By adding these two equations, it follows

$$\left(\alpha^{n+2} + \beta^{n+2}\right) - l\left(\alpha^{n+1} + \beta^{n+1}\right) + m\left(\alpha^{n} + \beta^{n}\right) = 0. \qquad (1)$$

Now $C_{n,r}$ is the absolute value of the coefficient of $l^{n-2r}m^r$ in $\alpha^n + \beta^n$. By considering coefficients in (1), we obtain at once the result

$$C_{n,r} + C_{n+1,r+1} = C_{n+2,r+1}$$

which was proved in Note 81.1.

2. Note 81.38 *Going dotty with vectors.*

There are two definitions of scalar (or dot) product in common use:

(1) $\mathbf{a.b} = a_1b_1 + a_2b_2 + \ldots + a_nb_n$, where $\mathbf{a} = (a_1, a_2, \ldots, a_n)$ etc

(2) $\mathbf{a.b} = |\mathbf{a}| \times |\mathbf{b}| \cos\theta$, where θ is the angle between **a** and **b**.

These are equivalent, that is, (1) $\Leftrightarrow$ (2). Most good elementary texts take one of these as a 'definition' and deduce the other as a 'property'.

In Note 81.38 Taverner gives a proof that (2) follows from (1) for the case of two-dimensional vectors. It is not clear to me in what sense this is a 'derivation'. The simplest proof of equivalence (for any dimension) makes use of the cosine rule in triangle *OAB*.

For the vector (or cross) product, there are also two equivalent definitions in common use. Treatments of the proof of their equivalence may be found in the references. [1] and [2] show that the geometric definition follows from the algebraic; [3] and [4] adopt the converse approach.

References

1. Howard Anton, *Elementary linear algebra (Fifth edition)*, John Wiley, New York (1987).
2. D. Griffiths, *Pure mathematics*, Harrap (1984).
3. R. I. Porter, *A school course in vectors*, Bell and Hyman (1970).
4. Murray R. Spiegel, *Advanced calculus*, McGraw-Hill, NewYork (1963).

Yours sincerely,

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DEAR EDITOR,

With reference to Note 81.49 on the Steiner-Lehmus Theorem, it would appear that the second sentence of the first paragraph is missing. The missing sentence should read.

'If two internal bisectors are equal in length, the triangle is also isosceles, but the demonstration in this case is more challenging.'

Without this second sentence, it appears that the first sentence of Note

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81.49 defines the Steiner-Lehmus Theorem, which is not the case. And the generalisation proposed in the second paragraph bears no relation to the proposition in the first sentence of Note 81.49.

Yours sincerely,

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DEAR EDITOR,

The dark green cover of my copy of [1] shows up traces of desert dust deposited during nine years in the Sultanate of Oman, when I came closest to experiencing the 'desert island' of [2]. My having been in Oman at the time (1991) provides the only excuse for my ignorance, belatedly rectified by [3], of the death of Theodor Estermann.

During the year 1957-58 Dr. T. Estermann had two students on his undergraduate course in functions of a complex variable. I had little idea at that time of the significance of being one of those two. When, nearly a decade later, I bought Estermann's book [1], it was mainly for nostalgia's sake since I had given up hope of becoming a mathematician. His matter-of-fact Gemanic accent spoke to me again from every sentence in the book.

It must be difficult for readers who have never met Estermann to appreciate the author's personality. For example, one reads on page 61 of [1], 'Jordan's own supposed proof (of the Jordan curve theorem) is ... very inadequate and marred by serious misconception.' Such language might remind one of a certain type of political polemicist, but when uttered by Estermann it sounded more like an apology, addressed to us students by a mathematician, for the shortcomings of his co-workers. I feel that having been a less than adequate student I at least owe Estermann this 'very inadequate' tribute.

References

1. T. Estermann, *Complex numbers and functions*, The Athlone Press (1962), (1965 reprint).
2. Tony Crilly and Colin Fletcher, Desert Island Theorems, *Math. Gaz.* **82** (March 1998) pp. 2-7.
3. Man-Keung Siu, Estermann and Pythagoras, *Math. Gaz.* **82** (March 1998) pp. 92-93.

Yours sincerely,

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DEAR EDITOR,

Even if a gifted schoolboy submits an article, surely the editor's first responsibility is to the *Gazette* readers. The author of Note 81.39, 'The singularity of Fibonacci matrices', should have been congratulated on his clear writing and his mastery of expansion of determinants by minors. Then