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THREE TOPOLOGICAL PROPERTIES FROM NOETHERIAN RINGS

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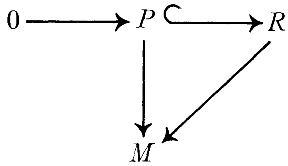
1. Introduction. The purpose of this paper is to study three concepts that deal with the topologies on ideals of commutative integral domains. We call a domain R *prime-injective* if for each torsion free R-module M, and all non-zero prime ideals



commutes implies that M is injective. From [**6**, Theorem 1 and the technique of Example 6] this is equivalent to all non-zero ideals of R being open in the topology defined by finite products of non-zero prime ideals as a base of neighborhoods around zero.

A domain is *strongly prime-injective* if for each (torsion theory) topology $\mathscr{F}$ and for φ the set of primes in $\mathscr{F}$, φ-injective implies $\mathscr{F}$-injective for $\mathscr{F}$ torsion free modules (see [**6**, **8**] for notation). As in the prime-injective case, this is equivalent to $\mathscr{F}$ being the topology generated by φ for all topologies $\mathscr{F}$. For our purposes we say that in a domain R the *Krull Intersection Theorem holds* for an ideal I, and write K.I.T. holds for I if for each finitely generated torsion free R-module M, $\bigcap_{n=1}^{\infty} I^n M = 0$. This means that the I-adic topology of M is Hausdorff [**9**].

The main results are Theorem 2.5, Corollary 2.7, Theorem 3.2, and Theorem 3.4. The first two of these give conditions when K.I.T. holds for an ideal I in terms of prime-injective. In Section 3 we study polynomial extensions. The main results are Theorems 3.2 and 3.4 which compare a domain R being prime-injective with the polynomial ring $R[X]$ being prime-injective.

A desired condition in completions of rings and modules is that the I-adic completion is Hausdorff, specifically when

$$\bigcap_{n=1}^{\infty} I^n = 0 \quad \text{or} \quad \bigcap_{n=1}^{\infty} I^n M = 0.$$

Thus knowing that the hypotheses of Theorem 2.5 or Corollary 2.7 hold

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automatically gives Hausdorff completions. Theorem 3.4 can be thought of as an attempt to answer the question of whether the Krull Intersection Theorem holding for each ideal I implies that it also holds in polynomial extensions.

The notation has been taken from [**8**], [**9**], and [**7**]. All rings are commutative with identity and all modules are unitary.

2. Prime-injective, K.I.T., and strongly prime-injective.

PROPOSITION 2.1. *Strongly prime-injective implies prime-injective.*

Proof. Let $\mathscr{F}$ be all non-zero ideals in R and apply the definition.

Example 2.2. Let V be a valuation domain with value group $\mathbf{Z} \oplus \mathbf{Q}$ (lexicographically ordered), then V is prime-injective (see the proof of Theorem 4 of [**6**]) but not strongly prime-injective since for M the maximal ideal $M^n = M$.

LEMMA 2.3. *Let (R, M) be a one-dimensional quasi-local domain, I a finitely generated ideal and A a torsion-free R-module. Let $N = \bigcap_{n=1}^{\infty} I^nA$. Then $IN = N$ and N is an injective R-module. If A is finitely generated, then $N = 0$.*

Proof. We can assume that $I \neq 0$. Let $0 \neq i \in I$, so $I^n \subseteq (i)$ for some n and hence $I^{nt} \subseteq (i)^t \subseteq I^t$. Thus

$$N = \bigcap_{n=1}^{\infty} I^nA = \bigcap_{n=1}^{\infty} (i)^nA,$$

so we can assume that I is principal. It is easily seen that $IN = N$. (This is true for any torsion-free R-module.) Hence $I^nN = N$ for all $n \geqq 1$. For any $I \neq j \in R$, $I^n \subseteq (j)$ for some n, so $M = jN$ and hence N is divisible and therefore injective. Thus N is a direct summand of A. Hence if A is finitely generated, so is N. But then $IN = N$ so $N = 0$ by Nakayama's Lemma.

COROLLARY 2.4. *Let R be an integral domain, I a finitely generated rank one ideal of R and A a finitely generated torsion-free R-module. Then*

$$\bigcap_{n=1}^{\infty} I^nA = 0.$$

Proof. Let $P \supseteq I$ be a rank one prime ideal. Pass to R_P. Then I_P is a finitely generated ideal in the one-dimensional quasi-local domain R_P and A_P is a finitely generated torsion-free R_P-module. Hence by Lemma 2.3,

$$\bigcap_{n=1}^{\infty} {I_P}^nA_P = 0.$$

Hence

$$\bigcap_{n=1}^{\infty} I^n A \subseteq \bigcap_{n=1}^{\infty} I_P{}^n A_P = 0.$$

THEOREM 2.5. *Let R be an integral domain, I an ideal of R and $\mathcal{F}$ the set of open ideals in the I-adic topology with φ the set of prime ideals in $\mathcal{F}$. Then if φ-injective implies injective,* K.I.T. *holds for I. Moreover, R is a G-domain.*

Proof. Let R, I, $\mathcal{F}$ and φ be as in the theorem. We may write

$$\varphi = \{P \in \text{Spec }(R) | \, P \text{ is open in the } I\text{-adic topology}\} = V(I).$$

Since φ-injective implies injective, every non-zero ideal contains a product of primes from φ. Thus, if J is any non-zero ideal of R, there exist $P_1, \ldots, P_s \in \varphi$ such that $P_1 \ldots P_s \subseteq J$. But $I \subseteq P_i$ for each i, so $I^s \subseteq P_1 \ldots P_s \subseteq J$. Hence every non-zero ideal is open in the I-adic topology. In particular, a power of I is contained in every non-zero prime ideal Q and hence $I \subseteq Q$. Thus I is rank 1. (It is also interesting to note that I is contained in only finitely many minimal primes since there exist primes $P_1, \ldots, P_t \supseteq I$ with $P_1 \ldots P_t \subseteq I$ which implies that if $Q \supseteq I$ then $Q \supseteq P_i$ for some i.) To show that we may assume I to be finitely generated, or even principal, let $i \in I$ and note that $(i) \subseteq I$, so $(i)^s \subseteq I^s$. Since (i) is open in the I-adic topology, $I^t \subseteq (i)$ for some t so $I^{ts} \subseteq (i)^s \subseteq I^s$. Hence for A an R-module

$$\bigcap_{n=1}^{\infty} (i)^n A = \bigcap_{n=1}^{\infty} I^n A.$$

To complete the proof that K.I.T. holds for I we let A be a torsion-free R-module and apply Corollary 2.4. R is a G-domain because each non-zero prime ideal contains I.

There is a partial converse to Theorem 2.5.

PROPOSITION 2.6. *Let I be a finitely generated ideal in a G-domain R with I contained in all non-zero prime ideals. Then* K.I.T. *holds for I in R.*

Proof. Let J be a non-zero ideal; then $I \subseteq \sqrt{I} \subseteq \sqrt{J}$. I is finitely generated, so $I^s \subseteq J$ for some s. We have that each non-zero ideal in R is open in the I-adic topology and the proof follows from the same argument as in Theorem 2.5.

COROLLARY 2.7. *If R is one dimensional quasi-local with maximal ideal M and R is prime-injective then* K.I.T. *holds for each non-zero ideal I of R.*

Proof. If I is any ideal of R then $I \subseteq M$ and we can apply Theorem 2.5.

Remark 2.8. K.I.T. may hold for all ideals I in R yet the hypothesis of

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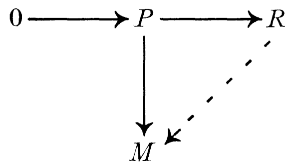
Theorem 2.5 need not be satisfied. If $R = K[x, y]$, K a field, then R is Noetherian and for $I = (y)$ the Krull Intersection Theorem holds. But $(x) \not\supseteq (y)^n$ for any n (i.e., (x) is not open in the I-adic topology), yet (x) is closed in the I-adic topology and the topology $\mathscr{F}$ generated by powers of I has the $\mathscr{F}$-injective module $E_F((x))$ which is not injective. Thus the torsion-theory topology generated by the $\{I^n\}$ need not contain all of the ideals of R for KIT to hold.

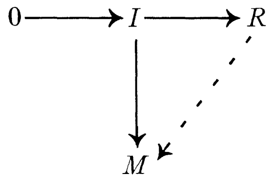
Example 2.9. A valuation domain V with value group $\mathbf{Z} \oplus \mathbf{Z}$ is strongly prime-injective by Theorem 4 of [**6**] and K.I.T. for $I = M$, the maximal ideal in V, does not hold.

3. R and $R[X]$.

LEMMA 3.1. *Let R be a graded ring which is (strongly) prime-injective and M a torsion free R-module, then R has the property that if*



commutes for each graded prime ideal P (in a topology $\mathscr{F}$ generated by graded ideals) in R then



commutes for each graded ideal (in the topology $\mathscr{F}$) in R.

Proof. It is sufficient to show the strongly prime-injective case since $\mathscr{F}$ may be taken to be all non-zero graded ideals in R. Let I be a graded ideal in a topology $\mathscr{F}$, then there exist prime ideals $\{P_i\}_{i=1}^n$ in $\mathscr{F}$ so that $I \subseteq \prod_{i=1}^n P_i$ since R is strongly prime-injective. Let $\{P_i^*\}$ be the set of graded prime ideals derived from each P_i by taking the ideal generated by the homogeneous elements in P_i. Each P_i^* is non-zero since the topology $\mathscr{F}$ is generated by graded ideals and so each ideal in $\mathscr{F}$ must contain a non-zero homogeneous element. We then have that

$$P_i^* \in \mathscr{F} \quad \text{and} \quad I \supseteq \prod_{i=1}^n P_i \supseteq \prod_{i=1}^n P_i^*.$$

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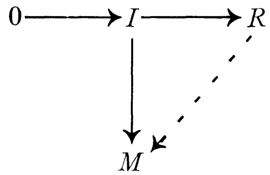
From [**6**, Theorem 1]



commutes.

THEOREM 3.2. *$R[X]$ prime-injective implies that R is prime-injective.*

Proof. Let I be an ideal in R and set $J = I \cdot R[X]$. We grade $R[X]$ by letting the degree of X equal one. The ideal J is graded so by Lemma 3.1 there exist graded prime ideals $\{P_i^*\}$ so that

$$J \supseteq \prod_{i=1}^{m} P_i^*.$$

We denote the contraction of P_i^* to R by $(P_i^*)^c$. We now wish to eliminate all $(P_i^*)^c$ which are zero from our consideration. Equivalently we wish to remove all $P_i^* = (X)$. So assume that $(X) \in \{P_i^*\}_{i=1}^m$, say $(X) = P_m^*$, then if

$$z \in \prod_{i=1}^{m} \{P_i^*\} \subseteq J$$

is homogeneous of degree n, $z = \alpha X^n$, $\alpha \in R$. This implies that

$$\alpha X^{n-1} \in \prod_{i=1}^{m-1} P_i^*$$

and since J is generated by homogeneous elements of degree zero, $\alpha X^{n-1} \in J$. In this manner we may eliminate all P_i^* equal to (X). Hence we may assume that $(P_i^*)^c$ is non-zero for each i and so

$$\prod_{i=1}^{m} (P_i^*)^c \subseteq I.$$

This proves that R is prime-injective.

We are able to obtain a partial converse of Theorem 3.2 by using an additional hypothesis:

(*) If $q(X) \in K[X]$, where K is the quotient field of R, then there exists a non-zero $s \in R$ (dependent upon $q(X)$) so that for all $h(X) \in K[X]$ with $h(X)q(X) \in R[X]$, $s \cdot h(X) \in R[X]$.

PROPOSITION 3.3. *If R is Noetherian or integrally closed, then* (*) *holds.*

Proof. If R is Noetherian, let I be the ideal $(q(X) \cdot K[X]) \cap R[X]$. I is finitely generated since R and hence $R[X]$ are Noetherian. Let $h_1(X)q(X)$,

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$h_2(X)q(X), \ldots, h_n(X)q(X)$ be generators for I with $h_i(X) \in K[X]$. So for each i there exists an $s_i \in R$ so that $s_i h_i(X) \in R[X]$. If we set $s = \prod_{i=1}^n s_i$, then for any $h(X) \in K[X]$ so that $h(X) \cdot q(X) \in R[X]$, $h(X) \cdot q(X)$ is in I and can be written in the form

$$\sum_{i=1}^{h} f_i(X)h_i(X)q(X)$$

with $f_i(X) \in R$. Since R is a domain

$$h(X) = \sum_{i=1}^{n} f_i(X)h_i(X).$$

Now

$$s \cdot h(X) = s \sum_{i=1}^{n} f_i(X)h_i(X) = \sum_{i=1}^{n} f_i(X)sh_i(X).$$

Each $f_i(X)$ is in $R[X]$ as is each $sh_i(X)$ and so $s \cdot h(X)$ is in $R[X]$.

Assume R is integrally closed. Let $h(X) \cdot q(X) \in R[X]$, then taking the content and applying the v-operation (see [**5**, Section 34])

$$c(h(X)) \cdot c(q(X)) \subseteq (c(h(X)) \cdot c(q(X)))_v = c(h(X)q(X))_v \subseteq R_v = R$$

since R is integrally closed. Since (*) may be restated in terms of finding an $s \in R$ so that $s \cdot c(h(X)) \subseteq R$ for each $h(X)$, we may choose $s \in R \cap c(q(X))$.

THEOREM 3.4. *If R is prime-injective and condition* (*) *holds, then $R[X]$ is prime-injective.*

Proof. Let $J \neq 0$ be an ideal in $R[X]$. If $J \cap R = I \neq 0$, then $I \subseteq \prod_{i=1}^m P_i$ in R. Thus for P_i^e, the prime ideal in $R[X]$ generated by P_i, $J \supseteq \prod_{i=1}^m P_i^e$. Thus J is open in the topology of $R[X]$.

If $J \cap R = 0$ let $f(X) \in J$ and assume, without loss of generality, that $J = (f(x))$. The prime ideals in $R[X]$ contracting to 0 in R are maximal ideals in $K[X]$ contracted to $R[X]$ where K is the quotient field of R. Since $f(X) \in K[X]$ and $K[X]$ is Noetherian then

$$f(X) \cdot K[X] \supseteq \prod_{i=1}^{n} (q_i(X))$$

for some set $\{q_i(X)\}_{i=1}^n$ of monic irreducible polynomials in $K[X]$.

Let $P_i(X) = (q_i(X)) \cap R[X]$. Then each $P_i(X)$ can be generated by elements of the form $r_{i\alpha}(X)q_i(X)$ with $r_{i\alpha}(X) \in K[X]$. Since

$$f(X) \cdot K[X] \supseteq \prod_{i=1}^{m} (q_i(X))$$

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then there exists an $l(X) \in K[X]$ so that

$$f(X) \cdot l(X) = \prod_{i=1}^{m} q_i(X).$$

Let $S_0 \in R$ so that $v(X) = S_0 \cdot l(X) \in R[X]$. Let $S_i \in R$ so that S_i is the element in condition (*) that corresponds to the polynomial q_i for $i = 1, \ldots n$ and let $S = \prod_{i=0}^{n} S_i$. Since R is prime injective then $(S) \subseteq \prod_{i=1}^{m} (A_i)$ where each A_i is a prime ideal in R. We claim that

$$(f(X)) \supseteq \prod_{i=1}^{n} (A_i^e) \cdot \prod_{i=1}^{n} P_i(X).$$

To see this let

$$u \in \prod_{i=1}^{n} (A_i^e) \cdot \prod_{i=1}^{n} P_i(X),$$

then

$$u = \left(\prod_{i=1}^{m} a_i(X)\right) \cdot \prod_{i=1}^{n} \left[\left(\sum_{j=1}^{k(i)} g_{ij}(X) r_{\alpha_{ij}}(X)\right) q_i(X)\right]$$

where $a_i \in A_i^e$, $g_{ij}(X) \in R[X]$, and $r_{\alpha_{ij}}(X) \in K[X]$. Thus

$$u = \prod_{i=1}^{m} a_i(X) \cdot \prod_{i=1}^{n} \left(\sum_{j=1}^{k(i)} g_{ij}(X) r_{\alpha_{ij}}(X)\right) \cdot \prod_{i=1}^{n} q_i(X)$$
$$= \prod_{i=1}^{m} a_i(X) \cdot \prod_{i=1}^{n} \left(\sum_{j=1}^{k(i)} g_{ij}(X) r_{\alpha_{ij}}(X)\right) \cdot l(X) \cdot f(X).$$

But $(S)^e \supseteq \prod_{i=1}^{m} (A_i e)$ and therefore there exists an $h(X) \in R[X]$ so that

$$\prod_{i=1}^{m} a_i(X) = S \cdot h(X).$$

Thus

$$u = \prod_{i=1}^{n} \left(\sum_{j=1}^{k(i)} g_{ij}(X) r_{\alpha_{ij}}(X)\right) h(X) \cdot S \cdot l(X) \cdot f(X)$$
$$= \prod_{i=1}^{n} \left(\sum_{j=1}^{k(i)} g_{ij}(X) r_{\alpha_{ij}}(X)\right) h(X) \cdot v(X) \cdot f(X) \cdot \left(\prod_{i=1}^{n} S_i\right).$$

But

$$\prod_{i=1}^{n} \left[\sum_{j=1}^{k(i)} g_{ij}(X) r_{\alpha_{ij}}(X)\right] \cdot \prod_{i=1}^{n} (S_i) = \prod_{i=1}^{n} \left[\left(\sum_{j=1}^{k(i)} g_{ij}(X) r_{\alpha_{ij}}(X)\right) S_i\right]$$

and $r_{\alpha_{ij}}(X) \cdot q_i(X) \in R[X]$ so $r_{\alpha_{ij}}(X) \cdot S_i \in R[X]$. Thus

$$\prod_{i=1}^{n} \left[\sum_{j=1}^{k(i)} g_{ij}(X) r_{\alpha_{ij}}(X)\right] \cdot \left(\prod_{i=1}^{n} S_i\right) \cdot h(X) \cdot v(X) \in R[X]$$

and therefore $u \in (f(X)) \cdot R[X]$. This completes the proof.

Example 3.5. A domain R which is strongly prime-injective but $R[X]$ is not strongly prime-injective.

Let R be an infinite dimensional valuation ring with value group $\bigoplus_{\mathbf{Z}^+} \mathbf{Z}$ (This example is from [**11**, Example 2.9]). Let $0 \subseteq P_1 \subseteq \ldots \subseteq M$ be the chain of prime ideals in R. By Theorem 4 of [**6**], R is strongly prime-injective. To show that $R[X]$ is not strongly prime-injective we use the construction of Ohm and Pendleton in [**11**]: Let

$$a_i \in P_{i+1} \backslash P_i, f_i(x) = a_i(a_1x - 1) \ldots (a_ix - 1) \quad \text{for each } i \geqq 1.$$

Let A' be the ideal generated by the f_i's and define

$$Q_i = P_i^e + (a_ix - 1) \quad \text{for each } i \geqq 1.$$

Let $\mathscr{F}$ be the topology generated by A' and its powers. The topology generated by the minimal primes Q_i cannot be the same as $\mathscr{F}$ since no finite product of the Q_i's is in A'. To see this let $I \supseteq (A')^n$ and P a prime in $R[X]$ containing I. Then $P \supseteq (A')^n$. By repeating the arguments in [**11**] we see that P must be one of the Q_i's and since their condition (FC) does not hold, $R[X]$ is not strongly prime-injective.

PROPOSITION 3.6. *If $R[X]$ is strongly prime-injective then so is R.*

Proof. Let $R[X]$ be strongly prime-injective and let $\mathscr{F}$ be a topology in R and $\mathscr{F}'$ the extended topology in $R[X]$ ($\mathscr{F}'$ is generated by the extended ideals of $\mathscr{F}$). Let I be an ideal in $\mathscr{F}$ and $J = I \cdot R[X]$. Then with X of homogeneous degree $= 1$ there exist, by Lemma 3.1, graded prime ideals $\{P_i^*\}_{i=1}^n$ so that $J \supseteq \prod_{i=1}^n P_i^*$. By reasoning similar to that in the proof of Lemma 3.1 we may assume that $P_i^* \cap R \neq 0$. Thus the 0th component of $\prod_{i=1}^n P_i^*$ contains the 0th component of J. Hence for $\{q_i = P_i^* \cap R\}_{i=1}^n, \prod_{i=1}^n q_i \subset I$. But each $q_i \in \mathscr{F}$ since P_i^* (the extended prime of q_i) $\in \mathscr{F}'$.

PROPOSITION 3.7. *If the* KIT *holds for each ideal I in $R[X]$ then the* KIT *holds for each ideal I in R.*

Proof. If M is a finitely generated R module, say $M = (f_i \cdot R)_{i=1}^n$, let

$$M' = (f_i \cdot R[X])_{i=1}^n.$$

Then for I^e, the extended ideal of I in $R[X]$,

$$0 = \bigcap_{n=1}^{\infty} (I^e)^n M' \supseteq \bigcap_{n=1}^{\infty} I^n M \supseteq 0.$$

PROPOSITION 3.8. *Let R be a domain, and J an ideal in $R[X]$. Define J^* as the ideal generated by the constant terms of elements of J. If $\bigcap_{n=1}^{\infty} (J^*)^n = 0$ then $\bigcap_{n=1}^{\infty} J^n = 0$.*

Proof. Let $f \in J^n$ for all n. We write f as a polynomial with lowest non-zero term $b_e x^e$. We claim that $b_e \in (J^*)^n = 0$. To see this we write

$$f = \sum_{j=1}^{m} \left(\prod_{i=1}^{n} f_{im}(x) \right) \in J^n \quad \text{for each } n$$

where $f_{im} \in J$ and m is a function of n. The lowest degree and lowest term of f remain fixed as n increases. Therefore, for $n > 1$, the coefficient b_e must come from the sum of the products of at least $n - e$ non-zero constant terms in the f_{im}'s. Hence $b_e \in (J^*)^{n-e}$ for each n. Thus

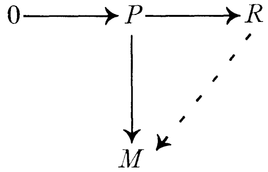
$$b_e \in \bigcap_{n=1}^{\infty} (J^*)^n = 0.$$

This implies that f must be zero so

$$\bigcap_{n=1}^{\infty} J^n = 0.$$

Remark 3.9. Proposition 3.8 is a rather incomplete answer to the question "when does KIT in R imply KIT in $R[X]$?," but it does give conditions on an ideal J in $R[X]$ that will guarantee that the J-adic topology on $R[X]$ will be Hausdorff. The proof that is given for Proposition 3.6 can be used to show that if J is an ideal in $R[[X]]$ so that $\bigcap_{n=1}^{\infty} (J^*)^n = 0$ then $\bigcap_{n=1} J^n = 0$.

Note. J. Golan has pointed out an error in Theorem 1 of [**6**]: the module M from



must be $\mathscr{F}$ torsion free. This means that throughout [**6**] the module M must be torsion-free with respect to the topology in question at that time.

4. Examples and open questions.

Example 4.1. Any strongly Laskerian ring will be prime-injective hence there exist examples of non-Noetherian prime-injective domains [**3**] and by Corollary 2.7 all one dimensional quasi-local strongly Laskerian domains satisfy K.I.T. for all ideals.

Example 4.2. An example of a one dimensional quasi-local domain with maximal ideal M so that $\bigcap M^n = 0$ yet not all ideals are open was relayed to the author by P. Eakin. Let V_1 be a valuation ring in $K(x, y)$, K a field, x, y indeterminates so that $v_1(x) = 1$ and $v_1(y) = \sqrt{2}$. Let V_2 be

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a valuation ring in $K(x, y)$ with $v_2(x) = v_2(y) = 1$. Then writing $V_1 = K + M_1$ and $V_2 = K + M_2$ where M_1 and M_2 are the maximal ideals, the example is $R = K + (M_1 \cap M_2)$. Here the ideal $y \cdot R$ is neither open nor closed in R under the $(M_1 \cap M_2)$-adic topology.

Open question 4.3. If R, M is a one dimensional quasi-local domain where every ideal is closed in the M-adic topology are all non-zero ideals open? (This is asking whether K.I.T. implies prime-injective under the one dimensional quasi-local domain condition.)

Open question 4.4. In Theorem 2.5 we use the fact that every non-zero ideal of R is open in the I-adic topology. The question is whether every non-zero ideal of R is closed in the I-adic topology implies K.I.T. for I. If R is quasi-local and the maximal ideal is finitely generated then the answer is yes since R must be Noetherian [**2**, Theorem 4.1].

Open question 4.5. If K.I.T. holds for each ideal I in R does it hold for each ideal J in $R[X]$?

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