

## Atmospheric Angular Momentum Variations and Diurnal Polar Motion

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**Abstract.** The atmospheric effective angular momentum functions were used to study the excitation of the diurnal polar motion and nutation. The main effect on polar motion at the frequency of the  $S_1$  tide is up to  $10 \mu\text{as}$ , and on the annual prograde nutation term is up to  $0.1 \text{ mas}$ . The atmosphere and viscosity of the outer core of the Earth were taken into account in calculating the transfer function.

The atmosphere treated as a thin rotating layer gives two new eigenmodes or two new resonance frequencies in the Earth's transfer function, and one of them is in the diurnal frequency band. Viscosity of the fluid outer core and choice of the Earth's model change the nearly diurnal frequencies of the normal modes.

### 1. Introduction

The extremely high precision with which the orientation of the Earth in space and the position of the instantaneous rotation axis inside the Earth can now be measured by space geodetic methods provides an opportunity to test new hypotheses about the Earth's deep interior. Both nutation, that is the motion of the celestial ephemeris pole (CEP) in the inertial reference frame, and short-periodic terms of polar motion, the position of the CEP in the terrestrial reference frame, are excited by the nearly diurnal forcing. The nutation is seen as a retrograde motion of CEP, and the retrograde terms have frequencies which are less than  $-1$  cycles per sidereal day. The main reason for nutation is the lunisolar gravitational torque on the Earth's equatorial bulge.

Polar motion is a prograde motion. Diurnal terms in polar motion can be explained by the effect of the relative motions of the atmosphere and the oceans and by the global pressure field of the fluid on the Earth.

The direct effect of atmospheric and ocean tides on nutation was calculated by Sasao and Wahr (1981). The observable nutation,  $\zeta(\sigma)$ , is the function of the surface load potential  $\Phi_L$  and the relative angular momentum  $h$ :

$$\begin{aligned} \zeta(\sigma) = & -(2.686 \times 10^{-3} \frac{\Omega_0}{\sigma - \sigma_{CW}} + 2.554 \times 10^{-4} \frac{\Omega_0}{\sigma - \sigma_{RFCN}}) \Phi_L + \\ & + (1.124 \frac{\Omega_0}{\sigma - \sigma_{CW}} - 6.170 \times 10^{-4} \frac{\Omega_0}{\sigma - \sigma_{RFCN}}) \frac{h}{A\Omega}, \end{aligned} \quad (1)$$

where  $\Omega_0$  is the sidereal rotation rate of the Earth,  $A$  is the equatorial moment of inertia,  $\sigma_{RFCN}$ ,  $\sigma_{CW}$  are the complex-valued Retrograde Free Core Nutation

(RFCN) and Chandler Wobble (CW) frequencies. Polar motion parameters  $p(t) = x(t) - iy(t)$  reported by IERS are connected with nutation in the frequency domain by the simple equation

$$\zeta(\sigma) = -p(\sigma). \quad (2)$$

The main contribution to nutation and polar motion in the diurnal frequency band comes from the ocean tides (Gross, 1993). The effect of the atmosphere is much smaller. Nevertheless the atmosphere has a non-negligible effect on nutation, in particular on the prograde and retrograde annual nutation (Zharov and Gambis, 1996; Bizouard *et al.*, 1998). In this work we calculate the daily atmospheric polar motion variations. In order to do this we use the corrected frequency domain version of the extended polar motion equation. It was derived by Brzeziński (1994) using equations (1,2):

$$p(\sigma) = \frac{\sigma_{CW}}{\sigma_{RFCN} - \sigma} (a_p \chi^p + a_w \chi^w) + \frac{\sigma_{CW}}{\sigma_{CW} - \sigma} (\chi^p + \chi^w), \quad (3)$$

where  $a_p, a_w$  are dimensionless parameters depending on the Earth model. Equation (3) connects the temporal variations of polar motion with the effective angular momentum (EAM) functions  $\chi$  of the atmosphere;  $\chi^p, \chi^w$  are the “pressure” and “wind” terms, respectively. In order to calculate the diurnal variations of polar motion it is necessary to know the EAM functions with high temporal resolution. We used the homogeneous 29-year series of the 4-times daily estimates of the atmospheric EAM that were obtained by the U.S. National Center for Environmental Research (NCEP) and the U.S. National Center for Atmospheric Research (NCAR) (Kalnay *et al.*, 1996).

Why is it necessary to correct equation (3)?

Coefficients and frequencies in (1) and (3) were calculated for the 1066A model of the Earth. But it was found by analysis of very long baseline interferometry (VLBI) data that the frequencies differ from frequencies predicted by Wahr’s theory (Wahr, 1981).

In order to calculate the effect of the atmospheric tides on polar motion the theoretical frequencies  $\sigma_{RFCN}, \sigma_{CW}$  in (3) are replaced by the observed frequencies (Bizouard *et al.*, 1998). Really it means that the model 1066A is replaced by another model of the Earth.

Our goal was to find better agreement between model and observations. The main idea was to take into account in our computation both the atmosphere as a thin rotating layer and the viscosity of the fluid outer core (FOC). As shown below the atmosphere determines new normal modes of the Earth and one of them is in the diurnal frequency band. It means that resonance enhancement of the diurnal atmospheric tides is possible. Viscosity of the outer core significantly changes the diurnal frequencies of the normal modes and quality factor  $Q_{RFCN}$  of RFCN. Using the different models of viscosity one can fit the frequency and the quality factor of the RFCN to the observed ones.

Our methodology is to seek the best fit of the theoretical amplitudes of nutation terms to ones obtained from VLBI data analysis by changing the model of viscosity. After that the solution of the dynamical equations for the Earth can be used to recalculate equations 1 and 3.

2. Main equations

According to the theory of nutation of the elastic, oceanless, oblately spheroidal and spherically stratified and dissipationless model of the Earth (Mathews *et al.*, 1991) we can write the dynamical equations in the Earth-fixed reference frame:

$$\frac{\partial \mathbf{H}}{\partial t} + \boldsymbol{\Omega} \times \mathbf{H} = \mathbf{L}, \tag{4}$$

$$\frac{\partial \mathbf{H}_f}{\partial t} - \boldsymbol{\omega}_f \times \mathbf{H}_f = 0, \tag{5}$$

$$\frac{\partial \mathbf{H}_s}{\partial t} + \boldsymbol{\Omega} \times \mathbf{H}_s = \mathbf{L}_s. \tag{6}$$

The equations are written for the whole Earth, for the FOC and the solid inner core (SIC), respectively. If we take into account the atmosphere as an outer layer relative to the solid Earth, we have to add the fourth equation:

$$\frac{\partial \mathbf{H}_a}{\partial t} + \boldsymbol{\Omega} \times \mathbf{H}_a = \mathbf{L}_a. \tag{7}$$

In equations (4–7)  $\mathbf{H}$ ,  $\mathbf{H}_f$ ,  $\mathbf{H}_s$ ,  $\mathbf{H}_a$  are the angular momentum of the whole Earth, the FOC, the SIC and the atmosphere, respectively,  $\mathbf{L}$ ,  $\mathbf{L}_s$ ,  $\mathbf{L}_a$  are the torque on the Earth, on the SIC and on the atmosphere. We defined the instantaneous rotation vector of the Earth as  $\boldsymbol{\Omega}$  and the rotation vector of the FOC relative to the mantle as  $\boldsymbol{\omega}_f$ .

The system of equations (4–7) can be reduced to a system of coupled linear equations for small unknown parameters that represent the wobble of the mantle ( $\mathbf{m}$ ), the wobbles ( $\mathbf{m}_f$ ,  $\mathbf{m}_s$ ,  $\mathbf{m}_a$ ) of the FOC, SIC and the atmosphere, respectively, and the inclination of the polar axis of the SIC ( $\mathbf{n}_s$ ) and the atmosphere ( $\mathbf{n}_a$ ) from that of the rest of the Earth. The amplitude of the wobble ( $\mathbf{m}$ ) does not exceed  $4 \times 10^{-8}$  (Mathews *et al.*, 1991) in the diurnal frequency band. The leading terms in (4–7) are of the first order in  $m$ , or  $O(m)$ .

Let us consider the effect of viscosity of the FOC first. Equation (4) for the whole Earth does not change, if we take into account the viscous forces, because they are inner forces, and, consequently, cannot change the torque  $\mathbf{L}$ . But equations (5,6) have to be rewritten.

As shown in Appendix B (Mathews *et al.*, 1991) the dynamical equation for the FOC, written in an alternative form is

$$\frac{\partial \mathbf{H}_f}{\partial t} - \boldsymbol{\omega}_f \times \mathbf{H}_f = - \int_V \rho \mathbf{r} \times \mathbf{G} dV, \tag{8}$$

where force  $\mathbf{G}$  was obtained by the integration of the equations of the fluid motion in our reference frame for the dissipationless model of the Earth. The right-hand side of (8) can be set to zero with error of order of  $O(m\varepsilon^2)$ , where  $\varepsilon$  is the flattening of the Earth. We have to add the torque of the viscous forces:

$$\mathbf{L}_f^{(\eta)} = \oint_{S_f} \mathbf{r} \times \mathbf{p}_n^{(\eta)} dS, \tag{9}$$

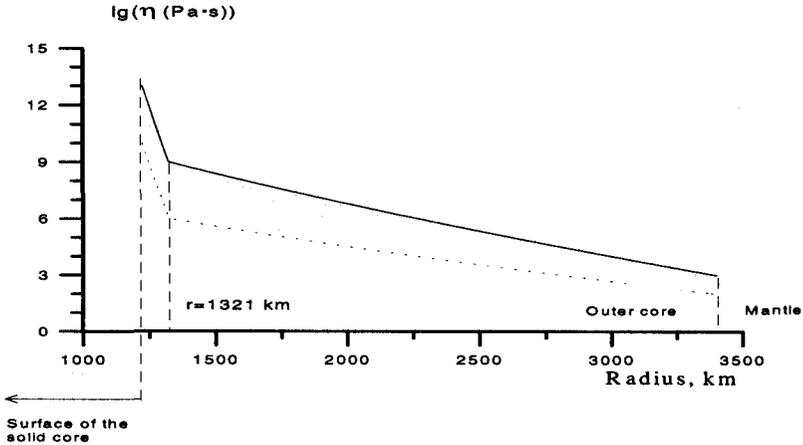


Figure 1. Model of viscosity

where

$$\mathbf{p}_n^{(\eta)} = \eta(r) \sum_{i,j=1}^3 \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) n_j \mathbf{e}_i, \tag{10}$$

$\eta(r)$  is the viscosity of the FOC,  $\mathbf{v} = \mathbf{v}(v_1, v_2, v_3) = \mathbf{v}^e + \mathbf{v}^v$  is the residual velocity field that includes non-rotational motion of a fluid, caused by the elastic deformational motion ( $\mathbf{v}^e$ ) and by the viscous motion ( $\mathbf{v}^v$ );  $\mathbf{n}$  is normal to the corresponding point of the FOC surface. One can write similar equations for the torque acting on the mantle ( $\mathbf{L}_m^{(\eta)}$ ) and the inner core ( $\mathbf{L}_s^{(\eta)}$ ). According to the conservation law we can write:

$$\mathbf{L}_s^{(\eta)} + \mathbf{L}_m^{(\eta)} + \mathbf{L}_f^{(\eta)} = 0.$$

The model of viscosity is shown on Fig. 1. The estimation of the viscosity of the FOC was reported by Brazhkin (1999). It was based on the experiments where the viscosity of the liquid was studied under high pressure (Brazhkin, 1998). The upper and lower curves in Fig. 1 correspond to the upper and lower possible values of viscosity assumed by Brazhkin.

For our model it is important that viscosity changes very significantly only on the boundary between the FOC and the SIC in a thin layer with thickness of order of 100 km.

We used the functional dependence of Brazhkin’s model, but reduced the viscosity values. We assumed that  $\eta = 10^7 - 5 \cdot 10^7 \text{ Pa} \cdot \text{s}$  for the viscosity value on the solid inner core – fluid outer core boundary. In this case the quality factor of the free core nutation is in the range  $Q_{RFCN} \simeq 30000 - 6000$ .

The torque approximately equals the surface integral of the product of viscosity and the gradient of velocity. Because the surface of the core–mantle boundary is only 10 times more than the surface of the SIC, we will omit the integrals on the surface of the core–mantle boundary. It means that  $\mathbf{L}_m^{(\eta)} \ll \mathbf{L}_s^{(\eta)}$ .

On the basis of this model we get

$$\mathbf{L}_s^{(\eta)} = -\mathbf{L}_f^{(\eta)}. \tag{11}$$

In order to calculate the torque  $\mathbf{L}_f^{(\eta)}$  of viscous forces (10) the integration over the FOC–SIC boundary (9) has to be estimated.

Briefly, if we should know the velocity field  $\mathbf{v}$  we could calculate the torque  $\mathbf{L}_f^{(\eta)}$  on the fluid core (9), and then, using equation (11) find the torque  $\mathbf{L}_s^{(\eta)}$  on the SIC. After correction of the right-hand sides of equations (5,6) we could solve the system of equations (4–6).

The main problem is the calculation of the velocity field  $\mathbf{v}$ . In order to avoid this hard work the special choice of the reference frame is necessary. The effect of  $\mathbf{v}$  has been eliminated in Mathews *et al.* (1991) by requiring that the relative torque is equal to zero:

$$\int_V \rho \mathbf{r} \times \mathbf{v}^e dV = 0.$$

In this case the reference system is close to the Tisserand mean axis for the mantle.

In order to do our calculations more simply we have to suppose some heuristic but logical requirements for the non-rotating velocity field  $\mathbf{v}$ . First, suppose that viscosity does not affect the motion of the particles of the fluid caused by elastic deformations. As shown below, the viscous torque has the order of  $O(m)$ . It means that the effect of viscosity on the deformational motion will be of the order of  $O(m^2)$ . We will neglect such terms.

Further approximation is possible on the base of this estimate. We will consider that the additional torque caused by the residual velocity field both from elastic deformational motion and from the viscous motion are equal to zero with necessary precision. This means that the reference system that rotates with the angular velocity of the FOC is the Tisserand axis system for the FOC.

Finally we shall consider that the velocity  $\mathbf{v}^v$  depends on the distance  $r$  from the center of the Earth:

$$\mathbf{v}^v(r) = \mathbf{\Omega}(r) \times \mathbf{r}, \tag{12}$$

$$\mathbf{\Omega}(r) = \mathbf{\Omega}_s + \frac{\mathbf{\Omega}' - \mathbf{\Omega}_s}{\delta r} (r - r_s) + O(\delta r^2), \quad r_s + \delta r > r > r_s.$$

where  $r_s = 1221$  km is the radius of the SIC,  $\delta r = 100$  km is the thickness of the layer of high viscosity. We assign a rotation vector  $\mathbf{\Omega}(r)$  to the fluid layer at distance  $r$  from the Earth's center. As mentioned above we consider that the FOC extends from the layer of low viscosity to the thin layer with thickness  $\delta r$  on the FOC–SIC boundary. So we define  $\mathbf{\Omega}_s$  as the rotation vector of the Tisserand axis system for the SIC relative to the inertial system and  $\mathbf{\Omega}'$  as the rotation vector of the FOC (outside the layer  $\delta r$ ).

The requirement that the relative torque has to be equal to zero leads to the equation:

$$\mathbf{\Omega}' = \mathbf{\Omega}_f + O(m\epsilon^2), \tag{13}$$

where  $\Omega_f$  is the rotation vector of the fluid core, or the angular velocity of the Tisserand axis system for the FOC.

Using the velocity field (12) together with (13) and (10) for the torque on the FOC (9) we get

$$L_f^{(\eta)} = -\eta(r_s) \frac{8\pi}{3} \frac{\Omega_f - \Omega_s}{\delta r} r_s^4. \tag{14}$$

The torque on the SIC can be calculated using relation (11) and written as:

$$L_s^{(\eta)} = W \Omega_0^2 (\mathbf{m}_f - \mathbf{m}_s), \quad W = \eta(r_s) \frac{8\pi}{3} \frac{r_s^4}{\delta r \Omega_0}, \tag{15}$$

so the viscous torque has the order of  $O(m)$ .

The equatorial component of the torque on the atmosphere  $\tilde{L}_a$  was calculated by Zharov (1997) and can be written as  $\tilde{L}_a = -iU(\tilde{c}_3^a + \tilde{h}/\Omega_0)$  where parameter  $U$  depends on the Earth's figure. Variations of the moment of inertia of the atmosphere  $\tilde{c}_3^a$  and relative angular momentum  $\tilde{h}$  are connected with the EAM atmospheric functions  $\chi = \chi^p + \chi^w$  by equation  $\chi = (\tilde{c}_3^a + \tilde{h}/\Omega_0)/(C - A)$ .

We keep the same notation for all parameters as in Mathews *et al.* (1991). With the inclusion of the viscous coupling at the SIC and the atmosphere, the system of equations is

where: 
$$Mx = y, \tag{16}$$

$$M = \begin{pmatrix} \sigma(1+\kappa) + \kappa - e & (1+\sigma)\left(\frac{A_f}{A} + \xi\right) & (1+\sigma)\left(\frac{A_s}{A} + \zeta\right) & (1+\sigma)\alpha_3 e_s \frac{A_s}{A} & (1+\sigma)\frac{A_a}{A} & (1+\sigma)\frac{A_a}{A} e_a (1+\kappa') \\ \sigma(1+\gamma) & 1 + e_f + \sigma(1+\beta) - i\frac{W}{A_f} & \sigma\delta + i\frac{W}{A_f} & -\sigma\alpha_1 e_s \frac{A_s}{A} & 0 & \sigma\left(\frac{\xi}{\tau} + h_f\right) \frac{A_a}{A} e_a \\ \sigma(1+\theta) - e_s \alpha_3 & \sigma\chi + \alpha_1 e_s + i\frac{W}{A_s} & \sigma\nu + 1 + \sigma - i\frac{W}{A_s} & (1+\sigma - \alpha_2) e_s & 0 & \sigma\left(\frac{\zeta}{\tau} + h_s\right) \frac{A_a}{A} e_a \\ \sigma - e_a & 0 & 0 & 0 & 1 + \sigma & \left(1 + \sigma + \frac{U}{\Omega_0^2}\right) e_a \\ 0 & 0 & 0 & 0 & 1 & \sigma \end{pmatrix}$$

$$x = \begin{pmatrix} \tilde{m} \\ \tilde{m}_f \\ \tilde{m}_s \\ \tilde{n}_s \\ \tilde{m}_a \\ \tilde{n}_a \end{pmatrix}, \quad y = \begin{pmatrix} (\kappa - e + \sigma\kappa)\tilde{\phi} - (1+\sigma)(1+\kappa')\frac{\tilde{c}_3^a}{A} \\ \sigma\gamma\tilde{\phi} - \sigma\left(\frac{\xi}{\tau} + h_f\right)\frac{\tilde{c}_3^a}{A_f} \\ (\sigma\theta - \alpha_3 e_s)\tilde{\phi} - \sigma\left(\frac{\zeta}{\tau} + h_s\right)\frac{\tilde{c}_3^a}{A_s} \\ 0 \\ -(1+\sigma + \frac{U}{\Omega_0^2})\frac{\tilde{c}_3^a}{A_a} \\ 0 \end{pmatrix}.$$

The parameters  $A, A_f, A_s, A_a$  are the equatorial moments of inertia,  $e, e_f, e_s, e_a$  are the dynamical ellipticities of the whole Earth, the FOC, the SIC and the atmosphere,  $\sigma$  is the frequency of harmonics of the tidal potential  $\tilde{\phi}$ . Other Greek symbols are the compliances representing the deformations of the Earth and core regions.

The frequencies of the normal modes of the Earth with the atmosphere and viscous outer core were found. Two of these belong to the Chandler Wobble (CW) of the Earth and its solid core (ICW) approximately:

$$\sigma_{CW} = \frac{A}{A_m}(e - \kappa), \quad \sigma_{ICW} = (1 - \alpha_2)e_s.$$

Viscosity does not change the values of these frequencies but affects the frequencies of the free core nutation (FCN) and free inner core nutations (PFCN) approximately:

$$\begin{aligned} \sigma_{FCN} &= -1 - \left(1 + \frac{A_f}{A_m}\right)(e_f - \beta) + i\left(1 + \frac{A_f}{A_m}\right)\frac{W}{A_f}, \\ \sigma_{PFCN} &= -1 + \left(1 + \frac{A_f}{A_m}\right)(e_f - \beta)(\alpha_2 e_s + \nu) + \frac{W^2}{A_f A_s} + \\ &+ i\left[\left(1 + \frac{A_f}{A_m}\right)(e_f - \beta)\frac{W}{A_s} - \left(1 + \frac{A_f}{A_m}\right)\frac{W}{A_f}(\alpha_2 e_s + \nu)\right]. \end{aligned}$$

The parameter  $W$  depending on viscosity  $\eta$  determines the dissipation of the tidal energy or quality factor.

Two frequencies associated with a wobble of the atmosphere  $\sigma_{ACW}$  and the nearly diurnal atmospheric nutation  $\sigma_{RFAN}$  are approximately equal to:

$$\sigma_{ACW} = \left(1 + \frac{U}{\Omega_0^2}\right)e_a, \quad \sigma_{RFAN} = -\left(1 + \frac{U}{\Omega_0^2}e_a + (\kappa + \beta - 2\xi)\frac{A}{A_m}\right).$$

Both frequencies are determined by the dynamical ellipticity of the atmosphere  $e_a$ . If the surface of the Earth is determined by the condition of hydrostatic equilibrium, then  $U = \Omega_0^2$ . According to the estimate of Sidorenkov (1973)  $e_a = 0.01476$  and it means that the frequency  $\sigma_{RFAN}$  differs significantly from the main diurnal tides.

In order to calculate the nutation amplitudes the transfer function was written as

$$f(\sigma) = R + R'(1 + \sigma) + \sum_{i=1}^6 \frac{R_i}{\sigma - \sigma_i},$$

where  $R, R', R_i$  are the resonance coefficients. The precise periods of the normal modes and the coefficients  $R, R', R_i$  are shown in Table 1. Amplitudes of the

Table 1. Periods P (in solar days) of the normal modes and resonance coefficients for  $\eta = 10^7 \text{ Pa} \cdot \text{s}, Q_{RFCN} \approx 30000$ .

	P	$R_i$
CW	400.64	$(-5.804 \cdot 10^{-4}, 1.1 \cdot 10^{-11})$
RFCN	-427.69	$(-1.176 \cdot 10^{-4}, -5.11 \cdot 10^{-8})$
ICW	2408.91	$(-4.6 \cdot 10^{-8}, -1.3 \cdot 10^{-11})$
PFCN	477.29	$(1.1 \cdot 10^{-6}, 5.054 \cdot 10^{-8})$
ACW	34.27	$(1.5 \cdot 10^{-10}, 0.0)$
RFAN	-69.54	$(-3.0 \cdot 10^{-10}, 0.0)$
		$R = (1.0504015, 1.7 \cdot 10^{-10})$
		$R' = (-0.2835477, -1.4 \cdot 10^{-12})$

main nutation terms (in-phase and out-of-phase) were calculated for viscosity

$\eta = 5 \cdot 10^7 \text{Pa} \cdot \text{s}$ ,  $Q_{RFCN} \approx 6000$  and  $\eta = 10^7 \text{Pa} \cdot \text{s}$ ,  $Q_{RFCN} \approx 30000$ . They were corrected for the effects of anelasticity and ocean tides and are shown in Table 2. The values of the dynamical ellipticity of the FOC  $e_f$  were calculated in order to give complete agreement with the observed amplitude of the retrograde annual nutation term.

Table 2. Amplitudes (in mas) of the main nutation terms and comparison with the VLBI estimate.

Period (days)	VLBI estimate	$\eta = 5 \cdot 10^7 \text{Pa} \cdot \text{s}$ $e_f = 2.6835 \cdot 10^{-3}$	$\eta = 10^7 \text{Pa} \cdot \text{s}$ $e_f = 2.6831 \cdot 10^{-3}$
-13.66	(-3.63,-0.03)	(-3.64,-0.01)	(-3.64,-0.01)
-182.62	(-24.57,-0.06)	(-24.52,-0.05)	(-24.52,-0.05)
-365.26	(-33.05, 0.35)	(-33.05,-0.11)	(-33.05,-0.07)
-6798.38	(-8024.88,1.43)	(-8025.48,0.57)	(-8025.62,0.85)
6798.38	(-1180.50,-0.07)	(-1180.28,-0.07)	(-1180.28,-0.11)
365.26	(25.64,0.12)	(25.68,0.00)	(25.71,0.01)
182.62	(-548.47,-0.51)	(-548.24,-0.44)	(-548.27,-0.50)
13.66	(-94.20,0.13)	(-94.05,0.00)	(-94.05,0.00)

### 3. Tidal analysis

The atmospheric tides are excited both by the lunisolar gravitational force and by the solar heating. They are the large scale waves that cause the periodic variations in pressure and in the wind velocity field, and, consequently, in the  $\chi$ -functions. In the frequency domain the EAM atmospheric functions  $\chi$  may be expressed as a series of discrete terms:

$$\chi(t) = \chi_1 + i\chi_2 = - \sum_{j=1}^n (\chi_j^c + i\chi_j^s) e^{i\Phi_j},$$

where  $n$  is the number of tidal terms considered,  $\chi^c$  and  $\chi^s$  are the cosine and sine amplitudes of these terms,  $\Phi_j$  is the tidal argument that is equal to the sum of the five fundamental arguments and of  $GMST + \pi$  ( $GMST$  is the Greenwich mean sidereal time).

To calculate the polar motion variation the corrected equation (3) has to be used:

$$p(\sigma) = \frac{\sigma_{CW}}{\sigma_{RFCN} - \sigma} (a_p \chi^p + a_w \chi^w) + \frac{\sigma_{CW}}{\sigma_{CW} - \sigma} (\chi^p + \chi^w) + \frac{\sigma_{CW}}{\sigma_{RFAN} - \sigma} (a'_p \chi^p + a'_w \chi^w).$$

Really the third term is very small. It is approximately six orders less than the first two terms. The amplitudes of the  $\chi$  tidal terms and effect on polar motion and nutation are shown in Table 3.

Table 3. Amplitudes (in  $\mu as$ ) of the atmospheric tides and contribution to polar motion and nutation.

Tide	Wind term		Pressure term		Polar motion	
	$\chi^c$	$\chi^s$	$\chi^c$	$\chi^s$	$p^c$	$p^s$
$K_1$	2	83	-159	369	0.4	-1.0
$P_1$	253	213	341	120	-1.4	-1.1
$S_1$	-158	-2020	-1958	-275	5.1	5.0
$O_1$	14	-5	-35	8	0.1	0.0
$Q_1$	42	-36	-27	-53	0.0	0.0
$\pi_1$	-30	-75	-251	112	0.7	0.2
$\psi_1$	-280	-23	-41	-55	0.7	0.1
$M_2$	51	-15	45	-152	0.0	0.1

Tide	Wind term		Pressure term		Nutation	
	$\chi^c$	$\chi^s$	$\chi^c$	$\chi^s$	$p^c$	$p^s$
$K_1$	16928	-3007	-436	350	71.3	-40.3
$P_1$	-18644	2686	88	383	-42.4	-3.9
$S_1$	-12812	2378	1545	-490	-88.9	25.1
$O_1$	116	-990	-149	-45	0.4	-2.2
$Q_1$	-214	69	21	40	-0.5	0.1
$\pi_1$	451	236	197	-183	-2.6	3.9
$\psi_1$	1857	-893	-28	-145	-4.6	-106.4
$M_2$	51	-15	45	-152	-0.1	0.1

4. Conclusion

The main atmospheric effect on polar motion and nutation are determined by the solar thermal tide  $S_1$ . Amplitude of the diurnal polar motion variations does not exceed  $10\mu as$ . The effect on the prograde annual nutation is of the order of 0.1 mas.

The effect of the viscosity of the FOC on nutation was calculated. It was shown that the dynamical ellipticity of the FOC and the frequency  $\sigma_{RFCN}$  depend on the model of viscosity and the Earth's model. Viscosity on the solid inner core – fluid outer core boundary is equal to  $\eta = 10^7 Pa \cdot s$ . In this case the quality factor of the free core nutation is equal to  $Q_{RFCN} = 30000$ .

The atmosphere as a rotating layer determines new normal modes, but the frequency  $\sigma_{RFAN}$  is far from the main tidal terms. This frequency is determined by the dynamical ellipticity of the atmosphere, and our estimate corresponds to an extra flattening of the atmosphere.

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