something new to say. This leads on to control theory and the study of the singularities of accessibility boundaries. An example given of control theory is tacking in sailing; as elsewhere, Arnold manages to find very simple but illuminating applications for the mathematics that he is discussing. Chapters on the projections of smooth surfaces onto planes and on the study of shortest paths that bypass obstacles illustrate other uses of singularity theory.

The penultimate chapter is on symplectic and contact geometries; these are topics that Arnold himself has done so much to bring to the forefront of modern geometry and its applications. These closely related topics have their roots firmly based in nineteenth-century work on analytical dynamics, a development culminating in E. T. Whittaker's influential treatise. In recent years the subject has been clarified enormously by the use of the axiomatic method (Arnold mentions Bertrand Russell's remark that this method has advantages similar to those that stealing has over honest work). Not only has Arnold given a masterly account of classical mechanics (in his book *Mathematical methods of classical mechanics*) from this viewpoint, but he has also developed variants of singularity theory that exploit the extra structure given by symplectic and contact geometries and has shown their relevance to the understanding of several interesting phenomena. These general ideas may even have more lasting significance than singularity theory itself; Arnold certainly believes in their importance as the final paragraph of this fourteenth chapter shows clearly.

The final chapter entitled "The mystics of catastrophe theory" begins with a slightly critical account of Thom's philosophy but ends with a more common brand of mathematical mysticism—why do the Dynkin diagrams appear in so many diverse branches of mathematics? The excuse for bringing this question in here is that Dynkin diagrams do appear in singularity theory.

The book is one of a rare kind amongst mathematics books: it is written for a general audience but does not talk down to the reader; additionally, the professional mathematician can return to it time and time again both for inspiration and for information about current mathematics. Arnold enjoys both doing mathematics and writing about it in all its forms and is fascinated by its internal mysteries and its applications; this enthusiasm is obvious on every page. It is something of a pity that the publishers forgot to proof-read it before printing; however, there is a loose errata sheet with further references. In many places the book can whet the appetite for more but tantalisingly often there are no adequate references. Fortunately the book *Singularities of differentiable maps,* Volume 1, by V. I. Arnold, S. M. Gusein-Zade and A. N. Varchenko (Birkhäuser 1985) has appeared in the meantime and contains mathematical details on most of the topics discussed in the present book as well as many references to the work of Arnold's students and collaborators.

Finally, I would like to say that I recommend that everyone should spend at least an evening enjoying this book.

E. G. REES

DUBROVIN, B. A., FOMENKO, A. T. and NOVIKOV, S. P., *Modern geometry—methods and applications. Part 2: The geometry and topology of manifolds* (translated by R. G. Burns, Graduate Texts in Mathematics 104, Springer-Verlag, Berlin-Heidelberg-New York 1985), xv+430 pp., DM 158.

This book is Part Two of a trilogy whose aim is no less than an exposition of modern geometry. The subject matter of this volume is the geometry and topology of manifolds, a quite enormous area, and the authors make no pretence at an encyclopedic treatment. Indeed, their philosophy is quite neatly encapsulated in the following quotation from their introduction. They claim to have striven to

> "minimise the degree of abstraction of the exposition and terminology, often sacrificing thereby some of the so-called 'generality' of statements and proofs: frequently an important result may be obtained in the context of crucial examples containing the whole essence of the

matter, using only elementary classical analysis and geometry . . . , while the result's most general formulation and especially the concomitant proof will necessitate a dramatic increase in the complexity and abstractness of the exposition."

This seems an admirable viewpoint to take when writing a text, rather than a reference work, and while not avoiding all of the associated dangers the authors have largely succeeded in producing an excellent book true to their philosophy. Before discussing this further, here is a brief description of the book's contents.

The first chapter is devoted to the definition and examples of smooth manifolds, and includes a *very* rapid introduction to the theory of Lie groups, a discussion of homogeneous spaces, and, perhaps surprisingly at this early juncture, a treatment of symmetric spaces. In Chapter Two there is a fairly standard treatment of foundational questions, which includes applications of partitions of unity to the existence of metrics and the general Stokes formula, approximation of continuous maps by smooth ones, Sard's theorem and transversality. It also includes the usual applications of transversality to the existence of Morse functions and Whitney's embedding theorem. There follows in Chapter Three a definition of the degree of a map and the intersection index of pairs of submanifolds of complementary dimension in a manifold. The treatment and the applications here are also fairly standard; applications include the classification of homotopy classes of mappings of compact n-manifolds to the n-sphere, the Gauss–Bonnet formula, Poincaré–Bendixson theorem, Brouwer fixed-point theorem and Jordan–Brouwer separation theorem. Chapter Four deals with the fundamental group and covering spaces, and there is a discussion of the discrete groups of motions of the Lobachevskian plane, although here most of the results are only stated. Chapter Five moves on to the higher homotopy groups, the exact sequence of a pair and a fibre space and includes a discussion of the Whitehead product. The last section uses Pontryagin's interpretation of homotopy groups of spheres as framed cobordism groups to calculate the groups $\pi_{n+1}(S^n)$, $\pi_{n+2}(S^n)$ and to give a geometric interpretation of the suspension map $\pi_{n+k}(S^n)\to\pi_{n+k+1}(S^{n+1})$. Chapter Six discusses fibre bundles, principal bundles, connexions on bundles, curvature, and characteristic classes via curvature forms. The final two chapters contain several recent results concerning dynamical systems and foliations, the general theory of relativity, and the theory of Yang–Mills and chiral fields. The resulting survey is of independent interest: certainly I am not aware of any similar treatment in any other text. Finally, a word about the other two volumes. The first volume is referred to for local analytic results, but the present volume can be profitably used without it. The final volume concerns itself largely with homology theory and it might have been better to introduce this topic earlier; its absence is rather annoying at times in Volume 2. For example, in the section on characteristic classes there are, not surprisingly, various references to Volume 3.

Although this is a volume of over 400 pages, the (incomplete!) listing of the contents given above shows that some of the topics must be treated in a rather superficial manner, and various sections do appear to be of very limited value. As an example there is an 11-page section concerned with knots, links and braids which occurs, strangely, in Chapter Six on fibre bundles. Similarly there is a 10-page review of foliations, with a mention of some of the fascinating and deep results concerning foliations of 3-manifolds due to the senior author Novikov. It is difficult to imagine what use such a section serves, other than to advertise the existence of an interesting and vast area of geometry. There are other sections of this type, but by and large the authors avoid the dangers of a superficial treatment.

There is a large number of interesting examples, and the diligent reader will gain much from this text, although it is rather difficult to judge how *easily* a novice would assimilate this breadth of material. There is little new by way of method or content in the first six chapters, but the unified treatment, the large number of examples, the resulting cross-fertilization of ideas are of great value. The authors claim to have had theoretical physicists very much in mind when writing this text. As they state in their introduction there has been a "rash" of non-trivial applications of topological methods here of late. The book is, for the mathematician or physicist, a good exposition of the *basic* ideas and methods available, and although any serious applications would undoubtedly require further reading, this is its great value.

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