



# Effect of shear on local boundary layers in turbulent convection

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In Rayleigh Bénard convection, for a range of Prandtl numbers  $4.69 \leq Pr \leq 5.88$  and Rayleigh numbers  $5.52 \times 10^5 \leq Ra \leq 1.21 \times 10^9$ , we study the effect of shear by the inherent large-scale flow (LSF) on the local boundary layers on the hot plate. The velocity distribution in a horizontal plane within the boundary layers at each  $Ra$ , at any instant, is (A) unimodal with a peak at approximately the natural convection boundary layer velocities  $V_{bl}$ ; (B) bimodal with the first peak between  $V_{bl}$  and  $V_L$ , the shear velocities created by the LSF close to the plate; or (C) unimodal with the peak at approximately  $V_L$ . Type A distributions occur more at lower  $Ra$ , while type C occur more at higher  $Ra$ , with type B occurring more at intermediate  $Ra$ . We show that the second peak of the bimodal type B distributions, and the peak of the unimodal type C distributions, scale as  $V_L$  scales with  $Ra$ . We then show that the areas of such regions that have velocities of the order of  $V_L$  increase exponentially with increase in  $Ra$  and then saturate. The velocities in the remaining regions, which contribute to the first peak of the bimodal type B distributions and the single peak of type A distributions, are also affected by the shear. We show that the Reynolds number based on these velocities scale as  $Re_{bs}$ , the Reynolds number based on the boundary layer velocities forced externally by the shear due to the LSF, which we obtained as a perturbation solution of the scaling relations derived from integral boundary layer equations. For  $Pr = 1$  and aspect ratio  $\Gamma = 1$ ,  $Re_{bs} \sim Ra^{0.375}$  for small shear, similar to the observed flux scaling in a possible ultimate regime. The velocity at the edge of the natural convection boundary layers was found to increase with  $Ra$  as  $Ra^{0.35}$ ; since  $V_{bl} \sim Ra^{1/3}$ , this suggests a possible shear domination of the boundary layers at high  $Ra$ . The effect of shear, however, decreases with increase in  $Pr$  and with increase in  $\Gamma$ , and becomes negligible for  $Pr \geq 100$  at  $\Gamma = 1$  or for  $\Gamma \geq 20$  at  $Pr = 1$ , causing  $Re_{bs} \sim Ra_w^{1/3}$ .

**Key words:** turbulent convection, turbulent boundary layers, Bénard convection

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## 1. Introduction

Turbulent Rayleigh–Bénard convection (RBC) is characterised by diffusive regions near the plates that have boundary layers and thermal plumes, and a turbulent bulk region that has an inherent large-scale flow (LSF) within it. In turn, the LSF, driven by plume columns created by the LSF itself, is expected to act on the boundary layers, thereby modifying their nature. Since the dominant resistance to heat transfer is in the boundary layers, a resulting change in the flux scaling from the classical  $1/3$  power law is also expected. However, the exact nature of this modification of the boundary layers due to LSF is not known conclusively. In this study, we bring new insights into how LSF modifies the boundary layers by presenting the evolution of the instantaneous probability density functions (p.d.f.s) of horizontal velocity magnitudes in a horizontal plane close to the hot plate with Rayleigh number. Novel scaling laws, based on the consideration of the modification of the local boundary layers on either sides of the plumes, embedded within a shear boundary layer due to the LSF, are shown to describe the observed variation of the peaks of these p.d.f.s.

It is well known that if heat flux in turbulent RBC has to be independent of the layer height  $H$ , then, dimensionally,  $Nu \sim Ra^{1/3}$ , where  $Nu = q/(k \Delta T/H)$  is the Nusselt number, and  $Ra = g\beta \Delta T H^3/\nu\alpha$  is the Rayleigh number. Here,  $q$  is the heat flux,  $k$  is the thermal conductivity,  $\Delta T = T_h - T_c$  is the total temperature drop across  $H$ , with  $T_h$  and  $T_c$  the top and bottom wall temperatures, respectively,  $g$  is the acceleration due to gravity,  $\beta$  is the coefficient of thermal expansion,  $\nu$  is the kinematic viscosity, and  $\alpha$  is the thermal diffusivity. However, the observed flux scaling is anomalous, showing  $Nu \sim Ra^n$  with  $n \leq 0.31$  for  $Ra < 10^{11}$  (Roche *et al.* 2004),  $n$  approaching  $1/3$  for  $10^{10} < Ra < 10^{13}$  (He *et al.* 2012; Iyer *et al.* 2020), and the value of  $n$  beyond  $Ra = 10^{13}$  being not known conclusively. For  $Ra > 10^{13}$ , He *et al.* (2012) and Zhu *et al.* (2018) observed  $n = 0.38$ , which, however, has been suggested to be due to non-Boussinesq effects (Skrbek & Urban 2015; Urban *et al.* 2019) or a low range of  $Ra$  (Doering, Toppaladoddi & Wettlaufer 2019), and is contested (Zhu *et al.* 2019). At the same time, for  $10^{11} < Ra < 10^{15}$ ,  $n = 1/3$  has been observed by Iyer *et al.* (2020) in low-aspect-ratio ( $\Gamma = L/H$ ) containers, where  $L$  is the horizontal dimension of the container.

The value of  $n$  being different from  $1/3$  implies that the heat flux becomes dependent on  $H$ , as expected, due to the modification of the boundary layers by the LSF, whose velocity ( $V_F$ ) depends on  $H$  through the Reynolds number ( $Re$ ) relation

$$Re = V_F H / \nu = 0.139 Ra_w^{0.447} Pr^{-0.7}, \quad (1.1)$$

at  $\Gamma = 1$  (Ahlers, Grossman & Lohse 2009). Here,  $Ra_w = Ra/2$  is the near-plate Rayleigh number based on the temperature drop near the plate,  $\Delta T_w = T_h - T_B = \Delta T/2$ , with  $T_B$  being the bulk fluid temperature. Numerous hypotheses about the nature of such modification of the boundary layers have been made, resulting in various boundary layer models in RBC. Among these, the major models are those that assume stable diffusion layers with a mixing zone above them (Castaing *et al.* 1989), a turbulent shear boundary layer (Shraiman & Siggia 1990), a laminar, zero pressure gradient, Prandtl–Blasius boundary layer (PBBL) with no buoyancy effects (Grossmann & Lohse 2000), a Falkner–Skan shear boundary layer with pressure gradient (Shishkina, Horn & Wagner 2013), a laminar mixed convection boundary layer (Ovsyannikov *et al.* 2016), and a turbulent forced convection boundary layer with small buoyancy perturbations (Ching *et al.* 2019). The first three of these models are able to match the flux scaling (Ahlers

*et al.* 2009; Chilla & Schumacher 2012; Roche 2020), but fail to reproduce the observed profiles of velocity and temperature (Reeuwijk, Jonker & Hanjalić 2008; Sun, Cheung & Xia 2008; Shi, Emran & Schumacher 2012; Wagner, Shishkina & Wagner 2012), as well as the boundary layer thicknesses (Qiu & Xia 1998*a,b*; Lam *et al.* 2002; Reeuwijk *et al.* 2008). The remaining models try to obtain the mean velocity and temperature profiles, but the resulting flux scaling does not match the observations (Tai *et al.* 2021); clearly, the nature of boundary layers in RBC, and more importantly, their interaction with LSF, remain unresolved.

All of these models assume a single, steady boundary layer that spans the plate width, and compare the resulting profiles of velocity and temperature with the spatio-temporal mean profiles from experiments and computations; similarly with  $Nu$ , where the average  $Nu$  is compared. However, in contrast, the dynamics near the hot plate is highly unsteady and spatially inhomogeneous with local boundary layers, possibly natural convection type (Stewartson 1958; Rotem & Classen 1969; Pera & Gebhart 1973*a*), becoming unstable at multiple locations to give rise to line plumes (Pera & Gebhart 1973*b*). These line plumes merge, and move along their length, as well as rise (Puthenveetil, Ananthakrishna & Arakeri 2005; Puthenveetil & Arakeri 2005; Gunasegarane & Puthenveetil 2014), with the LSF also aligning them in shear dominant regions (Rama Reddy & Puthenveetil 2011; Shevkar *et al.* 2019). This results in a complex network of line plumes on the hot surface, which act as channels of heat transport from the plate to the bulk (Shishkina & Wagner 2008), with local boundary layers in between them. Given this high spatial inhomogeneity and temporal unsteadiness, an averaging – which mixes up these local boundary layers and line plumes, along with the LSF – to infer about a global boundary layer, and from which the flux, could be the reason for the lack of success of the available phenomenological models of RBC.

In the absence of external shear, the critical thickness of these local boundary layers on either sides of the line plumes scales as

$$\delta_{nc} = \left(\frac{C_1}{2}\right)^{2/5} Z_w Pr^{0.54}, \tag{1.2}$$

where

$$Z_w = \left(\frac{\nu\alpha}{g\beta\Delta T_w}\right)^{1/3} = \frac{H}{Ra_w^{1/3}} \tag{1.3}$$

is a length scale near the plate (Theerthan & Arakeri 1998),  $Pr = \nu/\alpha$  is the Prandtl number, and  $C_1 = 47.5$  (Puthenveetil *et al.* 2011). On the other hand, the PBBL thickness at  $L$ , for  $\Gamma = 1$ , is

$$\delta_{pb} = \frac{0.922H}{\sqrt{Re}} \tag{1.4}$$

(Ahlers *et al.* 2009; Stevens *et al.* 2013). Then  $\delta_{nc}/\delta_{pb} \sim Ra_w^{-1/9}$ , with  $\delta_{nc}/\delta_{pb} \sim 0.1$  at  $Ra_w = 10^9$ ; the local boundary layers are then an order thinner than the shear boundary layer engendered by the LSF, and hence are embedded within them. If a major part of the temperature drop near the plate occurs across these local boundary layers, then (1.2) already implies that  $Nu = H/2\delta_{nc} \sim Ra_w^{1/3}$ , the classical flux scaling. An immediate possibility then arises that the anomalous flux scaling in RBC is due to the modification of these local boundary layers on either side of the line plumes, embedded within the shear boundary layers, by the shear boundary layer itself.

Such shear-induced modifications of these local boundary layers have been observed as changes in the spacings between these line plumes (Shevkar *et al.* 2019) due to shear-induced changes in the stability (Castaing *et al.* 1989) of the local boundary layers in between. The plume spacings that are log-normally distributed in the absence of shear (Puthenveetil & Arakeri 2005) become more narrowly spread around a single value when plumes align in the presence of shear (Shevkar *et al.* 2019). The mean plume spacing at zero shear,

$$\lambda_0 = C_1 Pr^{n_1} Z_w \quad (1.5)$$

(Theerthan & Arakeri 1998; Puthenveetil & Arakeri 2005; Puthenveetil *et al.* 2011), where  $n_1 = 0.1$ , is increased to

$$\lambda = \lambda_0 + \frac{Z_{sh}S}{D} \quad (1.6)$$

(Shevkar *et al.* 2019), where  $Z_{sh} = \nu/V_L$  is the viscous shear length, with  $V_L$  being the shear velocity on the local boundary layers,  $S = V_L^3 \alpha / g \beta \Delta T_w \nu^2 = Re_L^3 / Ra_w$  is a shear parameter, with  $Re_L$  being the Reynolds number based on  $V_L$  and  $H$ , and  $D$  is a function of  $Pr$ . However, it is not known how the shear due to the LSF changes the velocities within these local boundary layers, as well as their thicknesses. These are two crucial pieces of information necessary to understand the shear-induced changes to the nature of these local boundary layers, which could be the key missing piece in understanding the anomalous flux scaling and the near-plate profiles of velocity and temperature, in RBC.

With this aim, the present paper presents the instantaneous spatial velocity fields within these local boundary layers, obtained by stereo particle image velocimetry (PIV) in a horizontal plane, for  $10^5 \leq Ra \leq 10^9$ ,  $4.69 \leq Pr \leq 5.88$  and  $1.71 \leq \Gamma \leq 10$ . We avoid any spatio-temporal averaging of these highly spatially inhomogeneous and temporally unsteady velocity fields, which would mix up the velocities from the local boundary layers, plumes and the LSF. Instead, we study the p.d.f.s of the horizontal velocity magnitudes, which show predominantly a single peak at lower  $Ra$ , a double peak at intermediate  $Ra$ , and then again a single peak at higher  $Ra$ . The peaks of these p.d.f.s keep shifting to higher velocity values with increasing  $Ra$ , showing the increasing effect of shear due to the LSF on the velocity field within the local boundary layers. We also observe a corresponding increase in areas affected by shear, first qualitatively, as an increase in areas of regions having aligned velocity vectors of higher magnitudes, and then later quantitatively, as an increase in areas contributing to the second mode in the double-peaked p.d.f.s. The velocities from these shear dominant regions, reflected in the second peak of the double-peaked p.d.f.s and the lone peak from the single mode p.d.f. at higher  $Ra$ , scale in the same way as the strength of the LSF scales with  $Ra$ . However, interestingly, the first peak from the double-mode p.d.f.s, and the sole peak from the p.d.f.s at lower  $Ra$ , scale as  $Ra^{0.38}$ , possibly in a coincidental way, in the same way as the observed scaling of flux at  $Ra > 10^{13}$ . We then show that this scaling can occur when a natural convection boundary layer, embedded within a shear boundary layer due to the LSF, is forced externally by the same shear boundary layer. In such a scenario, with increasing shear, we show that these local boundary layers can become mixed convection boundary layers, and then later at large shear, PBBLs. Our model predicts that the shear at the edge of these local boundary layers increases with  $Ra$ , more than the increase of buoyant velocities within the boundary layers with  $Ra$ , thereby making these boundary layers more shear-dominant with increase in  $Ra$ . This prediction, however, needs to be qualified since changes in plume

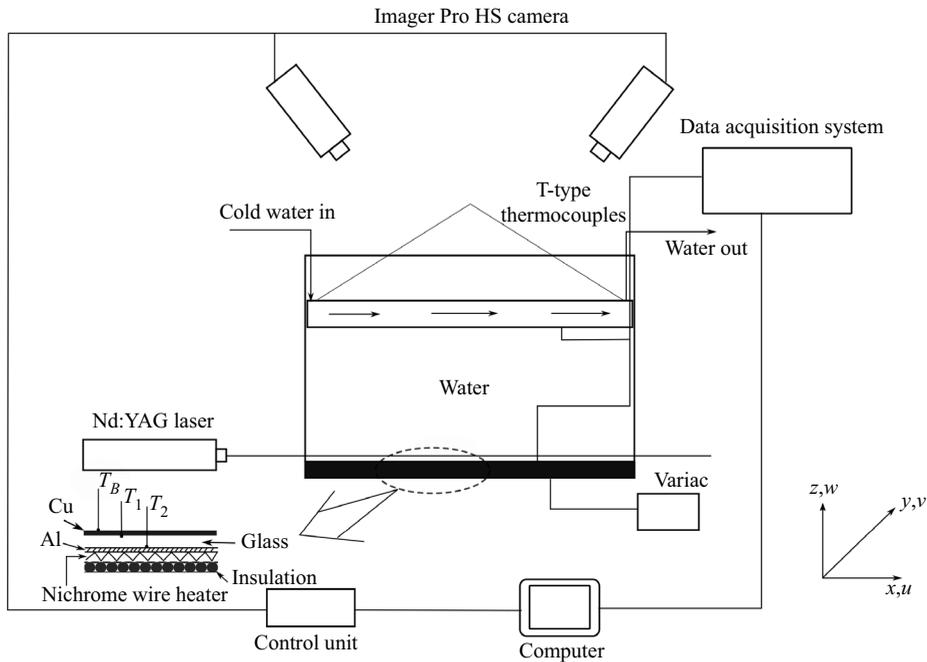


Figure 1. Schematic of the experimental set-up.

spacing and local boundary layer thicknesses, which could be significant at larger  $Ra$ , were neglected.

The paper is organised as follows. We first describe our PIV experiments in water at various  $\Gamma$  in § 2. The horizontal velocity fields within the local boundary layers, the characteristics of the p.d.f.s of horizontal velocity magnitudes, the evolution of the shear-affected areas, and that of the number of different types of p.d.f.s with  $Ra$ , as well as the experimental scaling of the peaks of the p.d.f.s, are described in § 3. Section 4 details the theoretical scaling analysis where the scaling equations (§ 4.1), obtained from the integral natural convection boundary layer equations forced by an external shear (Appendix A), are solved by perturbations methods (§ 4.2) to obtain the velocity boundary layer thickness (§ 4.3) and the dimensionless shear (§ 4.4) at the edge of the local boundary layers. These are then used to obtain the scaling of the characteristic Reynolds number within the local boundary layers, forced externally by the shear due to the LSF. The obtained scaling is shown to match the observations in § 4.5, before we conclude in § 5.

## 2. RBC experiments in water

Statistically steady, turbulent, RBC experiments were conducted in a glass tank, having cross-sectional area  $30 \text{ cm} \times 30 \text{ cm}$  and insulated from the sides, within which a water column of height  $H$  was confined between a hot copper plate at the bottom and a cool glass chamber at the top, through which water at constant temperature was circulated. The bottom Cu plate was maintained at a constant heat flux by a nichrome wire heater embedded within a plate assembly, as shown in figure 1, and connected to a variac. The heat flux into the Cu plate ( $q$ ) and  $H$  were varied to obtain the range of values of  $Ra$  and  $Pr$  given in table 1. The heat flux was estimated from the average temperature drop across the glass plate in the plate assembly, which was calculated from T-type thermocouples located

$Ra$	$Pr$	$\Delta T$ (°C)	$T_B$ (°C)	$H$ (cm)	$\Gamma$	$h_m$ (mm)	$h_F$ (mm)	$A_i$ (mm <sup>2</sup> )	$\delta_{pb}$ (mm)	$\delta_{nc}$ (mm)	$V_{bl}$ (mm s <sup>-1</sup> )	$V_L$ (mm s <sup>-1</sup> )	$V_F$ (mm s <sup>-1</sup> )	$w_{pl}$ (mm s <sup>-1</sup> )	$\Delta t$ (s)	$\nu$ (m s <sup>-2</sup> ) ( $\times 10^{-8}$ )
$5.52 \times 10^5$	5.88	0.99	26.52	3	10	2.7	—	$75 \times 64$	8.39	4.26	0.62	1.27	0.99	1.45	0.1	86.33
$3.32 \times 10^6$	5.21	4.86	31.63	3	10	1.5	7.5	$73 \times 61$	5.38	2.19	1.14	2.75	2.14	2.59	0.1	77.40
$2.37 \times 10^7$	5.69	3.17	27.97	7	4.29	1.8	—	$76 \times 65$	8.36	2.79	0.93	2.87	3.61	2.15	0.0667	83.60
$2.62 \times 10^7$	5.59	3.44	28.66	7	4.29	1.6	—	$76 \times 66$	8.12	2.67	0.97	2.99	3.76	2.18	0.1	82.30
$3.99 \times 10^7$	5.27	4.81	31.10	7	4.29	1.5	17.5	$75 \times 63$	7.24	2.25	1.12	3.56	4.49	2.53	0.1	78.22
$2.19 \times 10^8$	5.24	5.17	31.42	12	2.5	1.5	4	$75 \times 64$	8.46	2.18	1.15	4.42	7.58	2.62	0.0667	77.84
$4.41 \times 10^8$	5.30	5.33	30.90	15	2.0	1.0	—	$75 \times 63$	9.08	2.17	1.16	4.84	9.43	2.45	0.0667	78.63
$5.59 \times 10^8$	5.18	6.54	31.88	15	2.0	1.0	—	$74 \times 63$	8.55	1.98	1.26	5.35	10.42	2.68	0.0667	77.05
$6.30 \times 10^8$	5.00	7.12	33.38	15	2.0	1.0	4	$107 \times 87$	8.22	1.87	1.32	5.61	10.92	2.80	0.0667	74.73
$8.08 \times 10^8$	4.69	8.24	36.30	15	2.0	1.0	4, 10	$75 \times 63$	7.59	1.66	1.44	6.18	12.03	3.07	0.0667	70.60
$1.21 \times 10^9$	5.09	9.16	32.60	17.54	1.71	1.3	4	$76 \times 66$	8.35	1.77	1.40	6.43	13.66	3.19	0.0667	75.94

Table 1. Values of the experimental parameters and the parameters for the PIV measurements used in the present study. All the fluid properties were estimated at  $T_B$ , the bulk fluid temperature. The size of the square interrogation window ( $D_I$ ) was 32 pixels, and the overlap was 50% at all  $Ra$ .

$Ra_w$	$2.76 \times 10^5$	$1.66 \times 10^6$	$1.31 \times 10^7$	$1.10 \times 10^8$	$6.05 \times 10^8$
Number of p.d.f.s	4197	1399	11 192	11 192	11 192

Table 2. Total number of p.d.f.s at each  $Ra_w$ .

at three different horizontal positions across the glass plate. The temperatures of the hot Cu plate at the bottom and the cold glass plate at the top were measured at two different horizontal locations at the surface of the plates by T-type thermocouples. The average bottom and top plate temperatures,  $T_h$  and  $T_c$ , calculated from these measurements, were used to calculate the constant temperature difference  $\Delta T = T_h - T_c$  across the fluid layer, given in table 1. The error in temperature measurement was 0.02 °C, as estimated from repeated measurements of known temperatures.

The velocity field in a horizontal ( $x$ - $y$ ) plane close to the bottom hot plate was obtained by stereo PIV in a single-pulse, single-frame mode at all  $Ra$ . Since the height of the measurement plane ( $h_m$ ) was set so that  $Z_w/h_m \approx 0.2$ ,  $h_m$  was within the natural convection velocity boundary layer thickness  $\delta_{nc}$  (see (1.2)), and within the PBBL thickness  $\delta_{pb}$  (see (1.4)). The flow was seeded with neutrally buoyant polyamide particles (density  $\rho_p = 1.012 \text{ g cm}^{-3}$ ) of mean diameter 55  $\mu\text{m}$ , which were illuminated by a laser sheet of thickness  $l_l = 1 \text{ mm}$  from an Nd:YAG laser (Litron, 100 mJ pulse<sup>-1</sup>). The particles followed the flow since the Stokes number was less than  $4.2 \times 10^{-3}$ . Two Imager Pro HS (LaVision GmbH) double-shutter cameras (1024  $\times$  1280 pixels), oriented at 32.5° with the vertical from above, were used to capture particle images of area  $A_i = L_i \times W_i$  at the centre of the plate in a single-frame mode; the values of  $A_i$  for each  $Ra$  are given in table 1. Refraction errors were minimised by viewing the bottom plate through a water-filled prism placed over the top of the cold chamber, and by using a third-order polynomial mapping function obtained by imaging a calibration plate. The times between the laser pulses ( $\Delta t$ ), given in table 1, were chosen such that the highest out-of-plane particle displacements, due to the plume centre line vertical velocities, in the plane of measurement,

$$w_{pl} = 1.5 Pr^{0.27} \frac{\alpha}{h_m} Ra_{h_m}^{2/5} \quad (2.1)$$

(Gunasegarane & Puthenveetil 2014), where  $Ra_{h_m}$  is the Rayleigh number based on  $\Delta T$  and  $h_m$ , were not more than  $l_l/4$ . The maximum in-plane particle displacements corresponding to these  $\Delta t$  were approximately 10 pixels at each  $Ra$ . The frame rates of the single frames from each camera were varied from  $1/t_f = 10 \text{ Hz}$  at the lowest  $Ra$  to 15 Hz at the highest  $Ra$ , synchronous with the pulses from the laser.

A multipass adaptive window stereo cross-correlation method (DaVis, LaVision GMBH) was used on the successive frames from each camera to calculate the two-dimensional, three-component spatial vector fields in a horizontal plane at height  $h_m$  from the bottom plate with frequency  $1/t_f$ , for a time period of at least ten LSF circulation times. The size of the square interrogation window ( $D_I = 32 \text{ pixels}$ ) and the particle concentrations were chosen so that the displacement of particles was  $x_p \leq D_I/4$ , and at least 10 particles were present in an interrogation window at any time. Spurious vectors, removed by applying a median filter of 3 pix  $\times$  3 pix neighbourhood, were replaced by interpolated vectors. A Gaussian low-pass filter with kernel defined over a 9  $\times$  9 matrix was applied on the final vector field to reduce the noise. The spatial resolution  $L_v$  for all

the vector fields at each  $Ra$  was approximately 1 mm, except at  $Ra = 6.3 \times 10^8$ , where it was 1.4 mm; the parameters for PIV, used for each  $Ra$ , are listed in [table 1](#).

### 3. Probability density functions of horizontal velocities

#### 3.1. Procedure to obtain the p.d.f.s

To understand the velocities of dominant motions within the boundary layers, we now study the instantaneous p.d.f.s of  $U = \sqrt{u^2 + v^2}$ , the magnitude of the horizontal velocities within the local boundary layers, where  $u$  and  $v$  are the fluid velocities in the  $x$  and  $y$  directions, respectively. These instantaneous p.d.f.s were estimated from the instantaneous PIV velocity fields in horizontal planes at heights  $h_m < \delta_{nc} \ll \delta_{pb}$ , given in [table 1](#), at each  $Ra_w$ . An in-house code generated the instantaneous p.d.f.s using the following iterative procedure on each instantaneous spatial velocity field, separated temporally by  $t_f$ , from a set having 1399 such velocity fields; 1–8 such sets, separated temporally by 210 s, were used at each  $Ra_w$  to span at least 10 LSF times.

At any instant, an initial p.d.f. was first obtained from the spatial velocity field using 50 bins of equal sizes as

$$\text{p.d.f.} = \frac{N_U}{sb}, \quad (3.1)$$

where  $N_U$  is the number of occurrences of  $U$  in each bin,  $s$  is the sample size of  $U$ , and  $b$  is the bin size. We then proceed to estimate the total number of dominant peaks occurring in the p.d.f.s, so as to classify these p.d.f.s in the following way. In this calculation, since we are interested in the velocities of the dominant motions, peaks with  $N_U$  less than 10 % of the  $N_U$  of the highest peak are neglected. The minimum allowed separation between the peaks was kept equal to  $\frac{1}{4}(V_L/V_{bl} - 1)$ , to minimise the possibility of counting peaks due to fluctuations as dominant peaks. Here,  $V_L$  is the shear on the local boundary layers, estimated as the velocity of longitudinal motion of plumes by Gunasegarane & Puthenveetil (2014), given through the corresponding Reynolds number

$$Re_L = V_L H / \nu = 0.55 Ra_w^{4/9} Pr^{-2/3}. \quad (3.2)$$

Here,  $V_{bl}$  is the characteristic velocity in the local natural convection boundary layer (NCBL), given through the corresponding Reynolds number

$$Re_{bl} = V_{bl} H / \nu = 1.88 Ra_w^{1/3} Pr^{-0.98} \quad (3.3)$$

(Vipin 2013; Gunasegarane & Puthenveetil 2014). The rarely occurring fluctuations at the tails of the p.d.f.s, similar to that at  $U/V_{bl} \approx 7.75$  in [figure 2\(e\)](#), are also neglected in this estimation.

If the number of maxima detected by this procedure was greater than two, then the p.d.f. was recalculated by reducing the number of bins by one in successive iterations until the number of maxima became less than or equal to two. The iteration was stopped when the number of bins became 10, before two or fewer dominant peaks were detected. In such a case, peaks with frequencies less than 11 % of the frequency of the highest peak in the p.d.f. were neglected, and the above iterative process, with the initial number of bins equal to 50, was repeated. After this process, in the rare instances where the number of bins was still  $\leq 10$  with the number of peaks being  $> 2$ , the cut-off frequency was further increased to 12 % and the process repeated until a smooth p.d.f. with number of bins  $\geq 10$  and number of peaks  $\leq 2$  was obtained.

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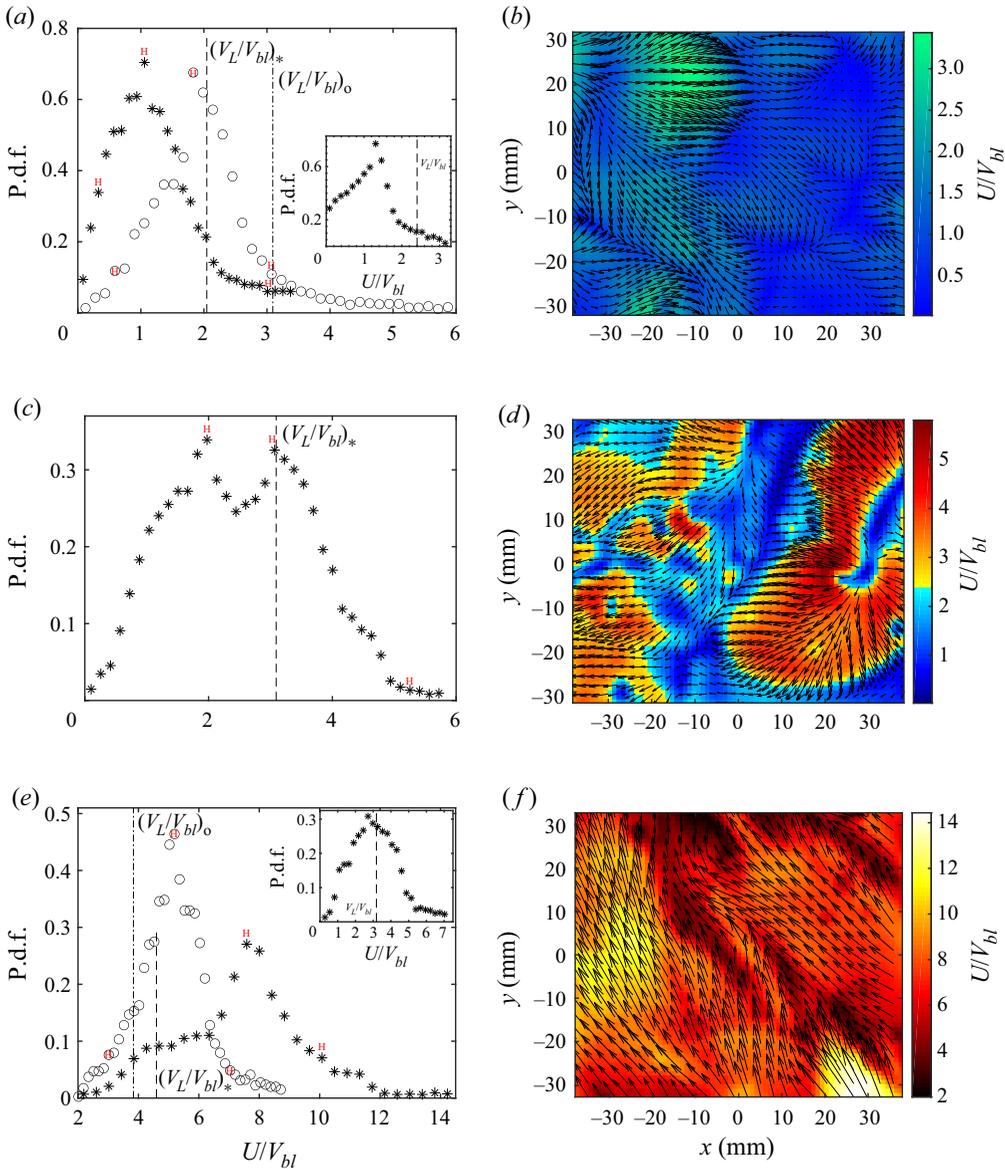


Figure 2. Instantaneous p.d.f.s of the dimensionless horizontal velocity magnitudes and the corresponding instantaneous spatial distributions of horizontal velocity vectors overlaid over the dimensionless horizontal velocity magnitudes at three  $Ra_w$ . Values of  $(Ra_w, \Gamma, Re_F)$  for  $*$  are: (a,b)  $(2.76 \times 10^5, 10, 34.42)$ ; (c,d)  $(1.31 \times 10^7, 4.3, 320)$ ; (e,f)  $(6.05 \times 10^8, 1.71, 3148)$ . Values for  $\circ$  are: (a)  $(1.31 \times 10^7, 4.3, 320)$ ; (e)  $(1.10 \times 10^8, 2.5, 1169)$ . In insets, values for  $*$  are: (a)  $(1.66 \times 10^6, 10, 82.86)$ ; (e)  $(2 \times 10^7, 4.3, 401.6)$ . The values of  $V_L/V_{bl}$  corresponding to  $*$  and  $\circ$  are shown by dashed lines and dash-dotted lines, respectively, in (a,c,e). The error bars in (a,c,e) show the maximum uncertainties 7.1%, 2.5% and 4% at  $U/V_{bl} = 0.6, 2$  and  $3$ , respectively. Details of error analysis can be found in Appendix C.

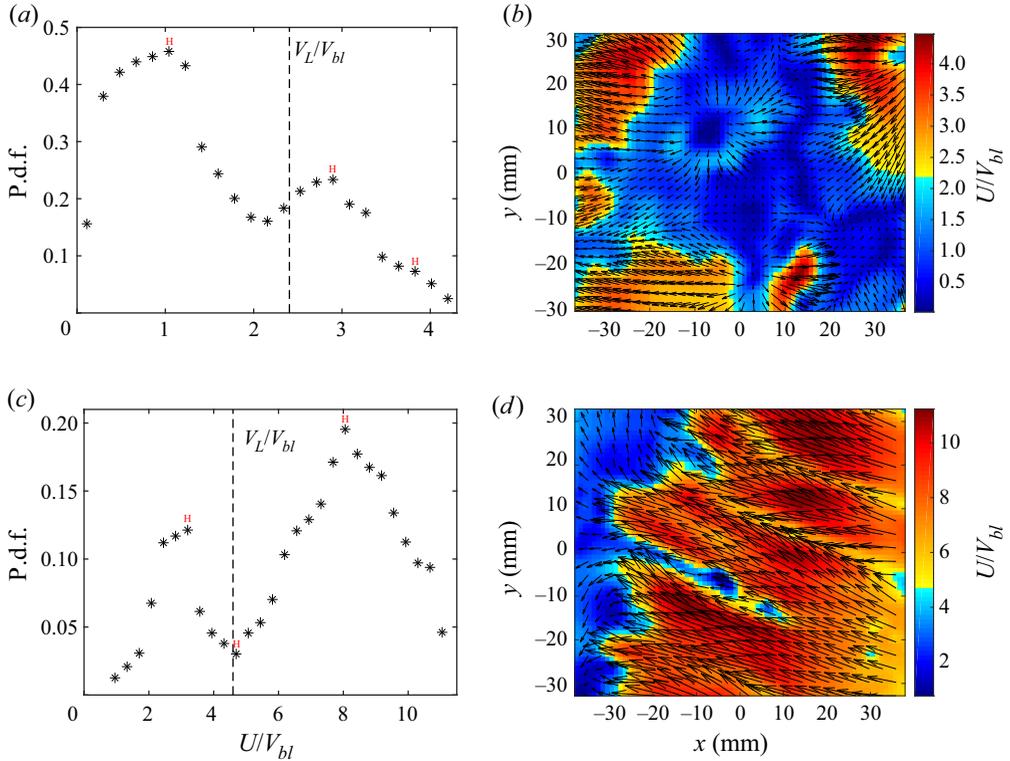


Figure 3. Instantaneous bimodal p.d.f.s of dimensionless horizontal velocity magnitudes, and the corresponding instantaneous spatial distributions of dimensionless horizontal velocity vectors, overlaid over the dimensionless horizontal velocity magnitudes, at intermediate and high  $Ra_w$ : (a,b)  $(Ra_w, \Gamma, Re_F) = (1.66 \times 10^6, 10, 82.86)$ ; (c,d)  $(Ra_w, \Gamma, Re_F) = (6.05 \times 10^8, 1.71, 3148)$ . The error bars in (a,c) show the maximum uncertainties of 3.8% and 4.9% at  $U/V_{bl} = 1.04$  and 3.2, respectively. Details of error analysis can be found in [Appendix C](#).

### 3.2. Types of p.d.f.s

In the present study,  $Ra_w$  was increased from the lowest value  $Ra_w = 2.76 \times 10^5$  to the highest value  $Ra_w = 6.05 \times 10^8$  by changing the temperature difference  $\Delta T$  and the cell height  $H$ . To increase  $Ra_w$ , (i) the cell height was kept constant and  $\Delta T$  was increased, or (ii) both  $\Delta T$  and  $H$  were increased. Then, with increasing  $Ra_w$ , the aspect ratio  $\Gamma = L/H$  in the experiments could either be a constant for the first case, or decrease monotonically for the second case. As shown in [table 1](#), the aspect ratio changed from 10 at  $Ra_w = 2.76 \times 10^5$  to 1.71 at  $Ra_w = 6.05 \times 10^8$ . However, in both of these cases, since the Reynolds number based on the LSF velocity  $V_F$  scaled as  $Re_F \sim Ra_w^{4/9} \Gamma^{-0.57}$  (see [\(B1\)](#)),  $V_F$  increased monotonically with increase in  $Ra_w$ , whether this increase in  $Re_F$  occurred due to the increase in  $\Delta T$  at a fixed  $\Gamma$  or due to a decreasing  $\Gamma$ , shown in [table 1](#). This increasing  $V_F$  with increasing  $Ra_w$  changed the shapes of the p.d.f.s and the locations of their peaks. So the change in the p.d.f. shapes and their peaks in the present study, shown in [figures 2, 3](#) and [4](#), was occurring primarily due to increase in  $Re_F$ , whether this increase in  $Re_F$  occurred due to increasing  $Ra_w$  and/or due to decreasing  $\Gamma$ .

Three different types of instantaneous p.d.f.s, obtained using the above procedure, at  $Ra_w = 2.76 \times 10^5$ ,  $1.31 \times 10^7$ ,  $6.05 \times 10^8$ , are shown in [figures 2\(a\), 2\(c\)](#) and [2\(e\)](#),

## Shear on local boundary layers in convection

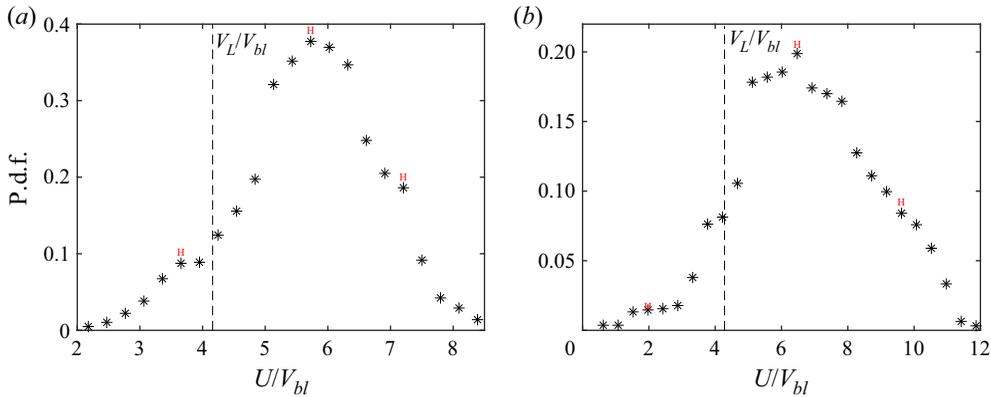


Figure 4. Instantaneous, type C p.d.f.s of dimensionless horizontal velocity magnitudes for two different  $Ra_w$  at fixed  $\Gamma$ : (a)  $(Ra_w, \Gamma, Re_F) = (2.21 \times 10^8, 2, 1800)$ ; (b)  $(Ra_w, \Gamma, Re_F) = (4.04 \times 10^8, 2, 2557)$ . The error bars show the maximum uncertainties 2.1 % and 7 % at  $U/V_{bl} = 3.66$  and 1.97, respectively. Details of error analysis can be found in [Appendix C](#).

respectively. The corresponding horizontal velocity fields in a horizontal plane at height  $h_m$  are shown in [figures 2\(b\), 2\(d\) and 2\(f\)](#), respectively. P.d.f.s in [figures 2\(a\) and 2\(c\)](#) are unimodal with peaks at  $U/V_{bl} = 1.06$  and 7.6, respectively, while the p.d.f. in [figure 2\(c\)](#) is bimodal with peaks at  $U/V_{bl} = 2$  and 3.1. The dashed vertical lines in [figures 2\(a\), 2\(c\) and 2\(e\)](#) show the values of the ratio  $V_L/V_{bl}$  of the shear velocity  $V_L$  (see (3.2)) and the boundary layer velocity  $V_{bl}$  (see (3.3)) at the corresponding  $Ra_w$ . We choose  $V_L$  as the relevant shear velocity, rather than  $V_F$  of (1.1), since  $V_L$  is measured closer to the plate at distances of the order of  $h_m$ , at which the present velocity fields are measured. The values of  $V_L$  and  $V_{bl}$  for the range of  $Ra_w$  in the present study are listed in [table 1](#).

It needs to be noted that at each  $Ra_w$ , all three types of p.d.f.s shown in [figure 2](#) were observed. However, as we discuss in § 3.4, the relative occurrence of these types of p.d.f.s changed with increasing  $Ra_w$ . At the lowest  $Ra_w = 2.76 \times 10^5$ , the p.d.f.s obtained were predominantly (45 %) of a unimodal nature with a peak at  $U/V_{bl} \approx 1$ , similar to that shown in [figure 2\(a\)](#); hereinafter, we refer to these as type A p.d.f.s. The most frequently occurring horizontal velocities at the lowest  $Ra_w$  are then of the order of  $V_{bl}$ , the NCBL velocity, given by (3.3). The spatial distribution of the velocities corresponding to these type A p.d.f.s, as shown in [figure 2\(b\)](#), shows a converging nature towards various randomly oriented lines.

At the highest near-plate Rayleigh number  $Ra_w = 6.05 \times 10^8$  also, we observed predominantly (71 %) unimodal p.d.f.s that were, however, peaking mostly at values greater than  $V_L/V_{bl}$ , as shown in [figure 2\(e\)](#) for a specific instant; hereinafter, we refer to these types of p.d.f.s as type C. The most frequently occurring horizontal velocities at the largest  $Ra_w$  are then of the order of, or greater than,  $V_L$ . The spatial distribution of velocities shown in [figure 2\(f\)](#), corresponding to these type C p.d.f.s, shows a more aligned nature, with larger magnitudes of velocity vectors compared to those with type A p.d.f.s at  $Ra_w = 2.76 \times 10^5$  in [figure 2\(b\)](#).

In contrast to p.d.f.s at low and high  $Ra_w$ , for an intermediate  $Ra_w = 1.31 \times 10^7$ , we obtain mostly (73 %) p.d.f.s of a bimodal nature. As shown in [figure 2\(c\)](#) for a specific instant, the first peaks of these bimodal p.d.f.s occur in between  $U/V_{bl} \approx 1$  and  $V_L/V_{bl}$ , while the second peaks occur at a value greater than or equal to  $V_L/V_{bl}$ . Hereinafter, the bimodal distributions of this form are referred to as type B. Unimodal p.d.f.s are

also observed at these intermediate  $Ra_w$ , but at only approximately 21% of the instants. Most of the unimodal p.d.f.s observed at this intermediate  $Ra_w$  peak at  $U/V_{bl} > 1 + \frac{2}{3}(V_L/V_{bl} - 1)$  or  $U/V_{bl} < 1 + \frac{1}{3}(V_L/V_{bl} - 1)$ . Horizontal velocities corresponding to the peaks of the former are much greater in magnitude compared to the velocities corresponding to the peaks of the latter. Further, the former type of p.d.f. is skewed to the left, similar to the p.d.f. shown in figure 2(e), while the latter type of p.d.f. is skewed to the right, similar to the p.d.f. shown in figure 2(a). For these reasons, we classify these unimodal p.d.f.s as type C and type A, respectively. Less than 1% of the total number of p.d.f.s at this intermediate  $Ra_w$  show a unimodal nature while peaking at  $1 + \frac{1}{3}(V_L/V_{bl} - 1) < U/V_{bl} < 1 + \frac{2}{3}(V_L/V_{bl} - 1)$ . For this very small number of unimodal p.d.f.s, we consider those peaking at  $U/V_{bl} < 1 + \frac{1}{2}(V_L/V_{bl} - 1)$  to be of type A. An instance of such a p.d.f. that occurs at the intermediate  $Ra_w = 1.31 \times 10^7$  is shown with the symbol  $\circ$  in figure 2(a), which peaks at  $U/V_{bl} = 1 + 0.4(V_L/V_{bl} - 1)$ . From these 1% p.d.f.s at intermediate  $Ra_w$ , those peaking at  $U/V_{bl} > 1 + \frac{1}{2}(V_L/V_{bl} - 1)$  are considered to be of type C; the p.d.f. shown by  $\circ$  in figure 2(e) is an example of such a case.

### 3.2.1. Effect of $Ra_w$

We now present the evolution of p.d.f.s with  $Ra_w$  at a fixed  $\Gamma$ . Figure 2(a) and its inset show the instantaneous p.d.f.s of the dimensionless horizontal velocity magnitudes at  $Ra_w = 2.76 \times 10^5$  and  $1.66 \times 10^6$  for  $\Gamma = 10$ . Both of the p.d.f.s are of type A, skewed to the right. With increasing  $Ra_w$ , the p.d.f. that peaked at  $U/V_{bl} = 1.06$  in figure 2(a) peaks at a higher magnitude  $U/V_{bl} = 1.31$  in the inset of figure 2(a). The horizontal velocity vector field corresponding to the p.d.f. shown in this inset is of a nature similar to that at the lowest  $Ra_w = 2.76 \times 10^5$  shown in figure 2(b).

Similarly, type C p.d.f.s for  $\Gamma = 2$  at  $Ra_w = 2.21 \times 10^8$  and  $4.04 \times 10^8$  are shown in figures 4(a) and 4(b). Both of these p.d.f.s are skewed to the left, similar to the type C p.d.f. at the highest  $Ra_w = 6.05 \times 10^8$ , shown in figure 2(e). With increasing  $Ra_w$ , the p.d.f. that peaked at  $U/V_{bl} = 5.73$  in figure 4(a) peaks at a higher magnitude  $U/V_{bl} = 6.47$  in figure 4(b). The horizontal velocity vector fields corresponding to the p.d.f.s shown in figures 4(a) and 4(b) are of a nature similar to that at the highest  $Ra_w = 6.05 \times 10^8$  shown in figure 2(f). Clearly, with increasing  $Ra_w$ , at a constant aspect ratio, the p.d.f.s of both type A and type C peak at increasing magnitudes of the dimensionless horizontal velocity  $U/V_{bl}$ .

### 3.2.2. Effect of aspect ratio $\Gamma$

We are unable to study the effect of varying aspect ratio on the p.d.f.s at the same  $Ra_w$ , since we have not explored such a parameter range in our experiments. However, by comparing three cases of type A p.d.f.s, at  $(Ra_w, \Gamma) = (2.76 \times 10^5, 10)$ ,  $(1.66 \times 10^6, 10)$  and  $(1.31 \times 10^7, 4.3)$  – shown with  $*$  in figure 2(a) and its inset, and with  $\circ$  in figure 2(a), respectively – we can get an idea of the effect of decreasing  $\Gamma$ . For  $\Gamma = 10$ , the type A p.d.f., which peaked at  $U/V_{bl} = 1.06$  at  $Ra_w = 2.76 \times 10^5$  in figure 2(a), shifted to peak at  $U/V_{bl} = 1.31$  at  $Ra_w = 1.66 \times 10^6$  in the inset of figure 2(a); the shift in the peak of the p.d.f. was then by 24% for a change in  $Ra_w$  by a factor of six. This type A p.d.f. shifted to peak at  $U/V_{bl} = 1.83$  at  $Ra_w = 1.31 \times 10^7$  when  $\Gamma$  decreased to 4.3 from 10 in figure 2(a); the shift in the peak of the p.d.f. was by 40% for a change in  $Ra_w$  by a

factor of eight. This larger shift of type A p.d.f., compared with the former case at the same  $\Gamma$ , occurred possibly due to the decrease of  $\Gamma$  from 10 at  $Ra_w = 1.66 \times 10^6$  to 4.3 at  $Ra_w = 1.31 \times 10^7$  in the latter case.

Similar effects of decreasing  $\Gamma$  can also be found on type C p.d.f.s by comparing three cases of type C p.d.f.s at  $(Ra_w, \Gamma) = (2 \times 10^7, 4.3)$ ,  $(1.10 \times 10^8, 2.5)$  and  $(6.05 \times 10^8, 1.71)$ . These cases are shown in the inset of [figure 2\(e\)](#) and with the symbols  $\circ$  and  $*$  in [figure 2\(e\)](#), respectively. The p.d.f. at  $Ra_w = 2 \times 10^7$ , shown in the inset of [figure 2\(e\)](#), is skewed to the left, similar to the p.d.f. at  $Ra_w = 6.05 \times 10^8$ , shown in [figure 2\(e\)](#). The velocity field corresponding to the p.d.f. shown in the inset of [figure 2\(e\)](#) is similar to that at the highest  $Ra_w = 6.05 \times 10^8$  shown in [figure 2\(f\)](#). However, the type C p.d.f. that peaked at  $U/V_{bl} = 2.7$  at intermediate  $Ra_w = 2 \times 10^7$  shifted to peak at  $U/V_{bl} = 5.2$  at  $Ra_w = 1.10 \times 10^8$ , with  $\circ$  in [figure 2\(e\)](#). The shift in the peak of the p.d.f. was by 93 % for a change in  $\Gamma$  by a factor of 1.7, and in  $Ra_w$  by a factor of 5.5. With increase in  $Ra_w$  to  $Ra_w = 6.05 \times 10^8$  by the same factor of 5.5, the type C p.d.f. shifted to a higher value of  $U/V_{bl} = 7.6$ , with  $*$  in [figure 2\(e\)](#). The shift in p.d.f. was by 46 % for a change in  $\Gamma$  by a factor of 1.45, when  $Ra_w$  increased by the same factor of 5.5. This larger shift of the p.d.f. when  $Ra_w$  increased from  $2 \times 10^7$  to  $1.10 \times 10^8$ , compared to the case when  $Ra_w$  increased by the same factor from  $1.10 \times 10^8$  to  $6.05 \times 10^8$ , is possibly due to the larger decrease in  $\Gamma$  by a factor of 1.7 in the former case, as against the decrease in  $\Gamma$  by a factor of 1.45 in the latter case. Clearly, these larger shifts in type A and type C peaks to higher magnitudes of the dimensionless horizontal velocity  $U/V_{bl}$  are due to the increased cell height  $H$  or the decreased  $\Gamma$ .

In summary, the p.d.f.s of the boundary layer velocity field show peaks at velocities of different magnitudes based on the value of  $Ra_w$ . At low  $Ra_w$ , we obtain unimodal p.d.f.s (type A) that peak at approximately the NCBL velocities  $V_{bl}$  (see (3.3)). At high  $Ra_w$ , we again obtain unimodal p.d.f.s (type C) that peak predominantly at (or greater than)  $V_L$  (see (3.2)), which is the appropriate scale for shear velocities very close to the hot plate. At intermediate  $Ra_w$ , the p.d.f.s are bimodal (type B). The first peak of the bimodal distribution occurs at  $1 < U/V_{bl} < V_L/V_{bl}$ , which we expect to be due to the forcing of the NCBL velocities by the shear due to the LSF; the second peak corresponds to the shear velocities  $V_L$  (see (3.2)). A small number of unimodal p.d.f.s that occur at these intermediate  $Ra_w$  are classified as type A or type C based on whether their peak is closer to  $V_{bl}$  or  $V_L$ , respectively.

As mentioned earlier, and discussed in § 3.4, bimodal p.d.f.s are observed at lower  $Ra_w$ , as well as at higher  $Ra_w$ ; the frequency of occurrence of such bimodal p.d.f.s is, however, low at these  $Ra_w$ . Further, in such bimodal p.d.f.s observed at lower  $Ra_w$ , the likelihood of occurrence of the first peak velocities was much higher than that of the second peak velocities, as shown in [figure 3\(a\)](#) at  $Ra_w = 1.66 \times 10^6$ . In contrast, at the highest  $Ra_w = 6.05 \times 10^8$ , the bimodal p.d.f.s show the likelihood of occurrence of the second peak velocities to be much higher than that of the first peak velocities, as seen in [figure 3\(c\)](#). Hence, comparing the bimodal p.d.f.s at the lower, intermediate and highest  $Ra_w$  cases in [figures 3\(a\)](#), [2\(c\)](#) and [3\(c\)](#), respectively, we conclude that the likelihood of occurrence of  $V_{bl}$  decreases with increase in  $Ra_w$ , while that of  $V_L$  increases with increase in  $Ra_w$ .

As we saw earlier, the velocity fields corresponding to the type A p.d.f.s are oriented randomly (see [figure 2\(b\)](#)), implying absence of significant shear due to the LSF in most regions. In contrast, the velocity field corresponding to type C p.d.f.s, shown in [figure 2\(f\)](#), has aligned velocity vectors in most of the domain, implying that the velocity field is

strongly affected by the shear due to the LSF in most of the domain. In the case of bimodal p.d.f.s, we separate the velocity field into two regions at the value of  $U/V_{bl}$  at the trough between the two peaks. **Figure 2(d)** shows the regions corresponding to velocities of the first mode as the blue/cyan regions, and the regions corresponding to the velocities of the second mode as red/yellow regions. The velocities in the blue regions are of random nature, similar to the velocities in **figure 2(b)**, while those in the red region are of aligned nature, similar to the velocities in **figure 2(f)**. The decreasing likelihood of occurrence of  $V_{bl}$ , and the corresponding increasing likelihood of occurrence of  $V_L$  with increasing  $Ra_w$ , seen in the p.d.f.s in **figures 3(a), 2(c) and 3(c)**, can also be clearly seen in the corresponding velocity fields in **figures 3(b), 2(d) and 3(d)**. In these figures, with increase in  $Ra_w$ , the areas of the blue/cyan regions, which are mildly affected by shear, decrease, compared to the areas strongly affected by shear, in the red/yellow regions.

### 3.3. Area affected by shear

We now study the evolution of areas of such shear affected regions with  $Ra_w$ . At any instant, for bimodal p.d.f.s, we calculate the ratios of areas corresponding to the first mode ( $A_{bl}$ ) and the second mode ( $A_{sh}$ ) to the total area ( $A$ ). In unimodal p.d.f.s of type A and type C, a similar splitting of the total area  $A$  into areas strongly affected by shear ( $A_{sh}$ ) and areas mildly affected by shear ( $A_{bl}$ ) can be achieved as well, by using appropriate thresholds, as described below. In **figure 2(b)**, which is the velocity field corresponding to a p.d.f. of type A shown in **figure 2(a)**, the green region around  $(-10, 20)$  mm has velocity vectors of aligned nature with velocity magnitudes greater than  $V_L$ . The velocity vectors in this region are of a nature similar to that in regions corresponding to the second mode (red/yellow regions in **figure 2d**) in bimodal p.d.f.s, or to those in most of the regions corresponding to type C p.d.f.s (**figure 2f**). Hence when unimodal p.d.f.s of type A occur, we separate the regions strongly affected by shear from the regions mildly affected by shear using a threshold at  $U/V_{bl} > V_L/V_{bl}$ . Since the velocities below  $U/V_{bl} = 1$  are not the shear velocities due to the LSF (see (3.2)), when p.d.f.s of type C occur, we separate the areas mildly affected by shear from the areas strongly affected by shear using the threshold  $U/V_{bl} < 1$ . In this way, at each  $Ra_w$ , we calculate the instantaneous area ratios  $A_{bl}/A$  and  $A_{sh}/A$  at all the instants, using the appropriate threshold from the above three thresholds, depending upon the type of p.d.f. that occurs at each instant.

**Figure 5** shows the variation of  $\langle A_{bl}/A \rangle$  and  $\langle A_{sh}/A \rangle$  with  $Ra_w$ , where  $\langle \cdot \rangle$  indicates the mean of the instantaneous area ratios over all the instants at each  $Ra_w$ . The values of  $\langle A_{bl}/A \rangle$  decrease with increase in  $Ra_w$ , while those of  $\langle A_{sh}/A \rangle$  increase with increase in  $Ra_w$ ; increase in  $Ra_w$  then increases the areas strongly affected by shear due to the LSF. This increase is drastic in the range  $1.66 \times 10^6 \leq Ra_w \leq 1.10 \times 10^8$ , with approximately 20 % of the area strongly affected by shear in the lower limit, changing to 90 % of the area strongly affected by shear in the higher limit; quite small variations are seen in the extent of areas strongly affected by shear beyond this range of  $Ra_w$ . The increase of  $\langle A_{sh}/A \rangle$  shows a trend opposite to the decrease of  $\langle A_{bl}/A \rangle$ , as expected, since  $\langle A_{bl}/A \rangle + \langle A_{sh}/A \rangle = 1$ . As shown in **figure 5**, the dependence of  $\langle A_{sh}/A \rangle$  on  $Ra_w$  can be approximated well by

$$\langle A_{sh}/A \rangle = 1 - (a e^{bRa_w} + c e^{dRa_w}), \tag{3.4}$$

where  $a = 0.7229$ ,  $b = -0.1416 \times 10^{-7}$ ,  $c = 0.09798$  and  $d = -0.5553 \times 10^{-11}$ . The relation (3.4) implies that at  $Ra_w \approx 10^{12}$ ,  $\langle A_{sh}/A \rangle = 1$ ; the whole area is then affected by shear. However, this specific limiting value of  $Ra_w$  needs to be verified further with measurements at higher  $Ra_w$  since (3.4) is not accurate beyond  $Ra_w = 6.05 \times 10^8$ .

## Shear on local boundary layers in convection

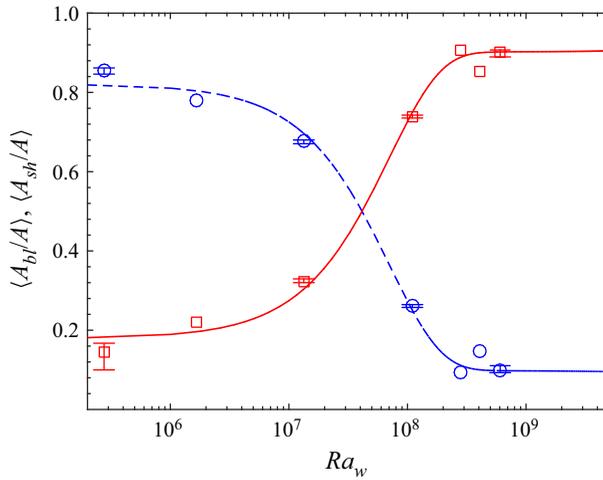


Figure 5. Variation of the mean fraction of areas mildly affected by shear  $\langle A_{bl}/A \rangle$ , and that of areas strongly affected by shear  $\langle A_{sh}/A \rangle$ , with  $Ra_w$ . Red squares indicate  $\langle A_{sh}/A \rangle$ ; blue circles indicate  $\langle A_{bl}/A \rangle$ ; red solid line uses (3.4); blue dashed line uses (3.5). The error bars show the estimated errors 10.8 % in  $A_{bl}/A$  and 31.6 % in  $A_{sh}/A$  at  $Ra_w = 6.05 \times 10^8$  and  $2.76 \times 10^5$ , respectively. Details of error analysis can be found in Appendix C.

Since  $A_{sh} + A_{bl} = A$ , the corresponding relation for  $\langle A_{bl}/A \rangle$  will be

$$\langle A_{bl}/A \rangle = 1 - \langle A_{sh}/A \rangle, \quad (3.5)$$

with  $\langle A_{sh}/A \rangle$  given by (3.4).

### 3.4. Number of occurrences of p.d.f.s

As mentioned earlier, at each  $Ra_w$ , all three types of p.d.f.s occur with varying relative occurrences. The percentages of the numbers of type A, B and C p.d.f.s in the total number of p.d.f.s, at each of the five different  $Ra_w$ , are shown in figure 6. At the smallest  $Ra_w = 2.76 \times 10^5$  in figure 6, the p.d.f.s are mostly type A and type B, with the number of type A p.d.f.s (46 %) being roughly equal to the number of type B p.d.f.s (53 %). The type A p.d.f.s peak at  $U/V_{bl} \sim 1$  at this  $Ra_w$  (figure 2a). Even though type B p.d.f.s are dominant at this  $Ra_w$ , since these type B p.d.f.s were similar to that in figure 3(a), the probabilities of occurrence of the first peak velocities were much higher than those of the second peak velocities. The velocities at this  $Ra_w$  were then mostly of the order of  $V_{bl}$  (see (3.3)).

As  $Ra_w$  increases from  $2.76 \times 10^5$  to  $1.31 \times 10^7$ , the percentages of both type B and type C p.d.f.s increase from 53 % to 73 %, and from 1 % to 6 %, respectively, at the expense of type A p.d.f.s, which decrease from 46 % to 21 %. With increase in  $Ra_w$ , the first peaks of the type B p.d.f.s shift from  $U/V_{bl} \sim 1$  at the lowest  $Ra_w$  (figure 3a), through  $U/V_{bl} \sim 2$  at the intermediate  $Ra_w$  (figure 2c), to  $U/V_{bl} \approx 3$  at higher  $Ra_w$  (figure 3c). Similarly, the type A p.d.f.s – which peaked at  $U/V_{bl} = 1$  at low  $Ra_w$  – shift with increase in  $Ra_w$ , to peak at  $U/V_{bl} = 1.83$ , as shown in figure 2(a). The local NCBLs, which contribute to the type A p.d.f.s and the first peak of the type B p.d.f.s, could then be changing to a mixed convection type with increase in  $Ra_w$ .

Comparing the last three  $Ra_w$  in figure 6, we see that the percentage of type C p.d.f.s increases from 6 % to 70 % with increase in  $Ra_w$  from  $1.31 \times 10^7$  to  $6.05 \times 10^8$ , at the expense of both the type A and type B p.d.f.s. Further, as shown in figure 2(e), peaks of

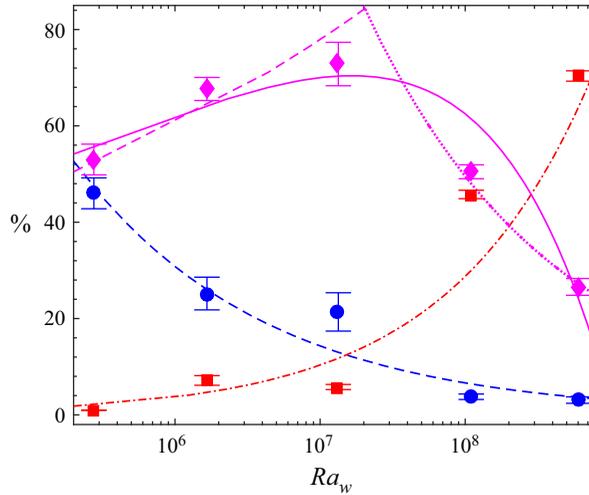


Figure 6. Variation of the percentages of type A, type B and type C p.d.f.s in the total number of p.d.f.s, with  $Ra_w$ . The total numbers of p.d.f.s at each  $Ra_w$  are given in table 2. Blue circles indicate type A; magenta diamonds indicate type B; red squares indicate type C; blue dashed line for  $3080 Ra_w^{-1/3}$ ; magenta solid line for  $14.8 Ra_w^{1/9} - 0.0145 Ra_w^{-4/9}$ ; red dash-dotted line for  $0.008 Ra_w^{4/9}$ ; magenta dashed line for  $13 Ra_w^{1/9}$ ; magenta dotted line for  $23\,000 Ra_w^{-1/3}$ . The error bars show the errors 7.1 % in type A p.d.f.s, 18.7 % in type B p.d.f.s, and 23.6 % in type C p.d.f.s, at  $Ra_w = 2.76 \times 10^5$ . Details of error analysis can be found in Appendix C.

type C p.d.f.s shift from  $U/V_{bl} = 5$  at an intermediate  $Ra_w = 1.10 \times 10^8$  to  $U/V_{bl} = 7.6$  at the highest  $Ra_w = 6.05 \times 10^8$ . At the highest  $Ra_w = 6.05 \times 10^8$ , the shear velocities caused by the LSF dominate the boundary layer flow field even though approximately 30 % of the p.d.f.s are still type A and type B, which have their sole/first modes showing mixed convection nature. Figure 5 shows that these mixed convection regions that are mildly affected by shear are limited to approximately 10 % of the area at the highest  $Ra_w$ .

We now look at the scaling of percentages of types of p.d.f.s with  $Ra_w$ . As shown in figure 6, as  $Ra_w$  increases, the percentages of type A peaks decrease from 46 % at  $Ra_w = 2.76 \times 10^5$  and become negligible (3 %) at the highest  $Ra_w$ . The figure shows that the percentages of type A peaks scale as

$$N_A = 3080 Ra_w^{-1/3}, \tag{3.6}$$

in the same way as  $\lambda_0$  (see (1.5)) scales with  $Ra_w$ . At the same time, the percentages of type C peaks increase from 1 % at  $Ra_w = 2.76 \times 10^5$  and become the highest (70 %) at the highest  $Ra_w$ , scaling as

$$N_C = 0.008 Ra_w^{4/9}, \tag{3.7}$$

in the same way as the shear velocities caused by the LSF  $V_L$  (see (3.2)) scale with  $Ra_w$ . The variation of percentages of type B peaks with  $Ra_w$  is non-monotonic, as shown in figure 6, and can be fitted with a combination of increasing and decreasing power-law functions, of the form

$$N_B = 14.8 Ra_w^{1/9} - 0.0145 Ra_w^{-4/9}. \tag{3.8}$$

Interestingly, as shown in figure 6, for the first three  $Ra_w$ , this relation has an effective exponent  $1/9 = 4/9 - 1/3$ , a sum of the power-law exponents of the variations of

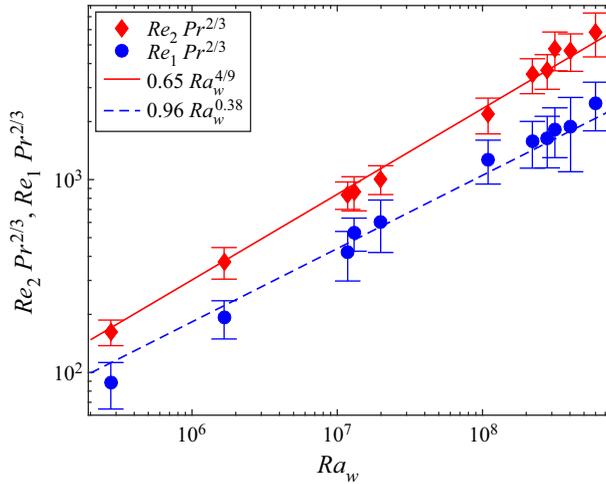


Figure 7. Variation of the mean Reynolds numbers based on the first peak velocities of type B p.d.f.s and the sole peak of the type A p.d.f.s ( $Re_1$ ), and that based on the second peak velocities of type B p.d.f.s and the sole peak of the type C p.d.f.s ( $Re_2$ ), with  $Ra_w$ .

percentages of type C and type A p.d.f.s, respectively. For the last two  $Ra_w$ , the effective power-law exponent for the variation of the percentages of type B p.d.f.s is  $-1/3$ .

### 3.5. Experimental scaling of peak velocities

At each  $Ra_w$ , from various instants, using all the second peaks of the bimodal p.d.f.s of type B, and the sole peaks of the unimodal p.d.f.s of type C, whichever occurs at each instant, we calculate the mean velocity  $V_2$ , and the corresponding Reynolds number  $Re_2 = V_2 H / \nu$ . In estimating  $V_2$ , we neglect the instants when type A p.d.f.s occur. Figure 7 shows the variation of  $Re_2$  with  $Ra_w$ , along with (3.2), with the error bars showing the  $2\sigma$  variation of the  $V_2$  values; it is clear that  $Re_2$  scales in the same way as  $Re_L$  given by (3.2), with a small increase in the prefactor. Similarly, at each instant, we calculate the mean from the first peak of the bimodal p.d.f.s and the unimodal peak of type A p.d.f.s, whichever occurs at that instant, for each  $Ra_w$ . We denote this mean velocity as  $V_1$ , with the corresponding Reynolds number as  $Re_1 = V_1 H / \nu$ ; p.d.f.s of type C are skipped in this calculation. Figure 7 shows the variation of  $Re_1$  with  $Ra_w$ ; interestingly,  $Re_1 \sim Ra_w^{0.38}$ , the same dependence as that of  $Nu$  on  $Ra_w$  observed by He *et al.* (2012) at high  $Ra_w$ . The variation of  $Re_1$  does not follow the  $Ra_w^{1/3}$  scaling of NCBL velocities (3.3), which are expected to occur in the absence of external shear; neither does it follow the scaling of  $Re_L$  in (3.2) due to LSF. As we have seen in § 3.2, the peaks of type A p.d.f.s and the first peak of the type B p.d.f.s occur at values larger than  $V_{bl}$  with increasing  $Ra_w$ , as expected due to the forcing of the boundary layers by the increasing LSF strength. We hence expect the deviation of  $Re_1$  from (3.3) to be due to the shear generated by the LSF changing the characteristic velocities within the boundary layer. Based on this expectation, we now look at the expected theoretical variation of the characteristic velocity within NCBLs when these local boundary layers are forced by the shear due to the LSF velocity  $V_F$ .

#### 4. Scaling of boundary layer velocity forced by shear

##### 4.1. Scaling relation for horizontal velocity

We consider a two-dimensional laminar NCBL on a horizontal surface, forced by an external shear due to an LSF of strength  $V_F$ , as shown in [figure 8](#). The external shear at the edge of the boundary layer is of strength  $AV_F$ , where  $A = u_{y=\delta}/V_F$  is the dimensionless shear at the edge of the boundary layers. We consider local boundary layers that occur on either side of the plumes on the hot plate, unlike the global, single boundary layer that is considered in many flux scaling theories ([Grossmann & Lohse 2000](#); [Ahlers \*et al.\* 2009](#); [Chilla & Schumacher 2012](#); [Scheel & Schumacher 2017](#)). These local boundary layers are an order thinner than the global boundary layers ( $\delta_{nc}/\delta_{pb} = 0.2$  at  $Ra = 1.21 \times 10^9$ ), which, due to their larger thickness, are more likely to be affected by the turbulent bulk. These local boundary layers on the hot plate become unstable to turn upwards and form the plumes. Since plumes are the outcome of the instability of these thin local boundary layers, on either side, it is natural to expect that before they become unstable, these boundary layers will be laminar. Further, successful scaling laws for the mean spacing between such line plumes ([Theerthan & Arakeri 1998, 2000](#); [Puthenveetil & Arakeri 2005](#)), their total lengths ([Puthenveetil \*et al.\* 2011](#)) and their mean dynamics ([Gunasegarane & Puthenveetil 2014](#)) have all been obtained assuming steady two-dimensional laminar NCBLs ([Rotem & Classen 1969](#); [Pera & Gebhart 1973a](#)) feeding these plumes. More importantly, the observed vertical distributions of fluctuations of velocities and temperature near the hot plate have been predicted well by a model that assumes laminar boundary layers giving rise to laminar plumes ([Theerthan & Arakeri 1998](#)). This means that the observed fluctuations in velocity and temperature near the hot plate are created by the spatial averaging of a spatially non-uniform field consisting of many laminar local boundary layers giving rise to many plumes, as well as by the lateral motion of such plumes ([Shevkar & Puthenveetil 2022](#)). Also, as an *a posteriori* justification, the scaling laws obtained in the present study using two-dimensional steady laminar local boundary layer equations, forced by shear, match well with our measurements in turbulent RBC shown later, in [figure 10](#). For these reasons, we expect that our assumption of a laminar mixed convection nature of the local boundary layers that occurs on either side of the plumes on the plate in turbulent convection, which we use to find the scaling of peak velocities in areas mildly affected by shear, to be valid, for at least  $Ra \lesssim 10^9$ . The integral boundary layer equations

$$\frac{\partial}{\partial x} \int_0^\delta u^2 dz - AV_F \frac{\partial}{\partial x} \int_0^\delta u dz - g\beta \int_0^\delta \frac{\partial}{\partial x} \int_0^\delta (T - T_B) dz dz + \nu \left. \frac{\partial u}{\partial z} \right|_{z=0} = 0 \quad (4.1)$$

and

$$\frac{\partial}{\partial x} \int_0^{\delta_T} u(T - T_B) dz + \alpha \left. \frac{\partial T}{\partial z} \right|_{z=0} = 0, \quad (4.2)$$

derived as [\(A8\)](#) and [\(A13\)](#), relate the horizontal velocity and temperature distributions within such boundary layers.

By using the characteristic scale for the driving temperature difference and the scales for the lengths in the horizontal and vertical directions in [\(4.1\)](#) and [\(4.2\)](#), with the external shear given by  $AV_F$ , we now obtain scaling relations for  $U_c$ , the characteristic horizontal velocities within NCBLs forced by the shear due to the LSF. The characteristic scale of temperature difference in [\(4.1\)](#) and [\(4.2\)](#) is  $\Delta T_w$ . The vertical length scale in [\(4.1\)](#) is  $\delta$ , while that in [\(4.2\)](#) is  $\delta_T$ , as shown in [figure 8](#). Since these boundary layers become unstable

## Shear on local boundary layers in convection

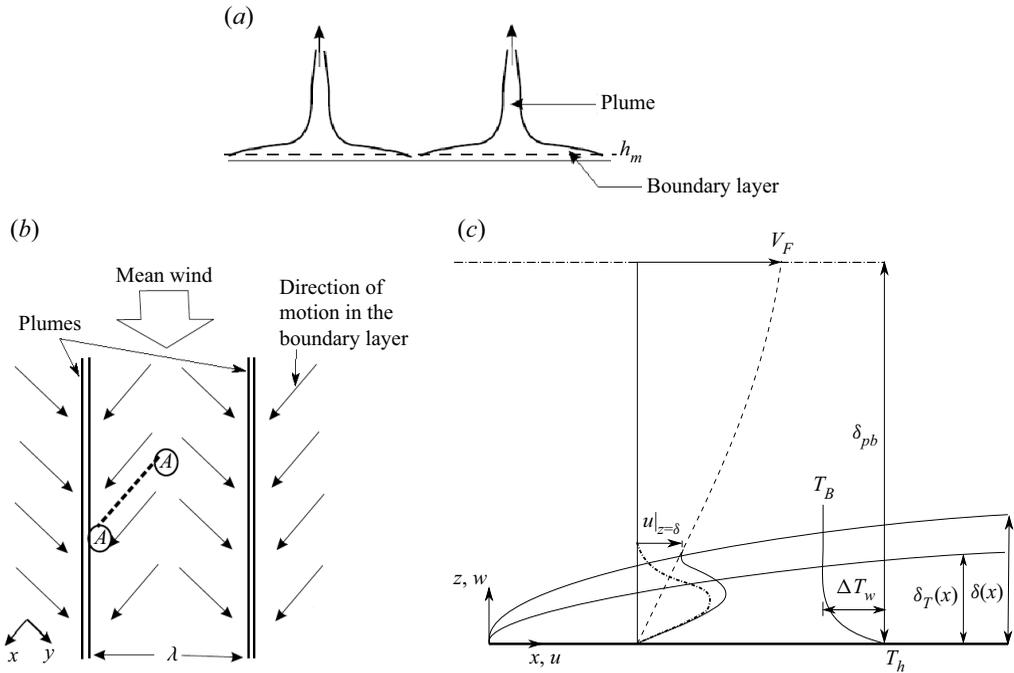


Figure 8. Schematic of the local boundary layer being forced by an external shear due to the LSF for  $Pr > 1$ : (a) side view; (b) top view, and (c) view along line A–A. Dash-dotted line indicates horizontal velocity profile within an NCBL; dashed line indicates horizontal velocity profile within a PBBL; solid line indicates horizontal velocity profile due to forcing by external shear of NCBL velocities.

at a horizontal distance to result in line plumes, the appropriate characteristic horizontal distance is half of the mean plume spacing  $\lambda$ , given by (1.6). However, for the present range of  $Ra_w$ , the second term in (1.6) becomes at least an order smaller than the first term and can hence be neglected; then  $\lambda \approx \lambda_0$ . Using these characteristic scales, the order of magnitude balances of (4.1) and (4.2) become

$$\frac{U_c^2 \delta}{\lambda_0/2} - \frac{AV_F U_c \delta}{\lambda_0/2} - \frac{g\beta \Delta T_w \delta^2}{\lambda_0/2} + \frac{\nu U_c}{\delta} \sim 0 \quad (4.3)$$

and

$$\frac{U_c \Delta T_w \delta_T}{\lambda_0/2} \sim \frac{\alpha \Delta T_w}{\delta_T}. \quad (4.4)$$

We now assume

$$\frac{\delta}{\delta_T} = C_2 Pr^n, \quad (4.5)$$

where  $C_2$  and  $n$  are constants. Eliminating  $\delta$  from (4.3) using (4.5) and (4.4), we obtain the scaling relation for  $U_c$  in terms of  $Pe_\lambda = U_c \lambda_0 / 2\alpha$  – the Péclet number based on  $\lambda_0$  – as

$$8E \frac{Pe_\lambda^{5/2}}{Ra_\lambda} - \epsilon \frac{4A}{Pr^n} Pe_\lambda^{3/2} - C_2 \sim 0, \quad (4.6)$$

where

$$\epsilon = Re_\lambda / Ra_\lambda, \quad (4.7)$$

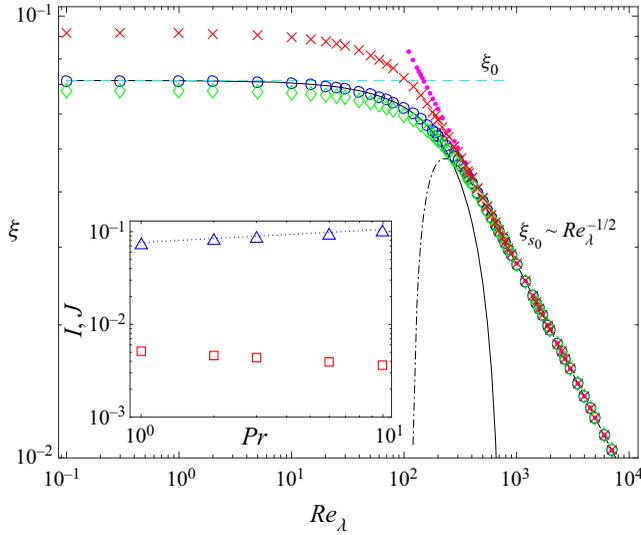


Figure 9. Variation of the critical, local dimensionless boundary layer thickness (4.18) with the shear Reynolds number based on the mean plume spacing (4.8) at  $Pr = 1$  and  $Ra_\lambda = 1.07 \times 10^5$ . Symbols and lines are: blue circles, numerical solution of (4.17); solid black line, (4.22); cyan dashed line, NCBL; magenta dots, PBBL; dash-dotted line, (D5); green diamonds, numerical solution of (4.17) at  $Pr = 0.7$  and  $Ra_\lambda = 9.63 \times 10^4$ ; red crosses, numerical solution of (4.17) at  $Pr = 6$  and  $Ra_\lambda = 1.83 \times 10^5$ . The inset shows the variations of  $I$  (see (4.26)) and  $J$  (see (4.27)) with  $Pr$  for  $n = 0.5$  and  $\Gamma = 1.71$ : blue triangles,  $I(Pr)$ ; red squares,  $J(Pr)$ ; blue dotted line,  $0.09 Pr^{0.14}$ . In computing solutions of (4.22) and (D5), terms of orders  $\epsilon^2$  and  $\epsilon_1^2$  were neglected, respectively.

with

$$Re_\lambda = V_F \lambda_0 / \nu \tag{4.8}$$

being the shear Reynolds number based on  $\lambda_0$ ,  $Ra_\lambda = g\beta \Delta T_w \lambda_0^3 / \nu \alpha$  being the Rayleigh number based on  $\lambda_0$ , and

$$E(Pr) = \frac{C_2^2 Pr^{2n-1} + 1}{C_2^2 Pr^{3n}}. \tag{4.9}$$

#### 4.2. Perturbation solution

The values of  $Re_\lambda$ ,  $Ra_\lambda$  and  $\epsilon$  for the range of  $Ra_w$  in the present study are listed in table 3. With increase in  $Ra_w$ ,  $Ra_\lambda$  remains of order  $10^5$ , while  $Re_\lambda$  changes by a factor 7 from 30 to 209, so that  $\epsilon$  increases by an order from  $10^{-4}$  to  $10^{-3}$ ;  $\epsilon$  is then a small parameter. We can then find the solution of (4.6) as a power series in  $\epsilon$  of the form

$$Pe_\lambda = Pe_{\lambda_0} + \epsilon Pe_{\lambda_1} + \epsilon^2 Pe_{\lambda_2} + O(\epsilon^3). \tag{4.10}$$

Substituting (4.10) in (4.6) and comparing the coefficients of similar powers of  $\epsilon$ , we obtain

$$Pe_{\lambda_0} = \left( \frac{C_2}{8E} \right)^{2/5} Ra_\lambda^{2/5}, \tag{4.11}$$

$$Pe_{\lambda_1} = \frac{1}{5E Pr^n} A Ra_\lambda \tag{4.12}$$

Shear on local boundary layers in convection

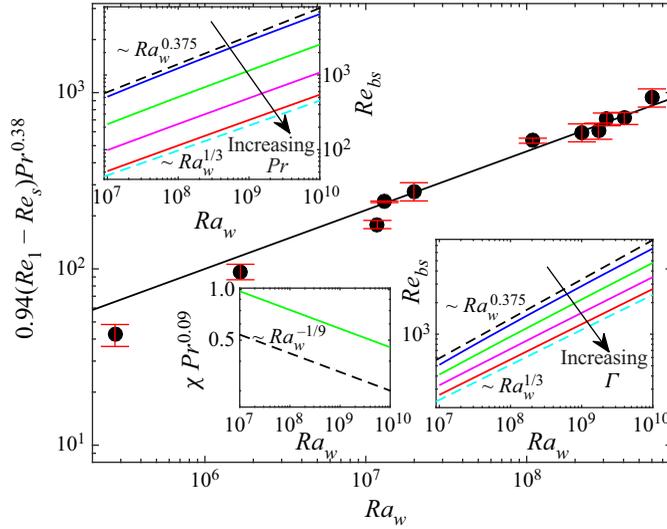


Figure 10. Variation of Reynolds number, based on the mean velocity from the sole peaks of type A p.d.f.s and the first peak of type B p.d.f.s, with  $Ra_w$ : black circles, (4.35) with  $Re_{bs} = Re_1$ ; black solid line,  $1.0 Ra_w^{1/3}$ . Bottom right inset shows the variation of  $Re_{bs}$  (see (4.33)) with  $Ra_w$  at  $Pr = 1$ : blue solid line,  $\Gamma = 1$ ; green solid line,  $\Gamma = 2$ ; magenta solid line,  $\Gamma = 5$ ; red solid line,  $\Gamma = 20$ ; black dashed line,  $Ra_w^{0.375}$ ; cyan dashed line,  $Ra_w^{1/3}$ . Top left inset shows variation of  $Re_{bs}$  (see (4.33)) with  $Ra_w$  at  $\Gamma = 1$ : blue solid line,  $Pr = 1$ ; green solid line,  $Pr = 5$ ; magenta solid line,  $Pr = 25$ ; red solid line,  $Pr = 100$ ; black dashed line,  $Ra_w^{0.375}$ ; cyan dashed line,  $Ra_w^{1/3}$ . Bottom left inset shows the variation of the dimensionless shear velocity  $\chi$  (see (4.32)) with  $Ra_w$ : green solid line,  $\Gamma = 1$ ; black dashed line,  $\Gamma = 10$ .

and

$$Pe_{\lambda_2} = \frac{0.069}{C_2^{2/5} E^{8/5} Pr^{2n}} A^2 Ra_\lambda^{8/5}. \quad (4.13)$$

Substituting (4.11)–(4.13) back in (4.10), we obtain

$$Pe_\lambda = \left(\frac{C_2}{8E}\right)^{2/5} Ra_\lambda^{2/5} + \frac{1}{5E Pr^n} A Re_\lambda + \frac{0.069}{C_2^{2/5} E^{8/5} Pr^{2n}} \frac{(A Re_\lambda)^2}{Ra_\lambda^{2/5}} + O(\epsilon^3). \quad (4.14)$$

Using (1.5) to replace  $\lambda_0$  appearing in  $Re_\lambda$  and  $Ra_\lambda$  in (4.14), then using the definitions of  $Ra_w$ ,  $Re_{bl}$ ,  $Re$  and  $Z_w$ , and dropping the second-order terms in  $Re_\lambda$ , we obtain the Reynolds number based on the characteristic velocity in the local NCBL, forced externally by the LSF, as

$$Re_{bs} = \frac{U_c H}{\nu} = \left(\frac{C_2}{E}\right)^{2/5} Re_{bl} + \frac{0.4}{E Pr^{n+1}} A Re + O(\epsilon^2), \quad (4.15)$$

where  $Re_{bl}$  is given by (3.3). Using (1.1) for  $Re$  in (4.15) will not capture the effect of  $\Gamma$  on the LSF strength, which is strong in the present case since  $\Gamma$  changes from 10 to 1.71 with increase in  $Ra$  in the present experiments. To account for the dependence of  $\Gamma$  on  $Re$ , we hence measure the LSF velocity in the present experiments and obtain the corresponding relation (B1) for  $Re(Ra, Pr, \Gamma)$  in Appendix B, which is then used in (4.15).

As per (4.15), the velocities in the local boundary layers, in the presence of LSF, are then a linear perturbation of the NCBL velocities by the large-scale velocities, the amount

$Ra_w$	$Re_\lambda$	$Ra_\lambda$	$\epsilon = Re_\lambda/Ra_\lambda$
$2.76 \times 10^5$	29.98	$1.82 \times 10^5$	$1.64 \times 10^{-4}$
$1.66 \times 10^6$	39.18	$1.76 \times 10^5$	$2.23 \times 10^{-4}$
$1.31 \times 10^7$	76.69	$1.80 \times 10^5$	$4.27 \times 10^{-4}$
$1.10 \times 10^8$	137.08	$1.76 \times 10^5$	$7.78 \times 10^{-4}$
$6.05 \times 10^8$	208.62	$1.75 \times 10^5$	$1.19 \times 10^{-3}$

Table 3. Values of the Rayleigh and Reynolds numbers based on the mean plume spacing, along with the values of the perturbation parameter for the present study.

of perturbation being dependent on the  $Pr$ . This perturbation also depends on the values of shear at the edge of these local boundary layers, given by the dimensionless shear

$$A = \frac{u|_{z=\delta}}{V_F} = 2 \frac{\delta}{\delta_{pb}} - \left( \frac{\delta}{\delta_{pb}} \right)^2, \tag{4.16}$$

obtained as (A5) in Appendix A, using the von Kármán velocity profile. Since  $V_F$ ,  $\delta_{pb}$  and  $\delta$  in (4.16) are all functions of  $Ra_w$  and  $Pr$ ,  $A$  is also a function of  $Ra_w$  and  $Pr$ . We now find this expression for the dimensionless shear ( $A$ ) at the edge of the velocity boundary layer. Since  $A$ , given by (4.16), is a function of  $\delta$ , whose dependence on  $Ra_\lambda$  and  $Re_\lambda$  is unknown, we first proceed to find the dependence of  $\delta$  on  $Ra_\lambda$  and  $Re_\lambda$ .

### 4.3. Velocity boundary layer thicknesses for small shear

Replacing  $\delta_T$  in (4.4) with  $\delta$  using (4.5), and substituting the resulting expression for  $U_c$  in (4.3), we obtain

$$\frac{4}{C_2^2} \xi^5 + 2A Pr^{2n} \epsilon \xi^2 - \frac{C_2^2 Pr^{5n} E}{Ra_\lambda} \sim 0, \tag{4.17}$$

where

$$\xi = \frac{\delta}{\lambda_0} = \frac{Ra_\delta^{1/3}}{C_1 Pr^{m_1}} \tag{4.18}$$

is the dimensionless local velocity boundary layer thickness when these boundary layers are perturbed by the LSF, with  $Ra_\delta = g\beta \Delta T_w \delta^3 / \nu\alpha$  being the Rayleigh number based on  $\delta$  and  $\Delta T_w$ . Let the solution for (4.17) be a power series in  $\epsilon$  of the form

$$\xi = \xi_0 + \epsilon \xi_1 + O(\epsilon^2). \tag{4.19}$$

Substituting (4.19) in (4.17) and comparing the coefficients of different powers of  $\epsilon$ , we get

$$\xi_0 = 0.76 Pr^n \left( \frac{C_2^4 E}{Ra_\lambda} \right)^{1/5} \tag{4.20}$$

and

$$\xi_1 = -0.17A \left( \frac{C_2 Ra_\lambda}{E} \right)^{2/5}. \tag{4.21}$$

Thus the perturbation solution of (4.17), obtained by substituting (4.20) and (4.21) in (4.19), after neglecting higher-order terms, is

$$\xi \sim 0.76 Pr^n \left( \frac{C_2^4 E}{Ra_\lambda} \right)^{1/5} - 0.17 \left( \frac{C_2}{E} \right)^{2/5} A \frac{Re_\lambda}{Ra_\lambda^{3/5}} + O(\epsilon^2). \quad (4.22)$$

Rewriting (4.22) in terms of  $Ra_\delta$  by using (4.18) and (1.5), we get

$$Ra_\delta^{1/3} = \frac{\delta}{Z_w} \sim 36.1 Pr^{n+0.1} \left( \frac{C_2^4 E}{Ra_\lambda} \right)^{1/5} - \frac{8.1}{Pr^{-0.1}} \left( \frac{C_2}{E} \right)^{2/5} A \frac{Re_\lambda}{Ra_\lambda^{3/5}} + O(\epsilon^2) \quad (4.23)$$

as the expression for the critical, local dimensionless boundary layer thickness, when these boundary layers are forced by an external shear due to the LSF. The first term on the right-hand side of (4.23) is the critical dimensionless boundary layer thickness in the absence of external shear, while the second term is the perturbation due to the external shear. For no external shear,  $Re_\lambda \rightarrow 0$ ,  $\lambda \rightarrow \lambda_0$  and  $n = 0.5$  (Puthenveetil *et al.* 2011). Then, using (1.5) and (1.3), for no external shear, (4.23) becomes

$$Ra_\delta \sim 45.1 \left( C_2^4 E \right)^{3/5} Pr^{1.62}, \quad (4.24)$$

which, being constant for a given fluid, is of the same form as the marginal stability condition proposed by Howard (1964).

Figure 9 shows the variation of  $\xi$  with  $Re_\lambda$  when  $A = 1$  for three different cases  $(Pr, Ra_\lambda) = (0.7, 9.63 \times 10^4)$ ,  $(1, 1.07 \times 10^5)$ ,  $(6, 1.83 \times 10^5)$ , as given by the numerical solution of (4.17). A similar plot, but for the  $Pr = 1$  case alone, was shown by Puthenveetil & Harsha (2010). It needs to be noted that in RBC,  $Re_\lambda$ ,  $Ra_\lambda$  and  $A$  are all actually interdependent through the LSF, and none of these can be fixed independent of others. Figure 9 could then be considered to show the expected variation of  $\xi$  when a small shear is forced externally and independently of the fixed  $Ra_w$ , as occurs in the case of mixed convection that causes negligible change of  $\lambda$  from its no-shear value  $\lambda_0$ . Figure 9 also shows the variations predicted by the perturbation solutions (4.22) at  $A = 1$  for  $(Pr, Ra_\lambda) = (1, 1.07 \times 10^5)$ . The perturbation solution (4.22) is a good approximation for the variation of  $\xi$  predicted by (4.17) for  $Re_\lambda < 250$ . The figure shows that for  $Re_\lambda < 400$ ,  $\xi$  is a function of  $Pr$  and takes larger values for higher  $Pr$ ; however,  $\xi$  becomes a very weak function of  $Pr$  for  $Re_\lambda > 400$ .

For large  $Re_\lambda$ , figure 9 shows that the variation of  $\xi$  is similar to that in the Blasius boundary layer. However, at large  $Re_\lambda$ ,  $\lambda$  will vary from  $\lambda_0$  (Shevkar *et al.* 2019), hence the actual variation of  $\xi$  will deviate from that shown in figure 9. The singular perturbation solution of (4.17) for large shear, given in Appendix D, shown by the dash-dotted line in figure 9, matches the numerical solution of (4.17) and the variation for PBBLs for large  $Re_\lambda$ . Figure 9 also shows that at  $Pr = 1$ , the value of  $\xi$  decreases by only 10.5 % as  $Re_\lambda$  increases from  $10^0$  to  $10^2$ , while it decreases by 57 % for further increase in  $Re_\lambda$  from  $10^2$  to  $10^3$ . Similarly, at  $Pr = 6$ , the value of  $\xi$  decreases by 20 % and 62 % for the increases in  $Re_\lambda$  from  $10^0$  to  $10^2$ , and from  $10^2$  to  $10^3$ , respectively. So at any  $Pr$ , the deviation of the values of  $\xi$  from its natural convection value in (4.20) is less, approximately, for  $Re_\lambda \leq 100$ . For  $Re_\lambda > 100$ , the value of  $\xi$  decreases more rapidly at higher  $Pr$  with increase in  $Re_\lambda$  so as to match the variation given by the large shear value  $\xi_{s_0}$  (see (D3)), which again is independent of  $Pr$ . A similar, larger increase of  $\lambda$  at larger  $Pr$  has been found by Shevkar

*et al.* (2019) at a given  $Ra_w$  and  $Re$ . Similarly, in sheared convection, with increasing wall shear velocity, Blass *et al.* (2020) found a more pronounced increase of  $Nu$  at larger  $Pr$ , which could be due to this larger decrease of  $\delta$  with increasing shear at higher  $Pr$ .

For  $Re_\lambda < 30$ , at any  $Pr$ , as shown in figure 9,  $\xi$  remains approximately independent of  $Re_\lambda$ , which is the expected variation for a pure NCBL. This constancy of  $\xi$  for small shear can be quantified by rewriting (4.22) in terms of  $Ra_w$ . Using  $\lambda_0$  from (1.5) in (4.22), and then using (1.3) and (B1), we obtain

$$\xi \sim I - JA Ra_w^{1/9} + O(\epsilon^2), \tag{4.25}$$

where

$$I(Pr) = C_3 Pr^{n-3/50} E^{1/5} \tag{4.26}$$

and

$$J(Pr) = \frac{C_4 \Gamma^{-0.57}}{Pr^{3/4} E^{2/5}}, \tag{4.27}$$

with  $C_3 = (C_2^4/4C_1^3)^{1/5}$  and  $C_4 = 0.27(C_2/C_1^2)^{2/5}$ .

For  $Pr > 1$ , since the velocity boundary layer thickness is more than the thermal boundary layer thickness, in (4.5),  $n$  is positive and less than 1 (Shishkina *et al.* 2010); we assume that  $n = 0.5$ , the same value as in the no-shear case (Puthenveetil *et al.* 2011), for the present case of small shear. The inset of figure 9 shows that for  $n = 0.5$ ,  $I(Pr)$  (see (4.26)) is more than an order larger than  $J(Pr)$  (see (4.27)). Then since  $A < 1$  (see (A5) and figure 8), the second term in (4.25), which has a weak dependence on  $Ra_w$ , gives negligible contribution to  $\xi$  when  $Pr > 1$ , for the range of  $Ra_w$  in the present study; (4.25) then implies that

$$\xi \sim I(Pr). \tag{4.28}$$

The relation (4.28) implies that in the presence of weak shear,  $\delta$  and  $\lambda_0$  have the same functional dependence on  $Ra_w$ , so that their ratio is only a function of  $Pr$ ; a similar dependence occurs in the no-shear case, as has been shown by Puthenveetil *et al.* (2011). As shown in the inset of figure 9, for  $n = 0.5$ , the rational function (4.26) can be approximated by a simple power law,  $I(Pr) \sim Pr^{0.14}$ ; (4.28) then becomes

$$\xi \sim C_5 Pr^{0.14}, \tag{4.29}$$

where  $C_5 = 0.09$ . Rearranging (4.29) using  $\lambda_0$  from (1.5), we get

$$\frac{\delta}{H} = C_6 Ra_w^{-1/3} Pr^{0.24}, \tag{4.30}$$

where  $C_6 = C_1 C_5 = 4.28$ . The  $Pr$  dependence of  $\delta$  in (4.30) is the same as that obtained by Lam *et al.* (2002) for the range  $6 \leq Pr \leq 1027$  in RBC.

#### 4.4. Dimensionless shear ( $A$ ) at the edge of the boundary layers

Now that the expression (4.30) for  $\delta$ , the NCBL thickness forced by small shear, has been obtained, we substitute (4.30) and (1.4) using (B1) in (A5), and simplify, to obtain the

dimensionless shear velocity acting on the edge of the local boundary layers:

$$A = \chi(2 - \chi), \tag{4.31}$$

where

$$\chi = \frac{\delta}{\delta_{pb}} = \frac{1.37C_6}{\Gamma^{0.285} Ra_w^{1/9} Pr^{0.09}}. \tag{4.32}$$

Interestingly, (4.31) and (4.32) show that for a given  $\Gamma$  and  $Pr$ , within the limit given by (E4), the dimensionless shear at the edge of the local NCBL decreases with increase in  $Ra_w$  as  $A \sim (2Ra_w^{1/9} - 1)/Ra_w^{2/9}$ . In the present study, since  $\Gamma$  and  $Ra_w$  increase much more than the decrease of  $Pr$  with increasing  $Ra_w$ ,  $A$  decreases from 0.87 at the lowest  $Ra_w$ , to 0.71 at the highest  $Ra_w$ . However, the velocity at the edge of these boundary layers, due to the shear by the LSF, is  $u|_{y=\delta} = AV_F$  from (A5), where  $V_F \sim Ra_w^{4/9}$  from (B1). Hence the actual shear forcing at the edge of these local boundary layers increases with  $Ra$  as  $u|_{y=\delta} \sim Ra_w^{1/3} - Ra_w^{2/9}$ , which has an approximate power-law dependence  $Ra^{0.35}$  in the range  $10^7 < Ra_w < 10^{10}$ . As per the present analysis, since  $V_{bl}$  scales as  $Ra_w^{1/3}$  (see (3.3)) with increasing  $Ra_w$ , the local NCBL would eventually become dominated by shear. However, the present analysis, limited to low shears, does not consider the change in plume spacing due to shear given by (1.6), as well as the change in boundary layer thickness with shear (see (4.30)). The above conclusion of local NCBLs becoming shear-dominant at large  $Ra_w$  has to be explored further by extending the present analysis to larger shears by including the changes in the plume spacing and the boundary layer thicknesses at larger shear.

#### 4.5. Reynolds number based on the characteristic velocity $U_c$

We now obtain the effect of external shear caused by the LSF on the velocities within the boundary layers. Substituting (4.31) and (B1) in (4.15), and simplifying, we obtain

$$Re_{bs} = Re_{bl} \left( \left( \frac{C_2}{E} \right)^{2/5} + \frac{F}{\Gamma^{0.855}} (2 - \chi) \right) \tag{4.33}$$

as the Reynolds number based on the characteristic velocity close to the hot plate in RBC when the local boundary layers are forced by the LSF. Here,

$$F(Pr) = \frac{0.46 C_6}{E Pr^{1.28}}, \tag{4.34}$$

with  $\chi$  being given by (4.32),  $Re_{bl}$  by (3.3), and  $E$  by (4.9), and we have chosen  $n = 0.5$  in (4.5), the value in the no-shear case (Puthenveetil *et al.* 2011). Comparing (4.33) and (4.15), we can rewrite (4.33) as

$$Re_{bs} = Re_b + Re_s, \tag{4.35}$$

i.e. the sum of a buoyant contribution and a shear perturbation, where

$$Re_b = \left( \frac{C_2}{E} \right)^{2/5} Re_{bl} = 1.07 Ra_w^{1/3} Pr^{-0.38} \tag{4.36}$$

and

$$Re_s = \frac{0.4}{E Pr^{1.5}} A Re = \frac{1.27}{\Gamma^{0.855}} Ra_w^{1/3} Pr^{-0.76} (2 - \chi), \tag{4.37}$$

with  $Re_{bl}$ ,  $Re$  and  $A$  given by (3.3), (B1) and (4.31), respectively.

Equation (4.35) gives the variation of the dimensionless boundary layer velocity as a function of  $Ra_w$ ,  $Pr$  and  $\Gamma$  in RBC for small shear by the LSF. The variation predicted by (4.35), shown as the solid line in figure 10, matches the variation of  $Re_1$ , the Reynolds number based on the sole peaks of type A p.d.f.s and the first peaks of type B p.d.f.s for a reasonable value  $C_2 = 0.72$ . The single-peak type A p.d.f.s occurred at the low shear at low  $Ra_w$ , while the first peaks of type B p.d.f.s were due to the low-shear regions at all  $Ra_w$ . The deviation of the data point at the lowest  $Ra_w$  in figure 10 is expected to occur since (1.4) may not be valid for  $Ra_w < 10^7$ .

It is interesting to see the effective power-law dependence of  $Re_{bs}$  on  $Ra_w$  corresponding to (4.35). The top left inset of figure 10 shows the variation of  $Re_{bs}$  with  $Ra_w$  given by (4.35) at  $\Gamma = 1$  for  $1 \leq Pr \leq 100$ . At  $Pr = 1$ , a power law  $Re_{bs} \sim Ra_w^{0.375}$  approximates (4.35) for  $10^7 \leq Ra_w \leq 3.3 \times 10^8$ . As  $Pr$  is increased from 1 to 100, the exponent of  $Ra_w$  is decreased from 0.375 to  $1/3$ ;  $Re_{bs}$  then scales with  $Ra_w$  in the same way as  $Re_{bl}$  scales with  $Ra_w$  (see (3.3)). Therefore, the present analysis suggests that for  $Pr \geq 100$ , the effect of external forcing on the local NCBLs become negligible close to the hot plate. Similarly, the bottom right inset of figure 10 shows that with increase in  $\Gamma$ , the dependence of  $Re_{bs}$  on  $Ra_w$  becomes weaker, and for  $\Gamma \geq 20$ ,  $Re_{bs} \sim Ra_w^{1/3}$ . It is interesting to note that at  $Pr = 1$  and  $\Gamma = 1$ ,  $Re_{bs}$  shows the same power law of  $Nu$  versus  $Ra_w$  observed at large  $Ra_w$  (He *et al.* 2012), which has been proposed as a sign of the ultimate regime, even though this inference has been disputed by Urban *et al.* (2012). Considering that at low  $Ra_w$ , the scaling of  $Re_b$  in (4.36) (or  $Re_{bl}$  in (3.3)) with  $Ra_w$  is the same as that of  $Nu$  with  $Ra_w$ , the power-law exponent 0.375 for the dependence of  $Re_{bs}$  on  $Ra_w$  at higher  $Ra_w$  is analogous to what occurs at low  $Ra_w$ ; the reason for such an interesting occurrence needs to be investigated.

## 5. Conclusions

The primary contributions of the present work include the novel observations of the three types of horizontal velocity p.d.f.s within the boundary layers, their evolution with Rayleigh numbers ( $Ra$ ), the evolution of the area affected by shear with  $Ra$ , and the novel scaling relations for the dominant horizontal velocities within the boundary layers. Together, these illustrate the effect of external forcing by the large-scale flow (LSF) – which we showed to be a function of  $Ra$ ,  $Pr$  and the aspect ratio  $\Gamma$  (see (B1)) – on the boundary layers on the plate in turbulent Rayleigh–Bénard convection (RBC).

At lower  $Ra$  ( $\sim 10^6$ ), in a horizontal plane close to the hot plate, we observed predominantly (figure 2a) unimodal, type A p.d.f.s of horizontal velocity magnitudes ( $U$ ) that had a peak at approximately the natural convection boundary layer (NCBL) velocities ( $V_{bl}$ ) (see (3.3)). At higher  $Ra$  ( $\sim 10^9$ ), these evolved into predominantly unimodal type C p.d.f.s (figure 2e), with a velocity peak at approximately double the value of  $V_L$  (see (3.2)), the LSF velocity at the edge of the boundary layers. This shifting of p.d.f.s with increase in  $Ra$  was also accompanied by more-aligned and larger-magnitude velocity vector fields (figures 2b,f). Not only the type of p.d.f.s but also their relative occurrences shifted with increasing  $Ra$ ; the occurrence of type A p.d.f.s decreased from approximately 50 % of instants at low  $Ra$  to approximately 10 % of instants at high  $Ra$ , while the occurrence of type C p.d.f.s increased from approximately 1 % to 70 % of the instants (figure 6). Interestingly, this change in percentage occurrences of type A and type C p.d.f.s followed a dependence of  $Ra^{-1/3}$  (see (3.6)) and  $Ra^{4/9}$  (see (3.7)), respectively; the magnitude of

the exponents was the same as those in the classical flux scaling and the scaling of the Reynolds number ( $Re$ ) (see (1.1)) based on the LSF, respectively.

This shift in types and numbers of p.d.f.s occurred through a stage at intermediate  $Ra$  where the p.d.f.s were predominantly bimodal (figure 2c). The numbers of these type B p.d.f.s increased from approximately 50 % of instants at low  $Ra$  to approximately 70 % of instants at  $Ra \approx 10^7$ , before decreasing to approximately 25 % of instants at  $Ra \approx 10^9$  (figure 6). The first mode of these bimodal type B p.d.f.s peaked at velocities slightly larger than  $V_{bl}$ , while the second mode peaked at  $V_L$  (figure 2c); the regions contributing to the second mode had more aligned velocity fields with larger magnitudes (figure 2d). Considering the regions that had aligned velocity vectors with larger magnitudes as shear-affected – which occurred in regions contributing to the second mode in type B p.d.f.s, in regions with  $U > V_L$  for type A p.d.f.s, and regions with  $U > V_{bl}$  for type C p.d.f.s – we obtained the variation of shear-affected areas with  $Ra$  (figure 5). The shear-affected areas increased exponentially with  $Ra$ , from approximately 20 % at  $Ra = 10^6$ , before saturating at approximately 90 % at  $Ra = 10^9$ ; the proposed scaling law (3.4) for this variation matched the observed variation. Thus we showed that the effects of increasing shear, due to increasing LSF strength with  $Ra$ , on the boundary layers on the plate occur in the following three simultaneous manifestations: (a) change of velocity p.d.f.s, from the ones that peaked at  $V_{bl}$  to those that peaked at velocities  $\geq V_L$ ; (b) an increase in the occurrence of such p.d.f.s that peaked at  $\geq V_L$ ; and (c) an exponential increase, and then saturation, of the extent of areas affected by shear, that had aligned velocity vectors with larger magnitudes.

In the regions where the effect of shear was felt strongly, i.e. in those regions that had aligned velocity vectors with larger magnitudes, the scaling of velocities – estimated as the scaling of the peaks of type C p.d.f.s and the second mode of the type B p.d.f.s – followed the scaling of  $V_L$ . The magnitudes of these velocities ( $V_2$ ) were, however, larger than  $V_L$ , but smaller than  $V_F$ , the LSF velocity far away from the plate. In contrast, in regions mildly affected by shear that did not have aligned velocity vectors with larger magnitudes, the scaling of velocities – estimated as the scaling of the peaks of type A p.d.f.s and the peaks of the first mode of type B p.d.f.s – followed neither the scaling of  $V_{bl}$  nor that of  $V_L$ . Instead, here, the velocities followed  $Ra^{0.38}$ , the same dependence of flux observed by He *et al.* (2012), suggested as a sign of the ultimate regime. Since the peak of the velocity p.d.f.s from these regions shifted to larger velocities with increase in  $Ra$ , and since the scaling anyway was different from that of  $V_{bl}$ , we proposed this  $Ra^{0.38}$  scaling to be due to the effect of shear by the LSF on the local NCBLs.

Using an order of magnitude balance of integral equations for NCBLs forced by small external shear, (A8) and (A13), we then quantified this effect of shear on the local boundary layers by obtaining a scaling relation (4.6) for the characteristic velocity ( $U_c$ ) within such boundary layers. Here, the boundary layers were assumed to be embedded within a Prandtl–Blasius boundary layer (PBBL) that had a von Kármán velocity profile (A4), and the plume spacings (1.5) assumed to be unchanged due to the small shear. The perturbation solution of this scaling relation showed that  $U_c$  is a sum of  $V_{bl}$  and  $u|_{y=\delta}$ , the shear at the edge of these local boundary layers, with functions of  $Pr$  as prefactors on each term of the sum (4.15). The effect of shear was then a linear perturbation of the natural convection velocities by the LSF velocities, with the amount of perturbation being dependent on  $Pr$  and the shear  $u|_{y=\delta}$  at the edge of the boundary layers.

Since  $u|_{y=\delta}$  was dependent on the velocity boundary layer thickness  $\delta$  of these local boundary layers in the presence of external shear, we obtained a scaling relation (4.22),

$\delta = f_1 Z_w + f_2 Z_w^2 / Z_{sh}$ , where  $f_1$  and  $f_2$  are functions of  $Pr$ ,  $Z_w$  is the natural convection length scale near the plate (see (1.3)), and  $Z_{sh} = \nu / u|_{y=\delta}$  is the viscous shear length at  $y = \delta$ . At the low shears considered in the present study, the second term in the above relation for  $\delta$  could be dropped so that  $\delta$  became the same as the velocity boundary layer thickness of NCBLs in the absence of shear. Using this asymptotic form of  $\delta$ , we obtained  $u|_{y=\delta}$ , which scaled as  $Ra_w^{1/3} - Ra_w^{2/9}$ , with an equivalent power law or  $Ra_w^{0.35}$ . Then, since  $V_{bl} \sim Ra_w^{1/3}$ , with increase in  $Ra_w$ , we proposed that these local NCBLs are expected to become shear-dominated at some large  $Ra_w$ , even though only an analysis that includes the effect of shear on  $\delta$  and plume spacing, at larger shear, can confirm this expectation. Using this  $u|_{y=\delta}$  in the perturbation solution for  $U_c$  gave the characteristic Reynolds number  $Re_{bs}$  within these boundary layers as a sum of buoyant contribution and shear perturbation (see (4.35)). The relation (4.35) for  $Re_{bs}$  – obtained using the derived relations for the shear at the edge of the local boundary layers (4.31) and the local velocity boundary layer thickness subjected to small shear (4.30) – matched the measurements of Reynolds number  $Re_1$  based on the peaks of type A p.d.f.s and the first peak of type B p.d.f.s of horizontal velocity magnitudes (figure 10), obtained from stereo PIV in a plane close to the hot plate. Similarly, the relations for the areas affected by shear (3.4) and the number of occurrences of various types of p.d.f.s (3.6)–(3.8) also matched our experimental observations for  $2.76 \times 10^5 \leq Ra_w \leq 6.05 \times 10^8$ ,  $4.69 \leq Pr \leq 5.88$  and  $1.71 \leq \Gamma \leq 10$ , thereby validating all these relations in this range of parameters.

These relations and the associated experimental observations help us to understand the varied effects of shear due to LSF on the local boundary layers on the hot plate. The scaling (4.35), obtained for the velocities in the mildly shear-affected regions in the boundary layers, appears to contribute progressively less with increase in  $Ra$ , in terms of the areas in which they occur, as well as in their number of occurrences. However, this could be because with a horizontal plane of measurement at a fixed height  $h_m$ , more areas of the boundary layers, which become progressively thinner with increase in  $Ra$ , will evade the measurement plane, as could be inferred from figure 8(a). In such a situation, the high-shear regions that show a velocity scaling the same as  $V_L$  could actually be outside the local boundary layers, with (4.35) being the scaling in all boundary layers. In such a scenario, the scaling (4.35) could be the key relation showing the effect of shear by LSF on the boundary layers on the hot plate. Application of (4.35) in that case, we hope, will lead to explanations for the anomalous flux scaling in turbulent convection, as well as clarifying the question of the ultimate regime. However, in obtaining these relations, we neglected the effect of shear due to the LSF on the plume spacings and the local boundary layer thicknesses; the present relations will then be valid only for the case of small shear, i.e.  $Ra_w < 5.5 \times 10^{11}$  (see Appendix E). To explain the anomalous flux scaling from these effects, we need to extend the present analysis to higher  $Ra$  by including these neglected effects.

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**Appendix A. Integral boundary layer equations**

The integral  $x$ -momentum equation is

$$\int_0^\delta u \frac{\partial u}{\partial x} dz + \int_0^\delta w \frac{\partial u}{\partial z} dz = -\frac{1}{\rho} \int_0^\delta \frac{\partial p}{\partial x} dz + \nu \int_0^\delta \frac{\partial^2 u}{\partial z^2} dz, \tag{A1}$$

where  $\delta(x)$  is the velocity boundary layer thickness, and  $p(x, z)$  is the pressure within the boundary layer. Rewriting the second term in (A1) as  $\int_0^\delta (\partial(wu)/\partial z - u \partial w/\partial z) dz$ , integrating, replacing  $\partial w/\partial z = -\partial u/\partial x$  from continuity, noticing that  $\int_0^\delta (u \partial u/\partial x) dz = \int_0^\delta (\partial u^2/\partial x) dz/2$ , and since  $(\partial u/\partial z)|_{z=\delta} \approx 0$  for small external shear, we obtain

$$\int_0^\delta \frac{\partial}{\partial x} u^2 dz + (wu)|_{z=\delta} = \frac{-1}{\rho} \int_0^\delta \frac{\partial p}{\partial x} dz - \nu \frac{\partial u}{\partial z} \Big|_{z=0}. \tag{A2}$$

The vertical velocity at the boundary layer edge in the second term in (A2) can be obtained by integrating the continuity equation and applying the Leibniz rule as

$$w|_{z=\delta} = -\frac{\partial}{\partial x} \int_0^\delta u dz + u|_{z=\delta} \frac{\partial \delta}{\partial x}. \tag{A3}$$

To obtain  $u|_{z=\delta}$  in (A2) and (A3), as shown in figure 8, we assume that the local boundary layers are embedded within a PBBL, which is driven by the LSF strength of  $V_F$ , acting at distance  $\delta_{pb}$  (see (1.4)) from the hot plate. Using the von Kármán velocity profile

$$\frac{u(z)}{V_F} = 2 \frac{z}{\delta_{pb}} - \left( \frac{z}{\delta_{pb}} \right)^2 \tag{A4}$$

for the PBBL, the dimensionless shear velocity acting on the upper edges of the local NCBLs, at height  $z = \delta$  from the hot plate, is then

$$A = \frac{u|_{z=\delta}}{V_F} = 2 \frac{\delta}{\delta_{pb}} - \left( \frac{\delta}{\delta_{pb}} \right)^2. \tag{A5}$$

Applying the Leibniz rule to the first term in (A2), substituting (A3) in (A2), simplifying, and finally replacing  $u|_{z=\delta}$  from (A5), we obtain

$$\frac{\partial}{\partial x} \int_0^\delta u^2 dz - AV_F \frac{\partial}{\partial x} \int_0^\delta u dz + \frac{1}{\rho} \int_0^\delta \frac{\partial p}{\partial x} dz + \nu \frac{\partial u}{\partial z} \Big|_{z=0} = 0. \tag{A6}$$

To replace the unknown pressure in (A6), we integrate the  $z$ -momentum equation,  $\partial p/\partial z = \rho g \beta (T - T_B)$ , to obtain,

$$p = -\rho g \beta \int_0^\delta (T - T_B) dz, \tag{A7}$$

where  $T(z)$  is the temperature distribution within the boundary layer. Substituting (A7) in (A6), we obtain

$$\frac{\partial}{\partial x} \int_0^\delta u^2 dz - AV_F \frac{\partial}{\partial x} \int_0^\delta u dz - g \beta \int_0^\delta \frac{\partial}{\partial x} \int_0^\delta (T - T_B) dz dz + \nu \frac{\partial u}{\partial z} \Big|_{z=0} = 0 \tag{A8}$$

as the integral momentum balance equation for the local NCBLs, forced externally by the LSF.

The integral energy equation is

$$\int_0^{\delta_T} u \frac{\partial T}{\partial x} dz + \int_0^{\delta_T} w \frac{\partial T}{\partial z} dz = \alpha \int_0^{\delta_T} \frac{\partial^2 T}{\partial z^2} dz, \quad (\text{A9})$$

where  $\delta_T(x)$  is the thermal boundary layer thickness of the local boundary layer. The first term in (A9) can be written as  $\int_0^{\delta_T} (\partial(uT)/\partial x) dz - \int_0^{\delta_T} T(\partial u/\partial x) dz$ . Applying the Leibniz rule to the first term in this equation, the first term in (A9) becomes

$$\frac{\partial}{\partial x} \int_0^{\delta_T} uT dz - (uT)|_{z=\delta_T} \frac{d\delta_T}{dx} - \int_0^{\delta_T} T \frac{\partial u}{\partial x} dz. \quad (\text{A10})$$

Similarly, rewriting the second term in (A9) as  $\int_0^{\delta_T} (\partial(wT)/\partial z) dz - \int_0^{\delta_T} T(\partial w/\partial z) dz$ , integrating the first term and using the continuity equation on the second term, the second term in (A9) becomes

$$(wT)|_{z=\delta_T} + \int_0^{\delta_T} T \frac{\partial u}{\partial x} dz. \quad (\text{A11})$$

Replacing  $w|_{z=\delta_T}$  in (A11) by  $-(\partial/\partial x) \int_0^{\delta_T} u dz + u|_{z=\delta_T} (d\delta_T/dx)$  obtained by integrating the continuity equation across the thermal boundary layer and applying the Leibniz rule, similar to (A3), the second term in (A9) becomes

$$-\frac{\partial}{\partial x} \int_0^{\delta_T} uT_B dz + u|_{z=\delta_T} T_B \frac{d\delta_T}{dx} + \int_0^{\delta_T} T \frac{\partial u}{\partial x}. \quad (\text{A12})$$

Substituting (A10) and (A12) in (A9) and simplifying, and since the last term in (A9) becomes  $-\alpha \partial T/\partial z|_{z=0}$  since  $\partial T/\partial z|_{z=\delta} \approx 0$ , we obtain

$$\frac{\partial}{\partial x} \int_0^{\delta_T} u(T - T_B) dz = -\alpha \left. \frac{\partial T}{\partial z} \right|_{z=0} \quad (\text{A13})$$

as the integral energy equation for the local NCBL forced by the LSF.

### Appendix B. Dependence of LSF velocities on aspect ratio ( $\Gamma = L/H$ )

For  $Ra \leq 10^{13}$ , the dependence of the Reynolds number based on the LSF velocities ( $Re$ ) on  $Ra$  and  $Pr$  has been well studied. Most studies find  $Re \sim Ra^a Pr^b$ , where  $a \approx 4/9$  and  $b \approx -2/3$  (Lam *et al.* 2002; Puthenveetil & Arakeri 2005; Ahlers *et al.* 2009; He *et al.* 2012; Gunasegarane & Puthenveetil 2014). However, the magnitude of the LSF velocities and the number of rolls are correlated to each other, and are also dependent on the aspect ratio  $\Gamma$  (Cuba, Emran & Schumacher 2010; Shi *et al.* 2012); this dependence of  $Re$  on  $\Gamma$  is not clear. In the present experiments, since  $Ra$  was changed by changing  $H$ , while keeping  $L$  constant,  $\Gamma$  decreased with increasing  $Ra$ ;  $\Gamma = 10$  at the lowest  $Ra$ , while at the highest  $Ra$ ,  $\Gamma = 1.71$ . Since the variation of  $\Gamma$  is significant in the present experiments, we now find the dependence of  $Re$  on  $\Gamma$  in our experiments.

From PIV, we obtain the velocity fields at various instants in horizontal planes at heights  $h_F \gtrsim \delta_{pb}$ , given in table 1, at six different  $Ra$ . At each of these  $Ra$ , we calculate the spatio-temporal average of these horizontal velocities to obtain the LSF velocity  $V_F = \langle \bar{U} \rangle$ . Figure 11 shows the variation of  $Re = V_F H/\nu$  with  $Ra$  for  $1.71 \leq \Gamma \leq 10$  and  $4.69 < Pr < 5.88$ . The data show a power-law scaling  $Re \sim Ra^{0.617}$ , giving the

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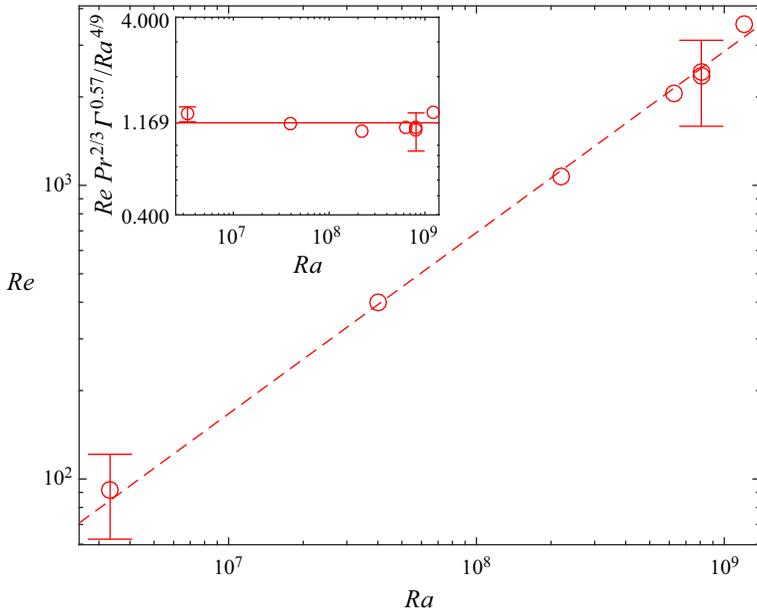


Figure 11. Variation of Reynolds number  $Re$  based on  $V_F$  with  $Ra$  for  $1.71 \leq \Gamma \leq 10$ : red dashed line,  $0.008 Ra^{0.617}$ . The inset shows the variation of normalised  $Re$  with  $Ra$ : red solid line,  $Re Pr^{2/3} \Gamma^{0.57} / Ra^{4/9} = 1.169$ . Error bars show the  $2\sigma$  variation of the  $V_F$  values.

impression that the dependences of  $Re$  on  $Pr$  and  $\Gamma$  are weak. However, this could be because of the opposite dependence of  $Re$  on  $\Gamma$  and  $Ra$ . Further, the range of our  $Pr$  is small, so the dependence of  $Re$  on  $Pr$  may also not be clearly seen. Since it is known that  $Re \sim Ra^{4/9} Pr^{-2/3}$ , as shown in the inset in figure 11 we plot the variation of  $Re Pr^{2/3} \Gamma^m / Ra^{4/9}$  with  $Ra$  and find the value  $m = 0.57$  that collapses the normalised  $Re$  onto a horizontal line. Then the Reynolds number based on the LSF velocities ( $V_F$ ) and  $H$  scales as

$$Re = 1.169 Ra^{4/9} Pr^{-2/3} \Gamma^{-0.57}. \quad (\text{B1})$$

Recent RBC experiments by Zhu & Zhou (2021) for  $1 \leq \Gamma \leq 4$ ,  $4.8 \times 10^7 \leq Ra \leq 4.5 \times 10^{10}$  and  $4.3 \leq Pr \leq 5.3$  found that  $Re \sim \Gamma^{-0.52}$ , the exponent of  $\Gamma$  being close to that in (B1). The prefactor in (B1) is also very close to the prefactor 1.1 in the expression for  $Re$  based on the oscillation frequency of the LSF, obtained by Lam *et al.* (2002). From (1.4) and (B1), it then follows that  $\delta_{pb}/H \sim \Gamma^{0.285}$ ; interestingly, the exponent 0.285 is the same as that in  $\delta_t/H \sim \Gamma^{0.29}$ , obtained by du Puits, Resagk & Thess (2013) for the scaling of thermal boundary layer thickness, for  $1 \leq \Gamma \leq 9$  at  $Pr = 0.7$ . Hence the new scaling of  $Re$  given by (B1), which is valid for the range of  $Ra$ ,  $Pr$  and  $\Gamma$  in the present experiments, is consistent with the available results.

Niemela & Sreenivasan (2003) observe that  $V_F/V_{FF}$  increases until  $Ra \approx 10^8$  and then decreases until  $Ra = 10^{13}$  in a cylindrical convection cell, where  $V_{FF} = \sqrt{g\beta \Delta T H}$  is the free fall velocity. If we rewrite (B1) in terms of  $V_{FF}$ , then  $V_F/V_{FF} = 1.169 Ra^{-1/18} Pr^{-1/6}$ , which decreases monotonically with  $Ra$ . This difference is most likely due to the difference in the shape of the cross-section of the convection cell in the two studies, compounded by the method by which  $V_F$  was measured in Niemela & Sreenivasan (2003). In a cylindrical

cell, the orientation of the large scale circulation (LSC) is constantly changing (Chilla & Schumacher 2012), while in a rectangular cell, the LSC is mostly oriented along the diagonal of the cross-section (Vishnu, De & Mishra 2020). Niemela & Sreenivasan (2003) used the correlation between two nearby temperature sensors to estimate the strength of the LSC. In the presence of the LSC that changes its orientation constantly, such an estimate would be accurate if the temperature sensors are aligned in the direction of the LSC, which would happen only for short time periods. Further, their measurements are made at a single point; however, the strength of the LSC does vary spatially. In contrast, our measurements are spatio-temporal averages of direct PIV measurements, which takes into account spatio-temporal non-uniformities as well as well as change in orientation of the LSC. We hence expect our measurements to be a more realistic, accurate and valid estimate in the present set-up.

### Appendix C. Error analysis

Errors in  $U/V_{bl}$ , at three points of different magnitudes of  $U/V_{bl}$ , in type A, type B and type C p.d.f.s, are shown in figures 2 and 3. Uncertainty in  $U/V_{bl}$  was assumed to be due to uncertainty in  $U$ . Uncertainty in  $U$  in a PIV interrogation window was calculated from the correlation statistics using the methodology of Wieneke (2015), using the Davis software (LaVision GmbH). In any p.d.f., the uncertainty at a specific magnitude of  $U$  is chosen as the uncertainty in horizontal velocity at any randomly chosen point of the vector field that has the same magnitude as  $U$ . The error bars on either side of any point on the p.d.f. are then plotted using half the total error in  $U/V_{bl}$ , and are shown either on or just above the points. Then at any point on the p.d.f., an error bar shows a 68.3 % confidence level range in  $U/V_{bl}$ . The typical values of uncertainties in  $U/V_{bl}$  at  $U/V_{bl} = 0.33, 1.06, 3$  in a type A p.d.f. at  $Ra_w = 2.76 \times 10^5$  were 7.1 %, 4.9 % and 1.2 %, respectively, as shown in figure 2(a). The maximum uncertainty in  $U/V_{bl}$  in type A p.d.f.s was 7.1 %, in type B p.d.f.s was 4.9 %, and in type C p.d.f.s was 7 %.

In figure 5, errors in  $A_{bl}/A$  and  $A_{sh}/A$  at  $Ra_w = 2.76 \times 10^5, 1.31 \times 10^7, 1.10 \times 10^8$  and  $6.05 \times 10^8$  are plotted. As described in § 3.1, the values of  $A_{bl}/A$  and  $A_{sh}/A$  were calculated at various  $Ra_w$  using peak separation value  $\frac{1}{4}(V_{sh}/V_{bl} - 1)$ . To estimate the errors in  $A_{bl}/A$  or  $A_{sh}/A$ , the ranges of the variation of the values of  $A_{bl}/A$  and  $A_{sh}/A$  at any  $Ra_w$  are now estimated using peak separation values  $0.3(V_{sh}/V_{bl} - 1)$  and  $0.2(V_{sh}/V_{bl} - 1)$ . The lower limit  $0.2(V_{sh}/V_{bl} - 1)$  was chosen, since values lower than this limit picked up peaks due to fluctuations present in turbulent flow, which do not represent the dominant motions in the flow. Similarly, values greater than the upper limit  $0.3(V_{sh}/V_{bl} - 1)$  gave unimodal p.d.f.s, and hence were not considered. At any  $Ra_w$ , the changes in mean peak velocities  $V_1$  and  $V_2$  at these limits were less than 2 % from that obtained using the peak separation value  $\frac{1}{4}(V_{sh}/V_{bl} - 1)$ . At a given  $Ra_w$ , these ranges of values of  $A_{bl}/A$  or  $A_{sh}/A$  were used as the errors at the corresponding points. The estimated errors in  $A_{bl}/A$  were 0.9 % and 10.8 % at  $Ra_w = 2.76 \times 10^5$  and  $6.05 \times 10^8$ , respectively, while the errors in  $A_{sh}/A$  were 31.6 % and 1.2 % at  $Ra_w = 2.76 \times 10^5$  and  $6.05 \times 10^8$ , respectively.

We now estimate the errors in percentages of type A, type B and type C p.d.f.s by estimating the numbers of these types of p.d.f.s that are obtained by using a range of peak separation values at  $Ra_w = 2.76 \times 10^5, 1.66 \times 10^6, 1.31 \times 10^7, 1.10 \times 10^8$  and  $6.05 \times 10^8$ . The percentages of type A, type B and type C p.d.f.s, in figure 6 were estimated using peak separation value  $\frac{1}{4}(V_{sh}/V_{bl} - 1)$ . To estimate errors, peak separation values

$Ra_w$	Type A	Type B	Type C
$2.76 \times 10^5$	7.1	18.7	23.6
$1.31 \times 10^7$	6.1	6.4	7.4
$6.05 \times 10^8$	5	12.9	1.7

Table 4. Percentage errors in total number of p.d.f.s at each  $Ra_w$ .

$0.2(V_{sh}/V_{bl} - 1)$  and  $0.3(V_{sh}/V_{bl} - 1)$  are now used to obtain the percentages of type A, type B and type C p.d.f.s at each  $Ra_w$ . The differences between these percentages of each type of p.d.f. are taken as the errors in the percentages of each type of p.d.f. at the corresponding  $Ra_w$ . The obtained errors in the percentages of the three types of p.d.f.s at three  $Ra_w$  are shown in table 4. Larger errors are observed at lower  $Ra$  due to the larger spatial non-uniformity of the velocity field, due to lower velocity strength of LSF.

**Appendix D. Velocity boundary layer thickness for large shear**

For large shear, using a new perturbation parameter  $\epsilon_1 = Ra_\lambda/Re_\lambda = 1/\epsilon$ , where  $\epsilon$  is given by (4.7), assuming that  $\lambda \sim \lambda_0$  remains valid, and following the same procedure as in § 4.1, we obtain from (4.17) that

$$\epsilon_1 \xi^5 + \frac{C_2^2 A}{2} Pr^{2n} \xi^2 - \frac{C_2^4 Pr^{5n} E}{4 Re_\lambda} \sim 0. \tag{D1}$$

Let the solution for (D1) be a power series in  $\epsilon_1$  of the form

$$\xi = \xi_{s_0} + \epsilon_1 \xi_{s_1} + O(\epsilon_1^2). \tag{D2}$$

Substituting (D2) in (D1) and comparing the coefficients of different powers of  $\epsilon_1$ , we get

$$\xi_{s_0} = \frac{0.707 Pr^{3n/2}}{A^{1/2}} \sqrt{\frac{C_2^2 E}{Re_\lambda}} \tag{D3}$$

and

$$\xi_{s_1} = -\frac{Pr^{4n}}{4A^3} \frac{C_2^2 E^2}{Re_\lambda^2}. \tag{D4}$$

Thus the perturbation solution of (D1), obtained by substituting (D3) and (D4) in (D2), after neglecting higher-order terms, is

$$\xi \sim \frac{0.707}{A^{1/2}} \left( \frac{C_2^2 Pr^{3n} E}{Re_\lambda} \right)^{1/2} - \frac{C_2^2 Pr^{4n} E^2}{4A^3} \left( \frac{Ra_\lambda}{Re_\lambda^3} \right) + O(\epsilon_1^2). \tag{D5}$$

Variation of (D5) with  $Re_\lambda$ , for  $Pr = 1$  and  $Ra_\lambda = 1.07 \times 10^5$ , is shown in figure 9 to match the variation from the PBBL and the numerical solution of (4.17) at larger values of  $Re_\lambda$ , implying larger shear.

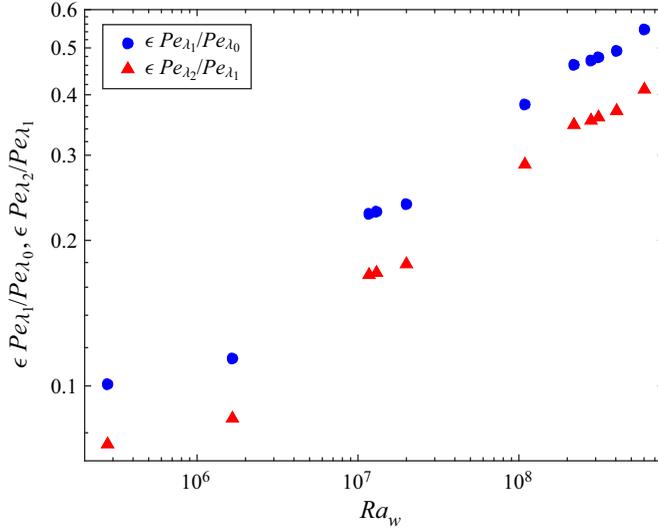


Figure 12. Ratio of terms on the right-hand side of (4.14): blue circles,  $\epsilon Pe_{\lambda_1}/Pe_{\lambda_0}$ ; red triangles,  $\epsilon Pe_{\lambda_2}/Pe_{\lambda_1}$ ; for the experimental data in the present study.

### Appendix E. Upper limit of the analysis

The  $Pe_{\lambda_0}$  term in (4.10) represents the no-shear contribution, while the other terms represent the shear contributions to  $Pe_{\lambda}$ . Figure 12 shows the variation of the ratios of the successive terms in (4.10) with  $Ra_w$ . The contribution of the shear term  $\epsilon Pe_{\lambda_1}$ , compared to the no-shear term  $Pe_{\lambda_0}$ , at the lowest  $Ra_w$ , was approximately 10%. This contribution increases with increase in  $Ra_w$ , and becomes close to 55% at the highest  $Ra_w$ . For the ranges of  $Ra_w$  and  $Pr$  in the present study, both the ratios are much less than 1, and  $\epsilon Pe_{\lambda_2}/Pe_{\lambda_1} < \epsilon Pe_{\lambda_1}/Pe_{\lambda_0}$ . The perturbation solution (4.14) is then a power series, and neglecting the second-order term to obtain (4.15) is justified.

We had limited the power-series solution (4.14) to order  $Re_{\lambda}$  to obtain the scaling (4.15) for  $Re_{bs}$ . The present scaling laws are then valid when  $\epsilon^2 Pe_{\lambda_2} \ll \epsilon Pe_{\lambda_1}$  in (4.10). In other words, from (4.14) with  $C_2 = 0.72$ , the scaling laws are valid when

$$Re_{\lambda} < 2.54 Ra_{\lambda}^{2/5} E^{3/5} Pr^n / A. \tag{E1}$$

Replacing  $\lambda_0$  in (E1) with (1.5), using (1.3) and (B1), and with  $n = 0.5$ , we obtain one of the limits of the present analysis as

$$Ra_w < 2.34 \times 10^7 Pr^{2.6} \Gamma^{5.13}, \tag{E2}$$

when  $A = 1$ .

Similarly, we had also limited the power-series solution (4.25) by eliminating the order  $\epsilon$  term to obtain the scaling (4.28) for  $\xi$ . The present scaling laws are then valid when  $I \gg JARa_w^{1/9}$ , i.e. when

$$Ra_w^{1/9} < \frac{I}{AJ}. \tag{E3}$$

Replacing  $I$  and  $J$ , using (4.26) and (4.27), for  $A = 1$ , we obtain another upper limit of the analysis as

$$Ra_w < 1.12 \times 10^9 Pr^{2.6} \Gamma^{5.13}. \quad (\text{E4})$$

At any  $Pr$ , the limit given by (E4) is higher compared to that given by (E2), and the upper limit of the present analysis at  $A = 1$  is then given by (E2). The upper limit of the analysis for  $A < 1$  will be higher than that given by (E4), as seen from (E3). For the values  $Pr = 5.09$ ,  $\Gamma = 1.71$  and  $A = 0.71$  in the present experiments, (E1) implies that the present analysis is valid for  $Ra_w < 5.5 \times 10^{11}$ ; the analysis is then well valid for the present experimental range.

#### REFERENCES

- AHLERS, G., GROSSMAN, S. & LOHSE, D. 2009 Heat transfer and large-scale dynamics in turbulent Rayleigh–Bénard convection. *Rev. Mod. Phys.* **81**, 503.
- BLASS, A., ZHU, X., VERZICCO, R., LOHSE, D. & STEVENS, R.J.A.M. 2020 Flow organization and heat transfer in turbulent wall sheared thermal convection. *J. Fluid Mech.* **897**, A22.
- CASTAING, B., GUNARATNE, G., HESLOT, F., KADANOFF, L., LIBCHABER, A., THOMAE, S., WU, X., ZALESKI, S. & ZANETTI, G. 1989 Scaling of hard thermal turbulence in Rayleigh–Bénard convection. *J. Fluid Mech.* **204**, 1–30.
- CHILLA, F. & SCHUMACHER, J. 2012 New perspectives in turbulent Rayleigh–Bénard convection. *Eur. Phys. J. E* **35**, 58.
- CHING, E., LEUNG, H.S., ZWIRNER, L. & SHISHKINA, O. 2019 Velocity and thermal boundary layer equations for turbulent Rayleigh–Bénard convection. *Phys. Rev. Res.* **1** (3), 033037.
- CUBA, B., EMRAN, M. & SCHUMACHER, J. 2010 Aspect ratio dependence of heat transfer and large-scale flow in turbulent convection. *J. Fluid Mech.* **655**, 152–173.
- DOERING, C.R., TOPPALADODDI, S. & WETTLAUFER, J. 2019 Absence of evidence for the ultimate regime in two-dimensional Rayleigh–Bénard convection. *Phys. Rev. Lett.* **123** (25), 259401.
- GROSSMANN, S. & LOHSE, D. 2000 Scaling in thermal convection: a unifying theory. *J. Fluid Mech.* **407**, 27–56.
- GUNASEGARANE, G.S. & PUTHENVEETIL, B.A. 2014 Dynamics of line plumes on horizontal surfaces in turbulent convection. *J. Fluid Mech.* **749**, 37–78.
- HE, X., FUNFSCHILLING, D., NOBACH, H., BODENSCHATZ, E. & AHLERS, G. 2012 Transition to the ultimate state of turbulent Rayleigh–Bénard convection. *Phys. Rev. Lett.* **108**, 024502.
- HOWARD, L.N. 1964 Convection at high Rayleigh number. In *Proceedings of the 11th International Congress of Applied Mechanics* (ed. H. Görtler), pp. 1109–1115. Springer.
- IYER, K.P., SCHEEL, J.D., SCHUMACHER, J. & SREENIVASAN, K.R. 2020 Classical 1/3 scaling of convection holds up to  $Ra = 10^{15}$ . *Proc. Natl Acad. Sci. USA* **117** (14), 7594–7598.
- LAM, S., SHANG, X.D., ZHOU, S.Q. & XIA, K.Q. 2002 Prandtl number dependence of the viscous boundary layer and the Reynolds numbers in Rayleigh–Bénard convection. *Phys. Rev. E* **65**, 066306.
- NIEMELA, J.J. & SREENIVASAN, K.R. 2003 Rayleigh-number evolution of large-scale coherent motion in turbulent convection. *Europhys. Lett.* **62** (6), 829–833.
- OVSYANNIKOV, M., KRASNOV, D., EMRAN, M.S. & SCHUMACHER, J. 2016 Combined effects of prescribed pressure gradient and buoyancy in boundary layer of turbulent Rayleigh–Bénard convection. *Eur. J. Mech. (B/Fluids)* **57**, 64–74.
- PERA, L. & GEBHART, B. 1973a Natural convection boundary layer flow over horizontal and slightly inclined surfaces. *Intl J. Heat Mass Transfer* **16** (6), 1131–1146.
- PERA, L. & GEBHART, B. 1973b On the stability of natural convection boundary layer flow over horizontal and slightly inclined surfaces. *Intl J. Heat Mass Transfer* **16**, 1147–1163.
- DU PUIITS, R., RESAGK, C. & THESS, A. 2013 Thermal boundary layers in turbulent Rayleigh–Bénard convection at aspect ratios between 1 and 9. *New J. Phys.* **15** (1), 013040.
- PUTHENVEETIL, B.A., ANANTHAKRISHNA, G. & ARAKERI, J.H. 2005 Multifractal nature of plume structure in high Rayleigh number convection. *J. Fluid Mech.* **526**, 245–256.
- PUTHENVEETIL, B.A. & ARAKERI, J.H. 2005 Plume structure in high Rayleigh number convection. *J. Fluid Mech.* **542**, 217–249.
- PUTHENVEETIL, B.A., GUNASEGARANE, G.S., AGRAWAL, Y.K., SCHMELING, D., BOSBACH, J. & ARAKERI, J.H. 2011 Length of near-wall plumes in turbulent convection. *J. Fluid Mech.* **685**, 335–364.

- PUTHENVEETIL, B.A. & HARSHA, N. 2010 Flux scaling from near-wall coherent structures in turbulent convection. In *Proceedings of the 13th Asian Congress of Fluid Mechanics, Dhaka, Bangladesh*.
- QIU, X.L. & XIA, K.Q. 1998a Spatial structure of the viscous boundary layer in turbulent convection. *Phys. Rev. E* **58**, 5816–5820.
- QIU, X.L. & XIA, K.Q. 1998b Viscous boundary layers at the sidewall of a convection cell. *Phys. Rev. E* **58**, 486–491.
- RAMA REDDY, G.V. & PUTHENVEETIL, B.A. 2011 The  $Pe \sim 1$  regime of convection across a horizontal permeable membrane. *J. Fluid Mech.* **679**, 476–504.
- REEUWIJK, M., JONKER, H. & HANJALIĆ, K. 2008 Wind and boundary layers in Rayleigh–Bénard convection. II. Boundary layer character and scaling. *Phys. Rev. E* **77**, 036312.
- ROCHE, P.E. 2020 The ultimate state of convection: a unifying picture of very high Rayleigh numbers experiments. *New J. Phys.* **22** (7), 073056.
- ROCHE, P.E., CASTAING, B., CHABAUD, B. & HÉBRAL, B. 2004 Heat transfer in turbulent Rayleigh–Bénard convection below the ultimate regime. *J. Low Temp. Phys.* **134**, 1011–1042.
- ROTEM, Z. & CLASSEN, L. 1969 Natural convection above unconfined horizontal surfaces. *J. Fluid Mech.* **39** (Part 1), 173–192.
- SCHEEL, J.D. & SCHUMACHER, J. 2017 Predicting transition ranges to fully turbulent viscous boundary layers in low Prandtl number convection flows. *Phys. Rev. Fluids* **2**, 123501.
- SHEVKAR, P.P., GUNASEGARANE, G.S., MOHANAN, S.K. & PUTHENVEETIL, B.A. 2019 Effect of shear on coherent structures in turbulent convection. *Phys. Rev. Fluids* **4**, 043502.
- SHEVKAR, P.P. & PUTHENVEETIL, B.A. 2022 Boundary layer velocity field in turbulent convection. *J. Fluid Mech.* (submitted).
- SHI, N., EMRAN, M.S. & SCHUMACHER, J. 2012 Boundary layer structure in turbulent Rayleigh–Bénard convection. *J. Fluid Mech.* **706**, 5–33.
- SHISHKINA, O., HORN, S. & WAGNER, S. 2013 Falkner–Skan boundary layer approximation in Rayleigh–Bénard convection. *J. Fluid Mech.* **730**, 442–463.
- SHISHKINA, O., STEVENS, R.J.A.M., GROSSMANN, S. & LOHSE, D. 2010 Boundary layer structure in turbulent thermal convection and its consequences for the required numerical resolution. *New J. Phys.* **12** (7), 075022.
- SHISHKINA, O. & WAGNER, C. 2008 Analysis of sheet-like thermal plumes in turbulent Rayleigh–Bénard convection. *J. Fluid Mech.* **599**, 383–404.
- SHRAIMAN, B.I. & SIGGIA, E.D. 1990 Heat transport in high Rayleigh number convection. *Phys. Rev. A* **42** (6), 3650–3653.
- SKRBK, L. & URBAN, P. 2015 Has the ultimate state of turbulent thermal convection been observed? *J. Fluid Mech.* **785**, 270–282.
- STEVENS, R.J.A.M., POEL, E.P., GROSSMANN, S. & LOHSE, D. 2013 The unifying theory of scaling in thermal convection: the updated prefactors. *J. Fluid Mech.* **730**, 295–308.
- STEWARTSON, K. 1958 On the free convection from a horizontal plate. *Z. Angew. Math. Phys.* **9** (3), 276–282.
- SUN, C., CHEUNG, Y. & XIA, K.Q. 2008 Experimental studies of the viscous boundary layer properties in turbulent Rayleigh–Bénard convection. *J. Fluid Mech.* **605**, 79–113.
- TAI, N.C., CHING, E., ZWIRNER, L. & SHISHKINA, O. 2021 Heat flux in turbulent Rayleigh–Bénard convection: predictions derived from a boundary layer theory. *Phys. Rev. Fluids* **6** (3), 033501.
- THEERTHAN, S.A. & ARAKERI, J.H. 1998 A model for near wall dynamics in turbulent Rayleigh–Bénard convection. *J. Fluid Mech.* **373**, 221–254.
- THEERTHAN, S.A. & ARAKERI, J.H. 2000 Planform structure and heat transfer in turbulent free convection over horizontal surfaces. *Phys. Fluids* **12**, 884–894.
- URBAN, P., HANZELKA, P., KRÁLÍK, T., MACEK, M., MUSILOVÁ, V. & SKRBK, L. 2019 Elusive transition to the ultimate regime of turbulent Rayleigh–Bénard convection. *Phys. Rev. E* **99** (1), 011101.
- URBAN, P., HANZELKA, P., KRÁLIK, T., MUSILOVA, V., SRNKA, A. & SKRBK, L. 2012 Effect of boundary layers asymmetry on heat transfer efficiency in turbulent Rayleigh–Bénard convection at very high Rayleigh numbers. *Phys. Rev. Lett.* **109**, 154301.
- VIPIN, K. 2013 Boundary layer velocity field in turbulent convection. Master's thesis, IIT Madras, Chennai.
- VISHNU, V.T., DE, A.K. & MISHRA, P.K. 2020 Dynamics of large-scale circulation and energy transfer mechanism in turbulent Rayleigh–Bénard convection in a cubic cell. *Phys. Fluids* **32** (9), 095115.
- WAGNER, S., SHISHKINA, O. & WAGNER, C. 2012 Boundary layers and wind in cylindrical Rayleigh–Bénard cells. *J. Fluid Mech.* **697**, 336–366.
- WIENEKE, B. 2015 PIV uncertainty quantification from correlation statistics. *Meas. Sci. Technol.* **26** (7), 074002.

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- ZHU, X., MATHAI, V., STEVENS, R.J.A.M., VERZICCO, R. & LOHSE, D. 2018 Transition to the ultimate regime in two-dimensional Rayleigh–Bénard convection. *Phys. Rev. Lett.* **120**, 144502.
- ZHU, X., MATHAI, V., STEVENS, R.J.A.M., VERZICCO, R. & LOHSE, D. 2019 Zhu *et al.* Reply. *Phys. Rev. Lett.* **123** (25), 259402.
- ZHU, X. & ZHOU, Q. 2021 Flow structures of turbulent Rayleigh–Bénard convection in annular cells with aspect ratio one and larger. *Acta Mechanica Sin.* **37**, 1291–1298.