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Letter to the Editor

Modulational instabilities of surface plasmons on metallic plasma surfaces with nanoparticles

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Abstract. The nonlinear coupling between high-frequency surface plasmons (SPs) and low-frequency ion oscillations on metallic plasma surfaces with charged nanoparticles is considered. It is shown that a finite-amplitude SP wave is modulationally unstable against the excitation of non-resonant ion oscillations. The growth rates and thresholds of the modulational instabilities are presented.

Heating as well as ablation of materials by intense light are of significant interest [1–3]. Intense light impinging on a metallic plasma–vacuum/dielectric interface can excite high-frequency surface plasmons (SPs) (also called surface plasma waves (SPWs)) owing to mode conversion [4] and parametric [5] processes. A large-amplitude SPW, which is an electron density wave, can ohmically heat electrons, and produce nonlinear absorption of the wave energy onto the metallic plasma–vacuum/dielectric interface. Nonlinear SPWs may appear in the form of localized excitations due to a balance between the medium nonlinearity and wave dispersion. Surface plasma wave solitons occurring at the interface between a dielectric medium (air) and a nonlinear material have indeed been observed in laboratory experiments [6, 7]. The promise [8–10] of plasmonics has been recognized in the context of its potential applications in improving the resolution of microscopes as well as in enhancing the efficiency of light-emitting diodes and chemical and biological sensors/detectors. Plasmonic circuits could help the designers of computer chips to build fast interconnects that could move large amounts of data across the chips.

In the present letter, we consider the amplitude modulation of a large-amplitude SPW that is propagating along a metallic plasma–vacuum/dielectric surface coated with charged nanoparticles. In such a condensed matter Fermi plasma, a quantum force associated with the Bohm potential [11] acts on the electrons, in addition to the electromagnetic forces. Thus, the dispersion properties of the high-frequency SPWs and low-frequency ion oscillations (IOs) are significantly affected by the quantum force. The SPWs interacting nonlinearly with the IOs would generate SP sidebands. The latter, in turn, would interact with the SP pump to produce a low-frequency ponderomotive force, which can eventually reinforce the IOs. As a result, there appear modulational instabilities [12] due to which the sidebands and the IOs grow at the expense of the SP pump energy. The modulational instability may be responsible for the formation of envelope solitons [13–15].

The dynamics of nonlinearly coupled SPWs and IOs on a metallic plasma–vacuum/dielectric surface is governed by the continuity equation

$$\frac{\partial n_{j1}}{\partial t} + n_{j0}\nabla\cdot\mathbf{v}_j = 0, \tag{1}$$

the momentum equation

$$m_j\left(\frac{\partial}{\partial t} + \mathbf{v}_j\cdot\nabla\right)\mathbf{v}_j = q_j\mathbf{E} - \frac{k_B T_{Fj}}{n_{j0}}\nabla n_{j1} + \frac{\hbar^2}{4m_j n_{j0}}\nabla\nabla^2 n_{j1}, \tag{2}$$

and the Poisson equation

$$\nabla\cdot\mathbf{E} = 4\pi(q_i n_{i1} + q_e n_{e1}), \tag{3}$$

where n_{j1} ($\ll n_{j0}$) is a perturbation of the density of the particle species j (j equals e for electrons and i for ions), n_{j0} is the unperturbed particle number density, $\mathbf{v}_j$ is the fluid velocity perturbation, $\mathbf{E} = -\nabla\phi$ is the electrostatic field, ϕ is the electrostatic potential, $q_j = -e$ ($Z_i e$) for the electrons (ions), e is the magnitude of the electron charge, Z_i is the ion charge state, m_j is the mass, k_B is the Boltzmann constant, and T_{Fj} is the Fermi temperature (in fact, for a condensed Fermi plasma, we have [16] $k_B T_{Fj} = (\hbar^2/2m_j)(3\pi^2)^{1/3} n_{j0}^{2/3}$, where $\hbar$ is the Planck constant divided by 2π). The third term on the right-hand side of (2) is the quantum force associated with the Bohm potential [11], which can cause electron tunneling. At equilibrium, we have $Z_i n_{i0} = n_{e0} + Z_d n_{d0}$, where Z_d is the number of electrons residing on nanoparticles. The latter are supposed to be immobile, since we are considering surface wave phenomena on time scales much shorter than the nanoparticle plasma period.

The frequency ω of the SPW on the metallic plasma–vacuum interface can be obtained from (1)–(3) by supposing that the ions do not participate in the SPW dynamics. Following the general approach of [17], we obtain

$$\omega \approx \frac{\omega_{pe}}{\sqrt{2}}\left(1 + \frac{kV_{Fe}}{\sqrt{2}\omega_{pe}}\sqrt{1 + \hbar^2 k^2/4m_e^2 V_{Fe}^2}\right), \tag{4}$$

where k is the wavenumber (along the direction of the plasma–vacuum/dielectric interface), $\omega_{pe} = (4\pi n_e e^2/m_e)^{1/2}$ is the electron plasma frequency, and $V_{Fe} = (k_B T_{Fe}/m_e)^{1/2}$ is the Fermi thermal speed. In the limit $\hbar k/2m_e V_{Fe} \gg 1$, we have from (4)

$$\omega \approx \frac{\omega_{pe}}{\sqrt{2}}\left(1 + \frac{\hbar k^2}{2\sqrt{2}m_e\omega_{pe}}\right) \equiv \frac{\omega_{pe}}{\sqrt{2}}(1+\beta). \tag{5}$$

The group velocity and the group dispersion of the SPWs, given by (5), are $V_g = \partial\omega/\partial k = \hbar k/2m_e$ and $\partial V_g/\partial k = \hbar/2m_e$, respectively.

We now consider the amplitude modulation of the SPWs, given by (5). Due to the nonlinear coupling between the SPW pump and the IOs, one encounters an envelope of the SPWs whose electric field varies slowly in space and time. The equation governing the envelope of the SPW in the presence of the IOs is

$$i\left(\frac{\partial}{\partial\tau} + V_g\frac{\partial}{\partial\xi}\right)E_z + P\frac{\partial^2 E_z}{\partial\xi^2} - \frac{\omega_p}{2\sqrt{2}}\frac{n_{es}}{n_{e0}}E_z = 0, \tag{6}$$

where $\partial E_z/\partial\tau \ll \omega E_z$, $\partial E_z/\partial\xi \ll kE_z$, E_z is the SPW electric field along the ξ-direction at the surface, $P \approx \hbar/4m_e$, and $\omega_p = (4\pi n_{e0}e^2/m_e)^{1/2}$ is the unperturbed plasma frequency. The slowly varying time and space variables in (6) are denoted here by τ and ξ, respectively. Furthermore, the electron number density perturbation associated with the low-phase speed IOs is denoted by $n_{\rm es}$ ($\ll n_{\rm e0}$). It is obtained from the inertialess electron momentum

$$\frac{e^2}{m_{\rm e}\omega_{\rm p}^2(1+\beta)^2}\frac{\partial|E_z|^2\exp(-2kx)}{\partial\xi}=e\frac{\partial\varphi}{\partial\xi}+\frac{\hbar^2}{4m_{\rm e}n_{\rm e0}}\frac{\partial^3 n_{\rm es}}{\partial^3\xi}, \tag{7}$$

where x is the direction normal to the surface, and φ is the electrostatic potential associated with the IOs. The left-hand side of (7) represents the ponderomotive force of the SPW. We note that (7) is valid for $\partial^2 n_{\rm es}/\partial\tau^2\ll(\hbar^2/4m_{\rm e}^2)\partial^4 n_{\rm es}/\partial\xi^4$ and $k_{\rm B}T_{\rm Fe}n_{\rm es}\ll(\hbar^2/4m_{\rm e})\partial^2 n_{\rm es}/\partial\xi^2$, so that the electron inertial and the electron pressure gradient are neglected, respectively.

The electrons are coupled to the ions via the electrostatic potential φ. The equations governing the ion dynamics supporting IOs are

$$\frac{\partial n_{\rm is}}{\partial\tau}+n_{\rm i0}\frac{\partial u}{\partial\xi}=0, \tag{8}$$

and

$$m_{\rm i}\frac{\partial u}{\partial\tau}=-e\frac{\partial\varphi}{\partial\xi}, \tag{9}$$

where $n_{\rm is}$ ($\ll n_{\rm i0}$) is the ion number density perturbation, u is the ion fluid velocity, and $m_{\rm i}$ is the ion mass. The ion quantum force and the ponderomotive force of the SPWs acting on the ion fluid are smaller by a factor $m_{\rm e}/m_{\rm i}$ (in comparison to that on the electrons), and therefore ignored in (9).

We now combine (7)–(9) by using the quasi-neutrality condition $n_{\rm es}=n_{\rm is}$, to obtain the wave equation for the IOs [18] in the presence of the SPWs,

$$\left(\frac{\partial^2}{\partial\tau^2}+\frac{\alpha\hbar^2}{4m_{\rm e}m_{\rm i}}\frac{\partial^4}{\partial\xi^4}\right)\frac{n_{\rm es}}{n_{\rm e0}}=\alpha\frac{\exp(-2kx)}{4\pi n_{\rm e0}m_{\rm i}(1+\beta)^2}\frac{\partial^2|E_z|^2}{\partial\xi^2}, \tag{10}$$

where $\alpha=n_{\rm i0}/n_{\rm e0}$. We see that the effect of charged nanoparticles appears through α, which is larger than unity in our plasma.

Equations (6) and (10) are the desired equations for investigating the modulational instability of a constant-amplitude SP pump. Following the standard technique [19] of parametric instability investigations, we then obtain the nonlinear dispersion relation

$$(\Omega^2-\Omega_q^2)[(\Omega-KV_{\rm g})^2-P^2K^4]=\alpha PK^4\frac{\omega_{\rm p}|E_{z0}|^2\exp(-2kx)}{4\sqrt{2}\pi n_{\rm e0}m_{\rm i}(1+\beta)^2} \tag{11}$$

where $\Omega_q=\sqrt{\alpha}\hbar K^2/2\sqrt{m_{\rm e}m_{\rm i}}$, Ω and K are the frequency and the wavenumber of the IOs, respectively, and E_{z0} is the electric field of the SP pump.

Two comments are in order. First, for $\Omega\gg KV_{\rm g}$, we have from (11)

$$\Omega^4-\Omega^2\left(\Omega_q^2+P^2K^4\right)+\Omega_q^2P^2K^4-\alpha PK^4\frac{\omega_{\rm p}|E_{z0}|^2\exp(-2kx)}{4\sqrt{2}\pi n_{\rm e0}m_{\rm i}(1+\beta)^2}=0, \tag{12}$$

which has the solutions

$$\Omega^2=\frac{1}{2}\left(\Omega_q^2+P^2K^4\right)\pm\frac{1}{2}\left[\left(\Omega_q^2-P^2K^4\right)^2+\alpha PK^4\frac{\omega_{\rm p}|E_{z0}|^2\exp(-2kx)}{\sqrt{2}\pi n_{\rm e0}m_{\rm i}(1+\beta)^2}\right]^{1/2}. \tag{13}$$

Equation (13) admits a modulational instability, and the growth rate is obtained by letting $\Omega = \Omega_r + i\Omega_i$, where Ω_r and Ω_i are the real and imaginary parts of the modulation frequency. Second, for $\Omega \ll \Omega_q$, (11) reduces to

$$(\Omega - KV_g)^2 = PK^4 \left[P - \alpha \frac{\omega_p |E_{z0}|^2 \exp(-2kx)}{4\sqrt{2}\pi n_{e0} m_i (1+\beta)^2 \Omega_q^2} \right], \tag{14}$$

which predicts an oscillatory modulational instability if

$$|E_{z0}|^2 \exp(-2kx) > \frac{\sqrt{2}\pi m_i n_{e0} \hbar \Omega_q^2 (1+\beta)^2}{\alpha m_e \omega_p}. \tag{15}$$

To summarize, we have considered the nonlinear coupling between finite-amplitude SPWs and IOs at metallic plasma–vacuum/dielectric surfaces with charged nanoparticles. It is shown that such nonlinear interactions lead to modulational instabilities via which the SPW sidebands and the IOs grow on account of the SPW pump energy. We expect that modulationally unstable waves will evolve in the form of envelope SPW solitons. The latter are localized excitations, which can transport the wave energy at metallic plasma surfaces having charged nanoparticles.

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