PN) is not analytically solvable, the optimization problem for each repayment structure is addressed numerically using (PNA). As a reminder, this involves applying the optimal solutions to nominal consumption and investment in problem (A.2). Consequently, expressing J, specific to the repayment structure a^j:

$$\tilde{J}\left(\tilde{W}_0, t=0, D_0; \left(a_s = a_s^j\right)_{s\in[t,T]}\right) = \sup_{\mathbf{b}} \bar{V}\left(\tilde{W}_0, t=0, D_0; (\tilde{c}_s^*, a_s, \tilde{\boldsymbol{\pi}}_s^*)_{s\in[t,T]}\right),$$

where $\boldsymbol{b}$ is again the descriptive parameter vector of repayment structure a^j.

For illustrative purposes, we present in Table (4) the investment strategies and nominal consumption at time 0 for the inner optimization problem of J in equation (PNA). It should be noted that both $\tilde{\boldsymbol{\pi}}_t$ and $\tilde{c}_t$ will vary over time. Since the optimal investment strategy in the *separable*

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Table 4. The optimal nominal strategy based on the estimates in Table 1 at time $t = 0$

The Optimal Nominal Strategy	Value
${\tilde{\pi}_0}^L$	24.88%
${\tilde{\pi}_0}^K$	36.71%
${\tilde{\pi}_0}^B$	38.41%
$\tilde{c}_0$	7062.38 + a_0

problem is also applied in the *nominal* problem, the strategy shown in Table (4) is effectively the same as that in Table 2, albeit expressed in different nominal terms. Following the same logic, we can see that the loan is invested in the cash account to ensure it can be fully repaid at the terminal date. The initial optimal net consumption rate, $c_0 = 7062.38$ (7.1 % of initial wealth), can be interpreted, for simplicity, as the total net consumption in the first year, as $g(t)$ does not vary significantly over a short time period. However, a_0 does depend on the specific repayment structure.

We conclude this section with a discussion of the comparison metric used in the numerical analysis. In the following, we select the constant repayment structure as the baseline. This choice is motivated by its broad use in the real world and ease of understanding as well as the fact that it appears in all of our analyses.

While our problem formulation involves utility maximization, the utility values themselves are inherently subjective and hard to interpret as they have no intuitive meaning (see, for example (Butt and Khemka, 2015)). Consequently, researchers have devised numerous measures to quantify utility outcomes in an easier-to-understand manner. For example, Cocco et al. (2005) and Khemka et al. (2024) use certainty equivalent consumption to compare utility outcomes of different strategies, with results being represented as either the certainty equivalent values or its change over different strategies. Other studies have used changes in certain variables such that the utility is the same as that of the benchmark. For example, Huang et al. (2025) employ the extra management fee that could be charged such that the utility from different investment strategies are the same. Our metric stems from the latter strand of literature, where we calculate the equivalent initial net wealth such that the utility for a given repayment structure is the same as that of the baseline.

For each repayment structure denoted as a^j, we identify an equivalent initial net wealth value, denoted as ${W_0}^C$, that enables the farmer to attain the same lifetime utility as under the constant repayment structure a^C, i.e.,

$$\hat{J}\left(W_0, t = 0, D_0; \left(a_s = a_s^j\right)_{s\in[0,T]}\right) = \hat{J}\left(W_0^C, t = 0, D_0; \left(a_s = a^C\right)_{s\in[0,T]}\right)$$

for problem (PS) and

$$\tilde{J}\left(W_0 + D_0, t = 0, D_0; \left(a_s = a_s^j\right)_{s\in[0,T]}\right) = \tilde{J}\left(W_0^C + D_0, t = 0, D_0; \left(a_s = a^C\right)_{s\in[0,T]}\right)$$

for problem (PNA).

We also define, ΔW_0, as the difference between ${W_0}^C$ and W_0:

$$\Delta W_0 = W_0^C - W_0, \qquad (14)$$

to measure the enhancement or deterioration of a particular repayment structure compared with the constant one expressed in monetary units.

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4.3. Numerical results

In this section, we first provide the numerical output for the separable problem and then delve into the solution of the nominal problem for each repayment structure outlined in Table 3, first using (PNA), and then directly with (PN), all based on a dataset comprising 10, 000 simulations.

4.3.1. Results for separable problem (PS)

Given the form of the reward function (A.1) in the separable problem, which assigns utility to the repayment rate a_t and the terminal loan amount D_T directly, the reward function achieves negative infinity when either a_t or D_T equals zero. This scenario can arise in contingent repayment structures when the loan is fully repaid before time T. Consequently, we cannot evaluate either contingent or hybrid repayment structures under the separable problem, and only deterministic repayment structures can be assessed.

Hence, we limit the comparison between the optimal repayment structure obtained in Corollary 3.1 against a numerically optimized constant repayment structure.[8] The results are presented in Table 5 and Figure 1. While, visually, the optimal repayment structure exhibits an approximately linear increasing pattern, it is in fact non-linear, as indicated by its functional form ($a_t = D_t/h(t)$) in Corollary 3.1.[9] Moreover, we note that the amounts being paid during the term of the loan are quite close for the two repayment structures, though there is a significant difference between the terminal loan repayment amounts. The comparison also shows that the optimal repayment structure outperforms the constant one by approximately 0.78% in terms of equivalent initial wealth. This is an important result as it shows that the "optimal" constant repayment strategy is not "too sub-optimal" for the separable problem.

4.3.2. Results for nominal problem, (PN) and (PNA)

As we discussed before, the nominal problem (PN) is not analytically solvable. Hence, the true "optimal" solution of the form of Corollary 3.1 is unavailable.[10] In this section, we first introduce and motivate two ansatz as candidates for solutions of (PN). We then compare all proposals, that is, the two ansatz mentioned above, as well as the optimal solutions to (PNA) based on the repayment structures in Table 3.

For our first ansatz candidate solution, we draw inspiration from the form of the optimal consumption and repayment rate derived for the net problem (A.2). We postulate that the optimal nominal consumption in the nominal problem (PN) might exhibit a similar functional representation or can be approximated. The functional form of the solution could be written as:

$$\tilde{c}^S(W_t, D_t, t) = \frac{W_t}{f(t)} + \frac{D_t}{f(t)}.$$

The intuition behind this proposal is that as γ approaches zero, the nominal problem (PN) tends to converge toward the separable problem (PS).

Now, if we consider an approximation of $f(t)$ as $\kappa g(t)$, where $g(t)$ is obtained from Proposition Appendix A.1, and κ is a measure of the degree of similarity between $f(t)$ and $g(t)$, then the speculated optimal repayment rate for the nominal problem (PN) would be:

---

[8]The constant repayment structure can take many forms, from equal repayments in every period to no repayment over the life of the loan and full repayment at the end. The optimization procedure identifies the form of the repayment structure that maximizes the underlying reward function.

[9]We also note that approximately linear increasing pattern is a function of the current parametrization used in the analysis. Further investigations show that increasing the utility discount factor parameter, δ, to 0.04 leads a visually non-linear repayment structure.

[10]Searching for optimal numerical solutions in continuous-time models is quite time consuming and could be inaccurate. This is an avenue for exploration in future research.

Table 5. The characteristics of repayment structures and their equivalent initial net wealth $W_0{}^C$ in the separable problem

Repayment Structure	Optimal Descriptive Parameter b_1	D_T	$W_0{}^C$	ΔW
Constant (C)	3504.95	1757.79	100, 000.00	0
Optimal Solution	-	3884.24	100, 779.50	779.50

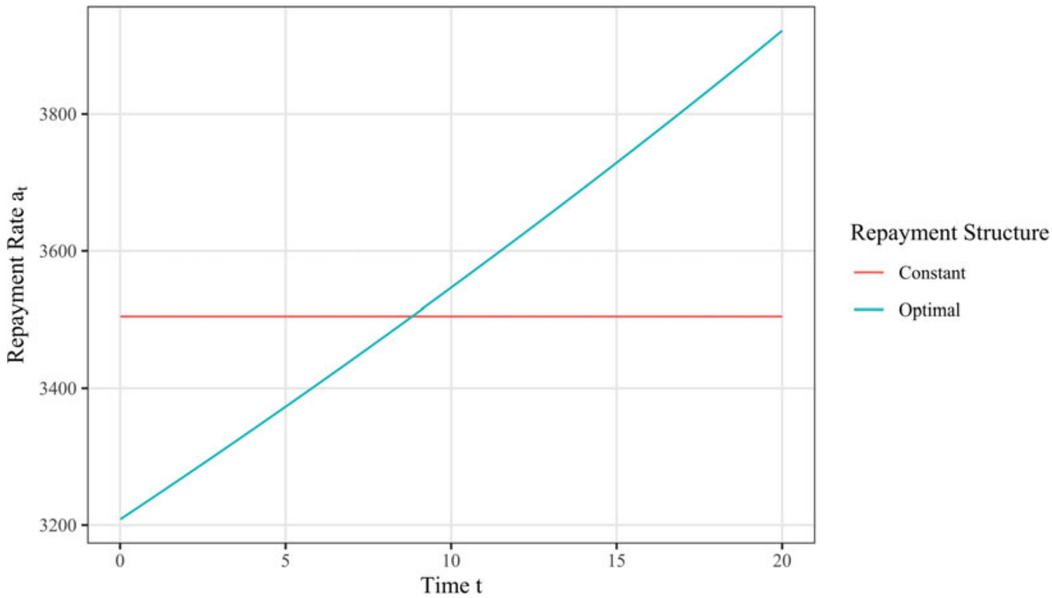

Figure 1. The repayment rates of the constant and optimal repayment structure in the separable problem.

$$\begin{aligned} a^S(W_t, D_t, t) &= \tilde{c}^S(W_t, D_t, t) - c^*(W_t, t) \\ &= \frac{W_t}{f(t)} + \frac{D_t}{f(t)} - \frac{W_t}{g(t)} \\ &= W_t\left(\frac{1}{\kappa g(t)} - \frac{1}{g(t)}\right) + \frac{D_t}{\kappa g(t)}. \end{aligned} \tag{15}$$

We call this the "Speculated Solution (S)."

For the second ansatz candidate solution, we introduce a two-stage solution for net consumption and repayment. We propose that, initially, before the loan is fully repaid, the farmer would focus solely on repaying the loan and set the net consumption to 0. Although the approach might not seem attractive to farmers due to the zero net consumption while the loan is being repaid, it signals a more rewarding strategy of minimal consumption at first. This aligns the theoretical solution to the optimal level derived from the net problem where we consider $W_t + D_t$ as the net wealth value according to net problem (A.2) and its solution provided in Proposition Appendix A.1. Once the loan is fully repaid in the subsequent stage, the nominal problem (PN) transforms into the net problem. In both stages, the constant proportion investment strategy, which is the optimal solution to the net problem, is applied to the net wealth W_t.

This two-stage solution ensures that the constraints in the nominal problem (PN) are satisfied. Note that "net" wealth is invested optimally (which must be superior to the risk-free strategy) while debt is invested in the risk-free asset. By regarding consumption and repayment as two outflows from W_t and D_t, respectively, the farmer contributes to the nominal consumption from

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Table 6. The characteristics of different repayment structure and their equivalent initial net wealth $W_0{}^C$ in problem (PNA) as an approximation of the nominal problem (PN)

Repayment Structure	Optimal Descriptive Parameters b_1	b_2	κ	Median of D_T	$W_0{}^C$	ΔW
Constant (C)	3530.17	-	–	1410.62	100, 000.00	0
Speculated Solution (S)	-	-	0.9812	2534.37	100, 050.12	50.12
Two-Stage Strategy (TS)	-	-	–	0	102, 521.83	2, 521.83
Capital Gain Contingent (CG)	-	0.5285	–	0	94, 924.75	− 5, 075.25
Revenue Contingent (R)	-	0.5018	–	0	93, 166.55	− 6, 833.45
Hybrid of Constant and Capital Gain Contingent (HC)	3462.04	0.0010	–	2798.95	100, 008.79	8.79
Hybrid of Constant and Revenue Contingent (HR)	3482.70	0.0066	–	2749.78	100, 006.11	6.44

the repayment as much as possible in the early stage to grow net wealth and thus achieve a higher level of expected nominal wealth overall. We call this the "Two-stage Strategy (TS)."

Table 6 provides the output of the numerical investigation for the nominal problem. The TS achieves the highest performance amongst all the repayment structures and outperforms the constant repayment strategy by 2.5 percent. Interestingly, in the speculated solution (S), the optimal value of κ is approximately 0.9812, which is very close to 1 and aligns with the hypothesis on the form of the repayment structure. Note that this repayment structure would initially be dominated by its deterministic component, $\frac{D_t}{\kappa g(t)}$. However, since $g(t)$ is a decreasing function, the impact of the stochastic component $W_t\left(\frac{1}{\kappa g(t)} - \frac{1}{g(t)}\right)$ becomes increasingly significant over time.

Although the performance of S is inferior to TS, it outperforms the hybrid structures where the enhanced flexibility only yields marginal improvements compared to the constant repayment structure. It is worth noting that the optimal b_2 values in the hybrid structures are relatively small, suggesting that the impact of the contingent components should be minimal. This observation aligns with the fact that both contingent repayment structures exhibit significantly poor performance.

We also perform sensitivity analysis for the two ansatz candidate solutions in Appendix C. The sensitivity analysis reveals that the performance of these strategies improves as the utility discount factor, δ decreases and the risk-free rate, r^B, increases.

The median values of the various repayment rates over time are illustrated in Figure 2. We see that the hybrid repayment structures closely resemble the constant repayment structure, as evidenced by the relatively small scale of b_2 in these hybrid structures. On the other hand, the speculated solution (S) deviates more from the constant repayment structure as it approaches the terminal date.

In contrast, the two-stage, optimal capital gain, and revenue contingent repayment structures exhibit a significantly wider range of repayment rates over time. The median paths show that early termination of the loan is highly likely under these three repayment structures. It could be seen that the median repayment period of the loan under the two-stage repayment structure is at about 5 years, while those of the capital gain and revenue contingent structures are at approximately 12.5 and 14 years, respectively.

Apart from the constant repayment structure, the repayment amounts within all the other repayment structures are path-dependent and, hence, stochastic in nature. Thus, the terminal repayment amount at time T, D_T is a random variable in these repayment structures, whose distributions are shown in Figure 3, while Table 6 provides the median values. Given the high likelihood of early repayment for the capital gain (CG) and revenue (R) contingent repayment structures, we see D_Ts centered around 0 for these structures. Note that given the nature of the TS,

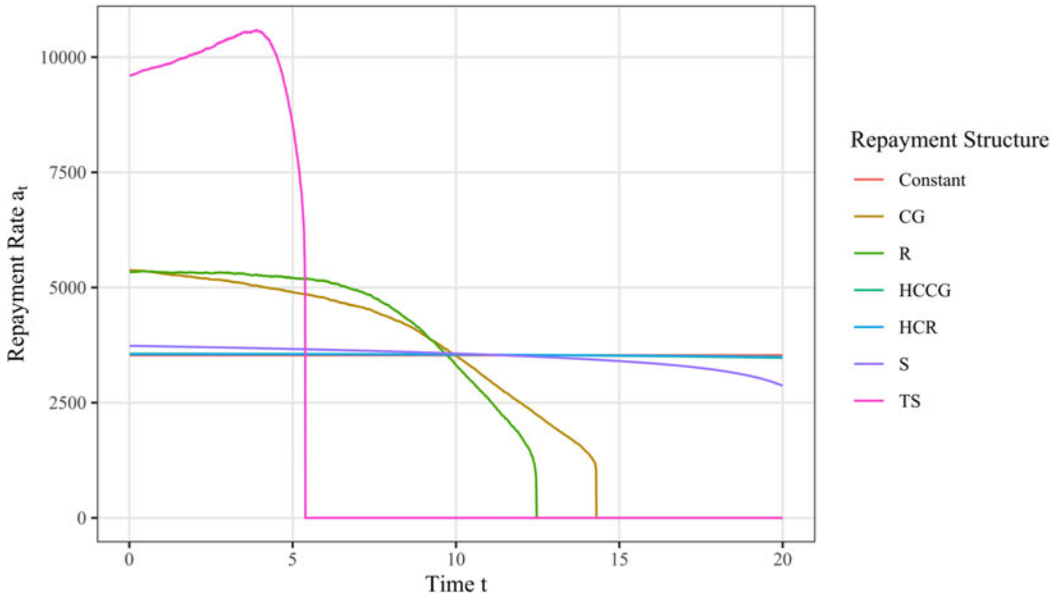

Figure 2. The median repayment rates of the optimal repayment structures in problem (PNA) over time.

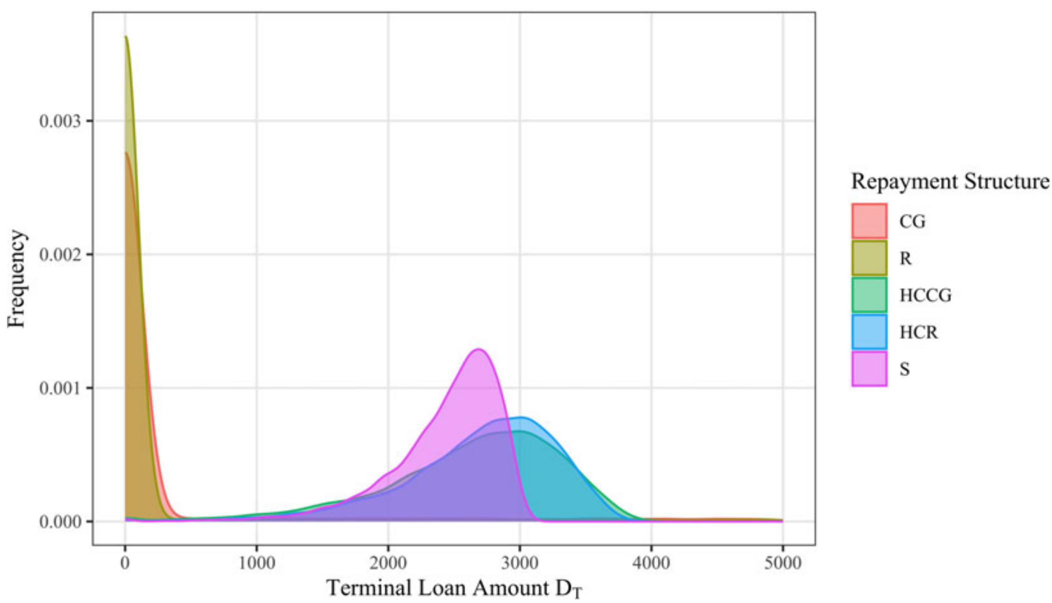

Figure 3. The distribution of terminal outstanding loan amount of the optimal repayment structures in problem (PNA) as an approximation to the nominal problem (PN).

early repayment is certain. This affects the scale of the graphs and is omitted for visualization purposes from Figure 3. We find that the hybrid structures (HCCG and HCR) have a similar distribution with medians of approximately $2,800. The speculated solution (S) also has a non-zero D_T but has a much narrower range and a slightly lower mean than the hybrid structures.

Given the numerical illustration above for the nominal problem, in the presence of a full repayment guarantee constraint, the two-stage repayment structure demonstrates a substantial

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outstanding performance, even though it entails net consumption being 0 until the loan is entirely repaid. Importantly, we see that the constant repayment structure significantly outperforms the contingent structures and attains a performance level comparable to the hybrid structures and the speculated solution (S), indicating that it is not "too sub-optimal." This is particularly appealing due to its straightforward structure and widespread use in the real world. A more practical scenario may allow for the financial risk that the farmer might default on the loan, which could serve as an advanced topic for future research.

5. Conclusion

In the first part of this study, we introduce a theoretical model for understanding the dynamics of a farmer's wealth in the presence of a loan. While the net revenue process depends on land and capital investments, the flexibility of its parameters allows us to capture various features of net revenue, such as diminishing marginal returns. We then propose two distinct reward functions that the farmer might seek to maximize, leading to the formulation of two constrained optimization problems: the separable problem and the nominal problem.

This paper solves the separable problem analytically in the presence of a full repayment guarantee. By doing so, we extend the classical results of Merton to a more flexible revenue process. The optimal investment strategy shows that the farmer should invest the loan amount in a risk-free manner, while the optimal repayment structure is deterministic and exhibits an approximately linear increasing pattern over time.

The nominal problem is our main objective, where deterministic, contingent, and hybrid repayment structures can be evaluated. The nominal problem is superior to the separable problem due to its flexibility, which allows us to investigate any repayment structure. Unfortunately, the nominal problem does not have a closed-form solution to the best of the authors' knowledge, and thus, the optimal repayment rates are found numerically. Additionally, we introduce two ansatz candidates for the optimal solution. The first is inspired by the optimal solution in the net problem, and the second is a two-stage solution derived from optimal investment theory. These solutions outperform all other structures, with the two-stage solution achieving significantly higher results.

An interesting result of our analysis is the performance of the widely used constant repayment structure. In both problems, the constant repayment performs relatively well and significantly outperforms the contingent structures. This shows that the constant repayment structure is not too sub-optimal. This is insightful to policymakers and lenders considering alternative repayment structures, albeit with a full repayment guarantee.

Our study allows for improvements to be addressed in future research. For instance, we assume full repayment of the loan without incorporating credit risk. The production function is modeled as a function of land and capital, excluding other economic factors such as commodity prices. Additionally, we assume farm assets are perfectly liquid, which might not be realistic for some assets. Moreover, we note that our conclusion of the comparable out-performance of the simpler repayment structures of the constant repayment schedule and the second Ansatz candidate solution is likely a byproduct of the problem formulation: the choice of the risk-averse utility function and our approximation to solution of the nominal problem, where we use the net consumption from the inner problem. Future research could explore solving the nominal problem holistically to generate a combined consumption and repayment policy functions, albeit, we suspect that it requires a significantly more complicated model that allows for credit risk in the model to generate a closed-form solution.

Data availability statement. The data that support the findings of this study are openly available in the Australian Bureau of Agricultural and Resouce Economics and Sciences (ABARES), at https://www.agriculture.gov.au/sites/default/files/documents/fdpperformance-by-size.csv.

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Appendix A. Motivation for objective function and preliminary result

To achieve this goal, we start with the reward function in (A.2) for the net wealth under a widely used utility framework. This will serve as a building block for addressing the reward functions of interest in (PS) and (PN).

If the farmer is assumed to gain utility from nominal consumption $\tilde{c}_s$, and nominal terminal wealth $\tilde{W}_T$, following a well-known Merton's problem setting with a hyperbolic absolute risk aversion (HARA) utility assumption (Merton, hereafter, Merton, 1969, 1975), we first consider the reward function below:

$$\begin{aligned} V\big(\tilde{W}_t, t, D_t; (\tilde{c}_s, a_s, \tilde{\boldsymbol{\pi}}_s)_{s\in[t,T]}\big) &= E_t\left[\int_t^T e^{-\delta(s-t)} u(\tilde{c}_s) \mathrm{d}s + \varepsilon e^{-\delta T} u(\tilde{W}_T)\right] \\ &= E_t\left[\int_t^T e^{-\delta(s-t)} \frac{(\tilde{c}_s - a_s)^{1-\gamma}}{1-\gamma} \mathrm{d}s + \varepsilon e^{-\delta T} \frac{(\tilde{W}_T - D_T)^{1-\gamma}}{1-\gamma}\right] \\ &= E_t\left[\int_t^T e^{-\delta(s-t)} \frac{(c_s - 0)^{1-\gamma}}{1-\gamma} \mathrm{d}s + \varepsilon e^{-\delta T} \frac{(W_T - 0)^{1-\gamma}}{1-\gamma}\right] \\ &= V\big(W_t, t, 0; (c_s, 0, \boldsymbol{\pi}_s)_{s\in[t,T]}\big), \end{aligned} \tag{A1}$$

where ε is a positive constant that denotes the relative weight the farmer places on terminal utility, and a high value of ε reflects that the farmer regards terminal wealth as important relative to the inter-temporal consumption. δ stands for the utility discounting factor, while γ is the coefficient of relative risk aversion.

It can be seen from equation (A.1) that the HARA construction on nominal terms is linked to a constant relative risk aversion (CRRA) on net terms, which implies that the farmer only gains utility from net consumption c_s and terminal wealth W_T. Thus, the objective function under (A.1) could be expressed as

$$\begin{aligned} J\big(\tilde{W}_t, t, D_t\big) &= \sup_{\left(\tilde{c}_s, a_s, \tilde{\pi}_s^L, \tilde{\pi}_s^K\right)\in\mathcal{B}_t} V\big(\tilde{W}_t, t, D_t; (\tilde{c}_s, a_s, \tilde{\boldsymbol{\pi}}_s)_{s\in[t,T]}\big) \\ &= \sup_{\left(c_s, \pi_s^L, \pi_s^K\right)\in\mathcal{A}_t} V\big(W_t, t, 0; (c_s, 0, \boldsymbol{\pi}_s)_{s\in[t,T]}\big) \\ &= J(W_t, t, 0). \end{aligned} \tag{A2}$$

where the second equality holds from the fact that when D_t and a_s become 0, the set $\mathcal{B}_t$ would be the same as $\mathcal{A}_t$. We name this as the *net* problem.

In this section, we will show that the net problem (A.2) has an analytical solution which could be interpreted as the farmer setting aside the loan and acting as per "Merton's solution" in the remaining wealth.[11] This analytical solution would be the same for any repayment structure, which implies that the net problem does not value the repayment structure and, thus, is not appropriate for the purpose of our study. Nonetheless, the optimal net consumption and net allocation obtained here will prove helpful in deriving the solution for problem (PS) and approximating the solution for problem (PN).

With the relationships $\tilde{W}_t = W_t + D_t$ and $\tilde{c}_t = c_t + a_t$, the solution to the constrained utility maximization net problem (A.2) can be expressed in nominal terms:

Proposition Appendix A.1. *If the parameters α and β in the revenue process (3) satisfy the relation for λ in equation (A.12), the solution to the net problem (A.2) with wealth dynamics (6) will be:*

[11]Our solution is an actual generalization of Merton's solution as we have allowed for a more flexible revenue process.

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the life-time utility is

$$J(\tilde{W}_t, t, D_t) = \frac{g(t)^\gamma}{1-\gamma}(\tilde{W}_t - D_t)^{1-\gamma}. \tag{A3}$$

the optimal nominal consumption rate is

$$\tilde{c}^*(\tilde{W}_t, t) = \frac{\tilde{W}_t - D_t}{g(t)} + a_t. \tag{A4}$$

the optimal asset allocation strategy is

$$\tilde{\boldsymbol{\pi}}^*(\tilde{W}_t, t) = \frac{1}{\gamma}\left(1 - \frac{D_t}{\tilde{W}_t}\right)(\boldsymbol{\sigma\rho\rho}^\top\boldsymbol{\sigma}^\top)^{-1}\tilde{\boldsymbol{\mu}}. \tag{A5}$$

the optimal nominal wealth dynamics is

$$\begin{aligned} \mathrm{d}\tilde{W}_t^* = &\left\{\tilde{W}_t^*\left[r^B + \frac{1}{\gamma}\left(1 - \frac{D_t}{\tilde{W}_t^*}\right)\tilde{\boldsymbol{\mu}}^\top(\boldsymbol{\sigma\rho\rho}^\top\boldsymbol{\sigma}^\top)^{-1}\tilde{\boldsymbol{\mu}}\right] - \frac{\tilde{W}_t^* - D_t}{g(t)} - a_t\right\}\mathrm{d}t \\ &+ \frac{1}{\gamma}(\tilde{W}_t^* - D_t)\tilde{\boldsymbol{\mu}}^\top(\boldsymbol{\sigma\rho\rho}^\top\boldsymbol{\sigma}^\top)^{-1}\boldsymbol{\sigma}\mathrm{d}\boldsymbol{z}_t, \end{aligned} \tag{A6}$$

where

$$A = \frac{\delta + r^B(\gamma - 1)}{\gamma} + \frac{1}{2}\frac{\gamma - 1}{\gamma^2}\tilde{\boldsymbol{\mu}}^\top(\boldsymbol{\sigma\rho\rho}^\top\boldsymbol{\sigma}^\top)^{-1}\tilde{\boldsymbol{\mu}}, \tag{A7}$$

and

$$g(t) = \frac{1}{A}\left(1 + \left[\varepsilon^{1/\gamma}A - 1\right]e^{-A(T-t)}\right). \tag{A8}$$

Proof. We can see from equation (7) that the dynamics of net wealth is not related to the amount of outstanding loan amount D_t or the instantaneous repayment $\mathrm{d}R_t$, which enables us to derive an elegant generic solution. Since the constraint about $\tilde{c}$ in the net problem (A.2) is $\tilde{c}_t > a_t$, the corresponding constraint of c_t becomes $c_t > 0$, which is effectively redundant due to the nature of maximizing the CRRA utility function $u(\cdot)$. Similarly, $W_t > 0$ in the net problem (A.2) would also be not binding by the nature of maximizing the CRRA utility function $u(\cdot)$. On the other hand, the proportions $\pi^i = \tilde{\pi}^i \frac{\tilde{W}}{W}$ for $i \in \{L, K\}$ are unconstrained since $\tilde{\pi}^i \in \mathcal{R}$ and $\frac{\tilde{W}}{W} > 1$ would result in $\pi^i \in \mathscr{R}$, which is not constrained. Thus, the feasible set for $\{c, \pi^L, \pi^K\}$ would be equivalent to the unconstrained set $\mathcal{A}_t$.

Then, the HJB equation of (A.2) is given by

$$\delta J(W,t) = \mathcal{L}^c J(W,t) + \mathcal{L}^{\boldsymbol{\pi}} J(W,t) + \frac{\partial J}{\partial t}(W,t) + r^B W J_W(W,t), \tag{A9}$$

where

$$\mathcal{L}^c J(W,t) = \sup_{c \geq 0}\{u(c) - cJ_W(W,t)\}, \tag{A10}$$

and

$$\mathcal{L}^{\boldsymbol{\pi}} J(W,t) = \sup_{\boldsymbol{\pi}}\left\{WJ_W(W,t)\boldsymbol{\pi}_t^\top\tilde{\boldsymbol{\mu}} + \frac{1}{2}J_{WW}(W,t)W^2\boldsymbol{\pi}_t^\top\boldsymbol{\sigma\rho\rho}^\top\boldsymbol{\sigma}^\top\boldsymbol{\pi}_t\right\}. \tag{A11}$$

Since $\boldsymbol{\rho}$ is non-singular, the matrix $\boldsymbol{\sigma\rho\rho}^\top\boldsymbol{\sigma}^\top$ would be real-valued symmetric positive-definite, and thus has a unique Cholesky decomposition, i.e.

$$\bar{\boldsymbol{\sigma}}\bar{\boldsymbol{\sigma}}^\top = \boldsymbol{\sigma\rho\rho}^\top\boldsymbol{\sigma}^\top,$$

where $\bar{\boldsymbol{\sigma}}$ is a lower triangular matrix with real and positive diagonal entries.

We could then define ϕ_t and $\boldsymbol{\lambda}$ in the following way:

$$\begin{aligned} \phi_t^\top &= \boldsymbol{\pi}_t^\top\bar{\boldsymbol{\sigma}}, \\ \boldsymbol{\lambda} &= \bar{\boldsymbol{\sigma}}^{-1}\tilde{\boldsymbol{\mu}} \end{aligned} \tag{A12}$$

It should be noted that λ should be a constant vector to make the following calculations work. Importantly, equation (A.12) conveys a relationship between α and β. To see this, take $\rho = I$, then we get $\alpha^L = \lambda_1\sqrt{(\sigma^L)^2 + (\beta^L)^2} - \mu^L + r^B$ and $\alpha^K = \lambda_2\sqrt{(\sigma^K)^2 + (\beta^K)^2} - \mu^K + r^B$. This implies β could be any state-dependent process, and consequently, α will be a similar state-dependent process, and vice versa. In particular, if β is constant, then α should also be constant, as per the numerical section of this paper.

Rewriting the HJB equation (A.11) with $\boldsymbol{\varphi}_t$ as the control:

$$\mathcal{L}^{\phi}J(W,t) = \sup_{\phi}\left\{ WJ_W(W,t)\boldsymbol{\phi}_t^{\top}\boldsymbol{\lambda} + \frac{1}{2}J_{WW}(W,t)W^2\boldsymbol{\phi}_t^{\top}\boldsymbol{\phi}_t \right\}. \tag{A13}$$

The first-order condition for the maximization of (A.10) is

$$u'(c) = J_W(W,t), \tag{A14}$$

Since $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ and $u'(c) = c^{-\gamma}$, we can solve for the optimal consumption c from equation (A.14):

$$c^* = (J_W(W,t))^{-\frac{1}{\gamma}}. \tag{A15}$$

Substituting c^* back to equation (A.10), we have

$$\mathcal{L}^c J(W,t) = u(c^*) - c^* J_W(W,t) = \frac{\gamma}{1-\gamma}J_W(W,t)^{1-\frac{1}{\gamma}}. \tag{A16}$$

The first-order condition for the maximization in (A.13) leads to

$$WJ_W(W,t)\boldsymbol{\lambda} + J_{WW}(W,t)W^2\boldsymbol{\phi}_t = 0. \tag{A17}$$

By solving (A.17) we can get the optimal investment strategy:

$$\boldsymbol{\phi}_t^* = -\frac{J_W(W,t)}{W_t J_{WW}(W,t)}\boldsymbol{\lambda}. \tag{A18}$$

and we can substitute $\boldsymbol{\varphi}_t{}^*$ back to equation (A.13) and get

$$\mathcal{L}\phi J(W,t) = -\frac{1}{2}\frac{J_W(W,t)^2}{J_{WW}(W,t)}\boldsymbol{\lambda}^{\top}\boldsymbol{\lambda}. \tag{A19}$$

With equations (A.16) and (A.19), the HJB equation (A.9) could be transferred to a second order PDE:

$$\begin{aligned}\delta J(W,t) &= \mathcal{L}^c J(W,t) + \mathcal{L}\phi J(W,t) + \frac{\partial J}{\partial t}(W,t) + r^B W J_W(W,t) \\ &= \frac{\gamma}{1-\gamma}J_W(W,t)^{1-\frac{1}{\gamma}} - \frac{1}{2}\frac{J_W(W,t)^2}{J_{WW}(W,t)}\boldsymbol{\lambda}^{\top}\boldsymbol{\lambda} \\ &\quad + \frac{\partial J}{\partial t}(W,t) + r^B W J_W(W,t).\end{aligned} \tag{A20}$$

with the terminal condition $J(W,T) = \varepsilon W^{1-\gamma}/(1-\gamma)$.

As the wealth dynamics (7) is linear in W, we could conjecture that if the strategy (c^*, $\boldsymbol{\varphi}^*$) is optimal with W and t, then the strategy (kc^*, $\boldsymbol{\varphi}^*$) would still be optimal with kW and t. If it is true, we have

$$\begin{aligned}J(kW,t) &= E_t\left[\int_t^T e^{-\delta(s-t)}u(kc_s^*)\mathrm{d}s + \varepsilon e^{-\delta T}v\left(kW_T^*\right)\right] \\ &= E_t\left[\int_t^T e^{-\delta(s-t)}\frac{(kc_s^*)^{1-\gamma}}{1-\gamma}\mathrm{d}s + \varepsilon e^{-\delta T}\frac{\left(kW_T^*\right)^{1-\gamma}}{1-\gamma}\right] \\ &= k^{1-\gamma}E_t\left[\int_t^T e^{-\delta(s-t)}\frac{(c_s^*)^{1-\gamma}}{1-\gamma}\mathrm{d}s + \varepsilon e^{-\delta T}\frac{\left(W_T^*\right)^{1-\gamma}}{1-\gamma}\right] \\ &= k^{1-\gamma}J(W,t).\end{aligned} \tag{A21}$$

Thus, the indirect utility function $J(W, t)$ is homogeneous of degree $1-\gamma$ in the wealth W. Let $k = 1/W_t$, we then have $J(1, t) = J(W, t)/W^{1-\gamma}$ and so $J(W, t)$ could be expressed as:

$$J(W,t) = W^{1-\gamma}J(1,t).$$

It can be seen that $J(W, t)$ is a product of $W^{1-\gamma}$ and a function solely related to t. For the convenience of solving the PDE (A.20), we define $g(t)$ as:

$$g(t)^{\gamma} = (1-\gamma)J(1,t)^{1/\gamma}.$$

Then we have

$$J(W,t) = \frac{g(t)^{\gamma}W^{1-\gamma}}{1-\gamma}; \tag{A22}$$

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$$J_W(W,t) = g(t)^\gamma W^{-\gamma}; \tag{A23}$$

$$J_{WW}(W,t) = -\gamma g(t)^\gamma W^{-\gamma-1}; \tag{A24}$$

$$\frac{\partial J}{\partial t}(W,t) = \frac{\gamma}{1-\gamma} g(t)^{\gamma-1} W^{1-\gamma} g'(t). \tag{A25}$$

By substituting equations (A.22), (A.23), (A.24), (A.25) back to the PDE (A.20), we get

$$\left(\frac{\delta}{1-\gamma} - r^B - \frac{1}{2\gamma}\boldsymbol{\lambda}^\top\boldsymbol{\lambda}\right) g(t) - \frac{\gamma}{1-\gamma} - \frac{\gamma}{1-\gamma} g'(t) = 0, \tag{A26}$$

or

$$g'(t) = Ag(t) - 1, \tag{A27}$$

with terminal condition $g(T) = \varepsilon^{1/\gamma}$, where

$$A = \frac{\delta + r(\gamma-1)}{\gamma} + \frac{1}{2}\frac{\gamma-1}{\gamma^2}\boldsymbol{\lambda}^\top\boldsymbol{\lambda}.$$

The ODE (A.27) has a unique solution:

$$g(t) = \frac{1}{A}\left(1 + \left[\varepsilon^{1/\gamma}A - 1\right]e^{-A(T-t)}\right).$$

Given the explicit form of $g(t)$, the indirect utility function $J(W, t)$ could be derived from equation (A.22):

$$J(W,t) = \frac{g(t)^\gamma W^{1-\gamma}}{1-\gamma} = \frac{W^{1-\gamma}}{1-\gamma}\left\{\frac{1}{A}\left(1 + \left[\varepsilon^{1/\gamma}A - 1\right]e^{-A(T-t)}\right)\right\}^\gamma.$$

By equation (A.15), the optimal consumption strategy is

$$C(W,t) = \frac{W}{g(t)} = A\left(1 + \left[\varepsilon^{1/\gamma}A - 1\right]e^{-A(T-t)}\right)^{-1} W,$$

and the optimal φ could be derived from (A.18):

$$\begin{aligned} \boldsymbol{\phi}^*(W,t) &= -\frac{J_W(W,t)}{W_t J_{WW}(W,t)}\boldsymbol{\lambda} \\ &= \frac{1}{\gamma}\boldsymbol{\lambda}. \end{aligned} \tag{A28}$$

The form of $C(W, t)$ and $\phi(W, t)$ could verify that our guess in equation (A.21) is correct.

According to our transformations $\tilde{W} = W + D$, $\tilde{c} = c + a$ and

$$\tilde{\boldsymbol{\pi}}^* = \frac{W}{\tilde{W}}\boldsymbol{\pi}^* = \frac{W}{\tilde{W}}\left(\tilde{\boldsymbol{\sigma}}^\top\right)^{-1}\boldsymbol{\phi}^*.$$

the optimal strategies $\tilde{c}^*$ and $\tilde{\boldsymbol{\pi}}^*$ could be expressed in terms of $\tilde{W}$:

$$\tilde{c}^*\left(\tilde{W}_t, t\right) = \frac{\tilde{W}_t - D_t}{g(t)} + a_t,$$

$$\begin{aligned} \tilde{\boldsymbol{\pi}}^*(W,t) &= \frac{1}{\gamma}\left(1 - \frac{D}{\tilde{W}}\right)\left(\tilde{\boldsymbol{\sigma}}^\top\right)^{-1}\boldsymbol{\lambda} \\ &= \frac{1}{\gamma}\left(1 - \frac{D}{\tilde{W}}\right)\left(\boldsymbol{\sigma}\boldsymbol{\rho}\boldsymbol{\rho}^\top\boldsymbol{\sigma}^\top\right)^{-1}\tilde{\boldsymbol{\mu}}. \end{aligned}$$

Thus, we see that a repayment structure with zero-quadratic variation would have no impact on the net consumption and investment strategies (in dollar amount) whatsoever. This conclusion can also be extended to a broader range of repayment structures according to our proof, where once we let the control variables be c_s, a_s and π_s, the dynamics of D_t become irrelevant. Intuitively we can say that regardless of the repayment structure, the farmer sets the loan aside, invests it in the bank account and repays the loan, which is the reminiscence of the well-known CPPI strategy in finance (see e.g., Balder and Mahayni, 2010). Under this kind of problem setup, the farmer never uses the loan to invest in the farm, as that may make the probability of default non-zero (recall the repayment guarantee constraint).

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Appendix B. Parameter estimation process

The data set from ABARES includes the average farm business data from 1990 to 2022 within different farming industries by farm size, with large farms classified as those with total cash receipts (in 2022-23 dollars) greater than \$1,000,000. The total cash receipts of the category medium size is between \$500,000 and \$1,000,000, while that of the small size is less than \$500,000. We consider large-size farms for our estimation. Since the data is on a yearly base, we would set $\mathrm{d}t = 1$ and treat all the parameters as constants.

For estimating μ^L and σ^L, we first calculate the land price per hectare year ${p_t}^L$ by dividing the terms "Value of land and fixed improvements (\$)" by "Area operated at 30 June (ha)" in the data set.

According to the land price process

$$\mathrm{d}\ln p_t^L = \mathrm{d}p_t^L/p_t^L - \frac{1}{2}(\sigma^L)^2\mathrm{d}t = \left(\mu^L - \frac{1}{2}(\sigma^L)^2\right)\mathrm{d}t + \sigma^L\mathrm{d}z_t^L.$$

If we use $\boldsymbol{x}^L = ({x_t}^L)_{t\,=\,1990,\,1991,\,\ldots,\,2021}$ to denote the observations $\ln p_{t+1}^L - \ln p_t^L$ for t from 1990 to 2021 based on the data set, we have

$$x_t^L \sim \mathcal{N}\left(\mu^L - \frac{1}{2}(\sigma^L)^2, (\sigma^L)^2\right).$$

The log-likelihood function for such a normal distribution would be

$$l(\boldsymbol{x}^L; \mu^L, \sigma^L) = -\frac{n}{2}\ln 2\pi(\sigma^L)^2 - \sum_{t=1990}^{2021} -\frac{\left(x_t^L - \left(\mu^L - \frac{1}{2}(\sigma^L)^2\right)\right)^2}{2(\sigma^L)^2}. \tag{B1}$$

where $n = 32$ is the dimension of $\boldsymbol{x}^L$. The estimates $\hat{\mu}^L$ and $\hat{\sigma}^L$ are obtained by maximizing the log maximum likelihood function (B.29).

The process for estimating μ^K and σ^K is similar because the form of the capital price process is exactly the same as that of the land price process. Note that we treat the summation of the terms "Depreciation (\$)" and "Total closing capital (\$)" in the data set as the "Initial capital amount" for each year. By assuming the initial price of capital in each year is \$1 per unit and the number of units during each year does not change, we have

$$\ln p_{t+1}^K - \ln p_t^K = \frac{\text{Total closing capital}}{\text{Initial capital amount}}.$$

which form the data points substituted into the log maximum likelihood function of $\boldsymbol{x}^K$.

For estimating α^L, α^K, β^L and β^K, we treat "Value of land and fixed improvements" in the data set as the land value ${p_t}^L L_t$ and "Initial capital amount" as the capital value ${p_t}^K K_t$ in equation (3), while the net revenue observation in each year ${x_t}^y = y_{t+1} - y_t$ would be "Profit at full equity (\$)" in the data set, which would be the inputs to the log likelihood function.

From (3), we have

$$x_t^y(p_t^L L_t, p_t^K K_t) \sim \mathcal{N}\left(\alpha^L p_t^L L_t + \alpha^K p_t^K K_t,\ (\beta^L p_t^L L_t + \beta^K p_t^K K_t)^2\right).$$

Then, we could get the estimates for these four parameters by maximizing the corresponding log-likelihood function.

The covariance matrices of these estimates are approximated by the inverse of the corresponding Fisher information matrices (see, Van der Vaart, 2000), while the standard deviations are calculated by taking the square roots of the diagonal elements of the covariance matrices.

Appendix C. Sensitivity analysis

We conduct sensitivity analysis on the performance of the two speculated solutions S and TS. Our tests show that the two most influential parameters are the utility discount factor, δ, and the risk-free rate, r^B, whose baseline values are set to be $\delta = 0.02$ and $r^B = 0.04$ respectively. In the following tables, b_1 is the optimal constant repayment rate, while κ is the optimal descriptive parameter of S.

Appendix C.1 The Ansatz candidate solution S

Table C.1 demonstrates that as δ varies from 0.01 to 0.05 while keeping other parameters constant, the equivalent initial wealth ${W_0}^C$ consistently rises. Note that *delta* is independent of the optimal investment strategy, optimal b_1 and κ. In summary, the speculated solution has a better performance when δ is larger.

Table C1. The sensitivity analysis result about utility discount factor δ

		Optimal Investment Strategy		Optimal Parameters			
r^B	δ	π^L	π^K	b_1	κ	$W_0{}^C$	ΔW
0.04	0.01	0.3732	0.5506	3523.24	0.9812	100,007.86	7.86
0.04	0.02	0.3732	0.5506	3530.17	0.9812	100,050.12	50.12
0.04	0.03	0.3732	0.5506	3535.79	0.9812	100,114.71	114.71
0.04	0.04	0.3732	0.5506	3541.10	0.9812	100,201.38	201.38
0.04	0.05	0.3732	0.5506	3546.13	0.9812	100,309.81	309.81

Table C2. The sensitivity analysis result about the risk free rate r^B

		Optimal Investment Strategy		Optimal Parameters			
r^B	δ	π^L	π^K	b_1	κ	$W_0{}^C$	ΔW
0.02	0.02	0.5281	0.8058	2960.22	0.9622	100,762.74	762.74
0.03	0.02	0.4507	0.6782	3237.17	0.9725	100,294.52	294.52
0.04	0.02	0.3732	0.5506	3530.17	0.9812	100,050.12	50.12
0.05	0.02	0.2958	0.4231	3839.52	0.9887	99,995.82	−4.18
0.06	0.02	0.2183	0.2955	4165.50	0.9945	100,081.84	81.84

Table C3. The sensitivity analysis result about utility discount factor δ

		Optimal Investment Strategy		Optimal Parameters		
r^B	δ	π^L	π^K	b_1	$W_0{}^C$	ΔW
0.04	0.01	0.3732	0.5506	3523.24	102,505.22	2,505.22
0.04	0.02	0.3732	0.5506	3530.17	102,521.83	2,521.83
0.04	0.03	0.3732	0.5506	3535.79	102,559.12	2,559.12
0.04	0.04	0.3732	0.5506	3541.10	102,616.76	2,616.76
0.04	0.05	0.3732	0.5506	3546.13	102,695.79	2,695.79

Table C.2 shows the impact of changing the risk-free rate r^B from 2 to 6%. We find that the equivalent initial wealth W_0^C exhibits a U-shaped pattern with a low point at $r^B = 0.05$ where it underperforms the constant strategy. In general, we state that solution S outperforms the constant repayment structure. We also note that the other parameters exhibit a change when r^B changes.

In conclusion, the speculated optimal solution S outperforms the constant repayment structure in most cases, and the improvement is particularly pronounced when δ is larger and r^B is smaller.

Appendix C.2 The Ansatz candidate two-stage solution

We see from Table C.3 that when δ increases, the performance of the TS solution improves, but the improvement is marginal. In contrast, when there is a significant relationship with the risk-free rate r^B (Table C.4). We see that as r^B increases

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Table C4. The sensitivity analysis result about the risk free rate r^B

		Optimal Investment Strategy		Optimal Parameters		
r^B	δ	π^L	π^K	b_1	$W_0{}^C$	ΔW
0.02	0.02	0.5281	0.8058	3523.24	105,444.19	5,444.19
0.03	0.02	0.4507	0.6782	3530.17	103,845.90	3,845.90
0.04	0.02	0.3732	0.5506	3535.79	102,521.83	2,521.83
0.05	0.02	0.2958	0.4231	3541.10	101,504.02	1,504.02
0.06	0.02	0.2183	0.2955	3546.13	100,806.93	806.93

from 2 to 6%, the out-performance of the *TS* solution over the constant strategy decreases markedly. This is because of the fact that repayments have to be made for longer as r^B increases.

In conclusion, if δ is large while r^B is small, the proposed two-stage optimal solution would outperform the constant repayment structure substantially and vice versa, similar to our observation in *S* above. In addition, the two-stage solution also performs significantly better than the speculated solution.

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