LETTER TO THE EDITORS

Dear Sirs,

In a recent paper in your journal, EL-BASSIOUNI (1991) presents a mixed model for loss ratio analysis with a fixed parameter for each insurance company and a random parameter for each year. On p. 231 he says,

> The assumption of random-effects for the companies under study is appropriate when they are assumed to be a random sample of the companies in the population. However, if they constitute the whole population or if they are considered to represent themselves, they should be considered to have fixed-effects.

I do not agree with this statement.

A model with random effect for company was presented by RAMLAU-HANSEN (1982) and applied to data from 71 Dutch insurance companies. These data had also been analysed by DE WIT and KASTELIJN (1980). Ramlau-Hansen does not say whether the 71 companies were the whole population of Dutch insurance companies, but de Wit and Kastelijn say that this was not the case. However, does it really matter? Would we have had to apply fixed effect if the companies might have constituted the whole population? Obviously not. In the following we shall argue that an assumption of random effect for company will not necessarily depend on whether the companies consitute the whole population or not; it is only a question of how to interpret the random effect. Furthermore, we shall see that even if the companies consitute the whole population, random effect would often be preferable. To clarify the discussion, we drop the assumption of differences between years, that is, we assume a one-way model.

Let X_{ij} and p_{ij} denote the loss ratio and the earned premium, respectively, for company i in year j, and let $Y_{ij} = \ln X_{ij}$. Company i has been observed for n_i years; El-Bassiouni assumes that all companies have been observed for the same number of years, but we shall not make that restriction in the following. We assume that

$$Y_{ij} = \theta_i + Z_{ij},$$

where the Z_{ij}'s are independent normally distributed random variables with zero mean and variance inversely proportional to the earned premium, that is,

$$\operatorname{Var} Z_{ij} = \frac{\varphi}{p_{ij}}.$$

The index i in θ_i has been introduced as we believe that there might be differences between companies. Considered as fixed effects, they are assumed to be non-random parameters. In practice they would be unknown and have to be estimated. The maximum likelihood estimator of θ_i is

$$\hat{\theta}_i = \frac{1}{p_{i\cdot}} \sum_{j=1}^{n_i} p_{ij} Y_{ij}$$

with

$$p_{i\cdot} = \sum_{j=1}^{n_i} p_{ij}.$$

As

$$\operatorname{Var} \hat{\theta}_i = \frac{\varphi}{p_{i\cdot}},$$

we see that our estimates would be more uncertain for small companies and companies that have only been observed for a short period. Furthermore, in a fixed-effect model it is not possible to estimate the solvency margin of a new company that enters the market.

These deficiencies of the fixed-effect model are less severe in a random-effect approach. It is natural to believe that we can learn something about the loss ratios of one insurance company from loss ratios of other companies. In particular for small and new companies, where we have little experience, these collateral data could be important whereas the importance of collateral data will be lower for companies with greater experience. This intuitive feeling can be modeled by considering θ_i as the value of an unknown random variable Θ_i and assuming that the Θ_i's are mutually independent and identically distributed and independent of the Z_{kj}'s. To avoid El-Bassiouni's problem with the total population one could consider, say, the four companies in the Kuwaiti market as a random sample from an infinite population of possible companies that operate in Kuwait or might enter the market. However, personally I find that considering the companies as a random sample from a larger population, is more confusing than clarifying. I would rather consider the assumption of independent and identically distributed Θ_i's as just a way of modeling uncertainty about differences between companies under the assumption that the companies have something in common.

In the random-effect model, the posterior distribution of Θ_i is normal with

(1) $$\tilde{\mu} = E[\Theta_i | X_{i1}, \ldots, X_{i,n_i}] = \frac{p_{i\cdot}}{p_{i\cdot}+\kappa} \hat{\theta}_i + \frac{\kappa}{p_{i\cdot}+\kappa} \mu$$

$$\tilde{\lambda} = \operatorname{Var}[\Theta_i | X_{i1}, \ldots, X_{i,n_i}] = \frac{\varphi}{p_{i\cdot}+\kappa}$$

$$\mu = E\Theta_i \qquad \lambda = \operatorname{Var} \Theta_i \qquad \kappa = \frac{\varphi}{\lambda}.$$

The conditional distribution of Y_{i, n_i+1} given the observed loss ratios is thus normal with mean $\tilde{\mu}$ and variance $\frac{\varphi}{p_{i, n_i+1}} + \tilde{\lambda}$, and we determine the solvency margin of company i in year $n_i + 1$ from this distribution.

We see that within the random-effect model it is possible to estimate the solvency margin of a new company for which we have no experience. Furthermore, for companies with little experience we give the experience low weight and rely more on the average of the companies instead of relying totally on an uncertain estimate like we did in the fixed-effect model.

The parameters φ, λ, and μ would in practice be unknown and have to be estimated. Distribution-free estimators of these parameters are proposed in SUNDT (1983).

And here there seems to be a better argument than the one given by El-Bassiouni for using fixed effects for the Kuwaiti companies: As there are only four companies, we have only four realisations of the company variable, and these realisations are not even observable. Hence an estimate of their variance would be rather uncertain. However, by using prior knowledge, including knowledge about conditions in other comparable countries, one could apply Bayesian estimation. A similar problem occurs in RAMLAU-HANSEN (1982) with random effect for year as in his numerical example he has data from only three years.

Yours sincerely
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