rate of shear $\frac{\partial \gamma}{\partial t}$ plotted as a function of the shear stress τ. This curve shows the yield stress of ice at 0° C. to be certainly no more than 0·1 kg./cm.[2] which is a tenth of the expected value. More accurate measurements carried out over a longer period may well prove that there is no detectable yield stress at all. On the other hand, the curve shows that creep only becomes really rapid at stresses approaching 1 kg./cm.[2], so that perhaps, as a first approximation at any rate, ice may be regarded as an ideally plastic material with a yield stress of that order.[1]

It should be mentioned that the experiment was carried out at an altitude of 3350 m., where the boundary between firn and ice lies at about 19 m. depth,[2] and where the entire glacier is at the pressure melting point, with the exception of a superficial crust of about 15 m. thickness which is penetrated by the winter cold wave.[3] A detailed account of this work will be published in due course.

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SOME COMMENTS ON GLACIER FLOW

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ABSTRACT. Drs. Orowan and Perutz have shown that glacier ice does not behave as a viscous fluid but is plastic like all crystalline materials. The present author discusses two observed types of ice flow:—(1) the normal, regular streaming flow in slow-moving glaciers; (2) *Block-Schollen* * movement in swiftly flowing ice. Mention is made of the shear plane or laminar flow theory of Philipp. It appears that Orowan's thesis is also applicable to *Block-Schollen* flow. The author agrees with Orowan on the fundamental point that when ice is subjected to shear stress a critical value of the shear stress exists beyond which the ice alters its consistency. But the author disagrees with Orowan in that he believes that below this critical value ice behaves as a viscous material, and he supports this view by reference to many phenomena, measurements and calculations.

ZUSAMMENFASSUNG. Ausgehend von der Feststellung von Dr. Orowan und Dr. Perutz, dass das Gletschereis sich nicht wie eine zähe Flüssigkeit bewegt, sondern ein plastisches Material wie alle kristallinen Körper ist, werden von den bisher bekannten Bewegungsarten des Eises: (1) die regelmässige in langsam fliessenden, (2) die Block-Schollen Bewegung in schnell bewegten Gletschern gekennzeichnet und die Philippsche Scherflächentheorie kurz gestreift. Dabei ergibt sich, dass die These von Dr. Orowan auf die Block-Schollen Bewegung anwendbar ist. Der Verfasser ist mit Dr. Orowan in einem entscheidenden Punkt einer Meinung, dass bei Überbeanspruchung des Eises durch Scherspannung ein kritischer Wert existiert, jenseits dessen das Eis seine Konsistenz ändert. Im Gegensatz zu Dr. Orowan ist der Verfasser aber der Meinung, dass sich unterhalb dieses kritischen Werts das Eis wie eine zähe Flüssigkeit bewegt, was durch eine Reihe von Erscheinungen, Messungen und Berechnungen sehr wahrscheinlich gemacht wird.

MY attention has been drawn to the discussion on "The flow of ice and of other solids" held at the Institute of Metals, London,† in April 1948, at which Dr. E. Orowan and Dr. M. F. Perutz made some important suggestions explanatory of glacier movement. They showed that ice does not behave as a liquid of constant viscosity but that it is a plastic material like all crystalline solids. May I be allowed to express my opinion on the very difficult and manifold problems and theories of glacier movement? I do so in the light of experience gained from the results of photogrammetric measurements on several glaciers of the Alps, Central Asia and Spitsbergen.[1]

The most important result of these researches was that there are two types of glacier flow which are fundamentally different:

* The term "*Block-Schollen*," as applied to glacier flow by Professor Finsterwalder in his original manuscript is left untranslated in this article since some English writers have used the term "Block Flow" in a somewhat different sense.—Ed.

† *Journal of Glaciology*, Vol. 1, No. 5, 1949, p. 231–40.

(1) In slow-moving glaciers there is a regular, streaming (*regelmässig*) flow like that of a viscous liquid.

(2) In fast-moving glaciers there is what we have termed "*Block-Schollen*" movement which exhibits several unusual phenomena.

Regular, Streaming Flow

In the opinion of the earlier glaciologists this kind of movement exists in the majority of the European glaciers and also in other glaciers flowing in wide, gently inclined valleys. It is characterized by great regularity of the streamlines, clear and well-formed moraines and an apparent continuity (*Kontinuität*) of movement throughout the whole mass of the glacier. Measurements of movement on the surface of these glaciers show an undisturbed increase of velocity from margin to centre.

This increase may be either linear or with a quadratic, or still higher, function depending on the cross section of the bed, but it is always regular and systematic except that there is a more or less broad band at the centre where flow is constant. To glaciers of this type Sebastian Finsterwalder applied his geometric theory of flow.[2] Ice, he said, moved from the firn basin throughout the glacier mass in regular and continuous streamlines. This theory was proved correct by measurements of velocity spread over the whole glacier surface during summer and winter and by borings to the glacier bed. The principal researches were carried out on the Hintereisferner and the Rhône Glacier. Measurements in the latter were made over a period of forty-one years.[3]

The ice was found to move smoothly and with regular changes from smaller to higher speeds, not only from margins to centre but also from firn region to tongue. In the lowest reaches of the ice stream the speed of flow was found to decrease. Apart from minor systematic fluctuations no difference in summer and winter speeds was detected in careful measurements on both glaciers.[4]

About this time C. Somigliana and M. Lagally also examined the nature of glacier flow, using the theory of viscous liquids according to Navier and Stokes.[5a, b] The phenomena observed on the surface and those calculated with the aid of this theory for the interior showed remarkable accord. It was possible, too, to determine the constant of viscosity μ, which is normally $0{\cdot}92 \times 10^{14}$ g. cm.$^{-1}$ sec.$^{-1}$. In debris-laden ice near the margin and the glacier bed the value of μ may be smaller. Using the formula of Lagally the depths of glaciers were calculated and checked by borings and by methods described below (see (*a*) and (*b*)). Geomorphological conclusions also corroborated the results. By this theory, too, Lagally was able to explain the formation of crevasses, their position, direction and depth. For this he assumed that ice possessed some degree of elasticity, an assumption confirmed by the transmission of sound waves in ice.

Recently it has been possible to test these theories afresh by modern methods of investigation, namely

(*a*) Seismic sounding.[6]
(*b*) Determination of the glacier economy.[7a, b]
(*c*) Speed measurements with modern theodolites of the Wild and Zeiss type [8] and by photogrammetry.[9]
(*d*) Pollen analysis.[10]

These techniques confirmed the main results of the classic explorations on slowly flowing glaciers. They also made it possible to discover facts at variance with previous findings. These occurred to some extent on slow glaciers but were much commoner on quickly moving ice. It was even possible to recognize the nature of this rapid movement on some of the Pamir and Nanga Parbat glaciers and in a few glaciers in Spitsbergen. It was decided to call this type of progression "*Block-Schollen* Movement."

Block-Schollen Movement

Block-Schollen movement, which manifests itself in various forms, is found on glaciers whose annual speed (v) is so great that its ratio (r) to the width (b) or to the depth of the glacier increases to much higher values than on slowly streaming glaciers. The ratio $r=v$ (m./year): b on quickly moving glaciers amounts to between $r=2:3$ and $r=1:6$ (Rakhiot Glacier on Nanga Parbat). On slow glaciers it is essentially smaller, for instance on the Pasterzenkees in Austria $r=1:20$ ($v=50$ m./year, $b=1000$ m.), or on the Fedchenko Glacier in Pamir $r=1:16$ ($v=180$ m./year, $b=2500$ m.). On these fast glaciers great changes of speed occur at relatively short intervals, and the consequent deformations bring the ice up to a critical point of stress-strain. These deformations cannot be transmitted by the ice in a normal viscous condition; zones of higher mobility, especially in the marginal ice, were shown to exist by photogrammetric measurements.[11] Such zones probably exist, too, near the beds of glaciers of this type. The greater part of the deformation of the ice takes place in these regions of higher mobility, so that the rest of the glacier moves on with uniform velocity, like a solid mass. This mass, however, breaks into blocks (*Schollen*) caused by irregulatities of the bed. If these are considerable and the speed high the whole mass of the glacier may, in extreme cases, be broken up into seracs falling forward with high velocity. These are the outward and visible sign of extreme *Block-Schollen* movement. Examples of this were to be found in the Alps in former times, for instance in the Vernagtferner which, in 1845, attained the speed of 11 m. a day. Other cases have been recorded quite recently in the Karakoram and the Andes and they are normal on the rapidly descending outflow glaciers of west Greenland and Patagonia. They occur, too, to some extent in Spitsbergen.

Photogrammetric speed measurements were made on the Rakhiot Glacier [11] in 1934 and in 1938 on Kongsbreen and elsewhere in Spitsbergen.[12] All these showed visible signs of the *Block-Schollen* phenomena, but it is also possible to recognize them from the theodolite speed measurements. Descriptions, too, of the Karajak [13] and of the Umiamako and Rink Isbræer in Greenland [14] point to the same form of flow.

As has been mentioned *Block-Schollen* movement is characterized by a rapid increase in small marginal zones of high mobility from zero to the maximum in one and the same reach of the glacier. Nearly the whole of the rest of the glacier shows a similar rate of movement, although small changes are caused by relative displacements of the blocks against each other. It is remarkable that there are also intermediate kinds of movement on fairly fast glaciers, one part of the glacier moving as a stream, the other as a solid mass. Indeed the same glacier can change from pure stream flow to pure *Block-Schollen* flow at different times. Instructive observations on this point were made by Mason and Helbling in the Karakoram and the Andes [15a, b] and by Pillewizer in Spitsbergen.

Movement along Shear Planes

Mention should be made of the flow theory of the geologist, H. Philipp, [16] based on a large number of inspections of alpine glaciers. According to this theory movement occurs along shear planes. This, regarded in the differential sense, is closely related to the theory of viscous liquids. Definite movement along shear planes doubtless takes place especially at the ends of tongues, but, in the view of the author, the effect differs in the main but little from ordinary streaming flow.

Comments on Dr. E. Orowan's Paper

The statements of Dr. E. Orowan on the plastic deformation of ice seem to be of great value in connexion with *Block-Schollen* movement. The zones of high mobility are, as mentioned above, caused by the fact that ice is overstrained beyond a critical value. Such a critical value does appear to exist. On this fundamental point I agree with Dr. Orowan. Nevertheless there is a difference between his diagram (Fig. 2, p. 232) of shear stress and rate of shear strain and the way in which I visualise it. According to Dr. Orowan the critical point is given by Y, but could it not be at A? If I may use Dr. Orowan's very instructive and simple diagram I can explain my view as follows: when a stress is applied from zero to the critical value, the rate $\frac{d\gamma}{dt}$ increases according to a linear function to A. In this line deformation occurs according to the theory of viscous liquids. As the slope of this line OA is the reciprocal of the coefficient of viscosity μ and as μ is extremely high, the slope of the line OA is extremely small, so that between Dr. Orowan's horizontal line OY and my supposed line OA there is only little, but nevertheless in principle considerable, difference. If with increasing stress the critical value of $\frac{d\gamma}{dt}$ is reached, the ice changes from a liquid of constant viscosity to another condition. What that condition is is open to doubt. My observations in areas of high glacial mobility lead me to think that it depends on local conditions whether it becomes plastic or of greatly accelerated viscosity, or whether it breaks up into incoherent units. In any event the regular streaming is disturbed and *Block-Schollen* movement begins. As the real nature of this type of flow is still to a great extent unknown it is important that Dr. Orowan has found and proved by independent methods, theoretically and in the laboratory, the existence of critical values of the shearing stress and the rate of shear of ice. This is decisive.

Dr. Orowan rightly states that the state of stress in a glacier is generally not uniaxial but bi- or tri-axial. This doubtless affects the critical value. The point must be clarified by further investigation. It seems to me of immense importance that glacier ice is normally more or less permeated with melt water. This may cause the ice to behave as a viscous liquid. As the coefficient of viscosity is very high and the differential movement correspondingly small it would be very difficult to demonstrate this in the laboratory. If we consider two points one metre apart near the margin of a glacier and on a transverse line across it, their relative displacement would only amount to 0·01 mm. in an hour, assuming the velocity increase from margin to centre being as that measured on the Fedchenko or Pasterzenkees.

In the firn basin special conditions obtain. Many facts point to the probability that there plastic yielding and permanent deformation occur, as is evidenced by folded layers in crevasses and by the form of the "ogives" found near the firn line. In this respect all investigations made (or to be made) in the firn area will be of the greatest value. But it must be said that, when in a normal valley glacier the ice from the firn basins reaches the tongue, further permanent deformations of a purely plastic nature do not seem to take place except for a few insignificant local exceptions. The ice streams regularly and all signs go to show that one can compare this streaming with that of a viscous liquid with a mean constant coefficient of viscosity in accordance with the line OA in the diagram. The regular distribution of velocities over the whole surface is well known. One can assume a similar regular distribution in the interior of the ice stream. If great irregularities and stresses beyond the critical point were present there one would expect some sign of these, such as disturbances, to appear on the surface as in the case of *Block-Schollen* movement.

To what extent ice in normal glaciers moves as a viscous liquid and not as a pure plastic material, can, in my opinion, be well understood from the excellent photograph of the Susitna Glacier on p. 175 of the *Journal of Glaciology* (Vol. 1, No. 4). The main stream suffers severe deformations caused by a tributary flowing in from the side. But these deformations are not permanent, as would be necessary in the case of plasticity; after overcoming the lateral pressure, the ice streams on undisturbed, without in any way having lost its continuity of inner coherence. This is shown by the medial moraines in the foreground which revert to their undisturbed course below the point where they became deformed. If plastic deformation had taken place the moraines would have shown permanent deformation in consequence of the former lateral pressure. The photograph of the Barnard Glacier (p. 174) also shows typical viscous streaming in the regular shape of the moraines of the main stream. This is particularly striking, too, in the moraines of the tributaries. Surely this is indicative of the predominance of regular and continuous viscous flow. Would it be possible to prove these phenomena by plastic yielding and deformation? Could the Lévy-Mises equations and the Maxwell-Mises quadratic yield condition have given us results with such a high degree of probability as Somigliana and Lagally have done by using the Navier-Stokes equations for viscous streaming? The latter have given clear numerical connexions not only between rate of flow and glacier depth but also between the form of the profile of the bed and the increase of velocity from the margin to middle in typical cases. They based their calculations on rather exact measurements on the glaciers themselves and brought their theories into close harmony with the actual facts. Nevertheless they have never stated that "ice is a viscous liquid" but only that "ice moves like a viscous liquid." A considerable part of the problem therefore seems to be as yet unsolved even from their point of view, so that we must pay very careful attention to the new aspects of the matter and to the theories presented to our attention by Dr. Orowan and adduced from his great metallurgical experience.

The foregoing considerations apply to plastic yield in normal-flowing glacier tongues, but I can, in many respects, confirm Dr. Orowan's assertions about large ice caps. Owing to the low temperature of the ice, the viscosity of the upper layers is reduced, and they may then behave as a rigid mass, whilst the bottom layers may be stressed beyond the critical value of stress-strain, causing extrusion through the gates of the marginal mountains. The great depths of ice caps undoubtedly cause stresses high enough to produce plastic flow and deformation with all its consequences.

There are further important statements in Dr. Orowan's paper about the longitudinal force Y which exerts pressure or tension if the inclination of the glacier bed changes. This force must also exist in the case of viscous movement. I also agree with Dr. Orowan's remarks about terraces and the excavation of hollows. On terraces greater velocities and overstraining of the ice may occur, resulting in the kind of movement Dr. Orowan explains. The reference to the geomorphological consequence—the excavation of hollows—is particularly important. The alpine glaciers of to-day with their decelerating tendency seem to stream fairly regularly down terraces, as was surprisingly shown in the long-continued measurements of the ice fall of the Rhône Glacier.[3] Nevertheless it is probable that when glaciers advance they change their mode of progress to *Block-Schollen* movement when flowing over terraces. They would thus excavate hollows in the manner Dr. Orowan suggests. This may have applied more particularly to glaciers of the Ice Age when they were larger and their influence on the shape of their beds was much greater than to-day, as, for instance, in the deep excavations in the fjords of Norway.

We have to thank Dr. Orowan for his most interesting and valuable paper which provides striking new aspects to those of us who may be more immersed than he in the glacier investigations of the past. It is our duty to bring together modern thought and the results of earlier research. In this respect Dr. Orowan and Dr. Perutz have greatly stimulated and helped us.

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EARLY DISCOVERERS

IV

ESMARK ON GLACIATION

The following is an extract from a paper by Professor Jens Esmark (1763–1839), *the Norwegian geologist and mineralogist*—"Remarks tending to explain the Geological History of the Earth." "*Edinburgh New Philosophical Journal,*" *Oct.–Dec.* 1826, *p.* 107–21. *The original was published in Norwegian in* "*Nyt Magazin for Naturvidenskaberne,*" 1824. *Place names which were misspelt in the English translation have been restored to the spelling of the Norwegian original. They should all be recognizable even though the orthography of modern maps differs from that of* 1824.

After discussing evidence of glaciation in the Alps, on the north German plain and in Denmark, he gives his original observations from Norway, which must be quoted at length:

". . . But it is particularly in Norway I have found many proofs of the operation of immense masses of ice which have now disappeared.

1. As in other countries we find large, loose rocky masses lying spread over pretty level plains; for example, in travelling from Morstuen, on the Mjösen, to Leuten in Hedemarken. These must have been brought from a great distance, for there are in the neighbourhood no mountains of the same character as these masses.

2. In no other satisfactory way than by the operation of ice can we explain how those prodigiously large loose stones, sometimes with sharp corners, have been brought up to the ridges and tops of high mountains, which are found in such numbers in the province of Christiansand. The first time I met with such single loose blocks lying on the ridge of the high mountains in Nummedalen, I thought they must be the remains of strata, or of masses which had covered the mountain, and which had in after-times been decomposed and carried off by water, leaving those traces of their former existence. But, on examining them more closely, I found that this account