Can. J. Math. Vol. XXVIII, No. 2, 1976, pp. 440–445

PLETHYSM OF S-FUNCTIONS

A. O. USHER

The S-function $\{\mu\} \otimes \{\lambda\}$, $\mu \vdash m$, $\lambda \vdash l$, where $\{\mu\} \otimes \{\lambda\}$ is the 'new multiplication' or plethysm of D. E. Littlewood [**1**], corresponds, in the sense defined below in (1), to the character afforded by a representation of the symmetric group $\mathbf{S}_{lm}$ induced from a representation of the subgroup $\mathbf{S}_m \wr \mathbf{S}_l$ [**3** § 6; **4** § 3.5]. The aim of this paper is to define the latter representation and deduce its character using a somewhat different approach from that in [**3**].

In Section 2, the character '$\{\mu\} \otimes \{\lambda\}$' of the general linear group, $\mathbf{GL}_n$, over the field of complex numbers, is introduced and expressed in a form given by H. O. Foulkes [**5**] which suggests that one might usefully consider a certain irreducible representation of the wreath product $\mathbf{S}_m \wr \mathbf{S}_l$. It is shown in Section 3 that the character of $\mathbf{S}_{lm}$ induced from the character afforded by this representation has corresponding S-function $\{\mu\} \otimes \{\lambda\}$. The connection between the plethysm of S-functions and wreath products of symmetric groups has been pointed out by several authors (e.g. [**9**, § 7; **10** p. 135]) but no proofs seem to be available. Finally, in Section 4 there is a brief summary of one of the possible methods of reducing $\{\mu\} \otimes \{\lambda\}$ into its irreducible components.

2. The S-function $\{\mu\} \otimes \{\lambda\}$. Let ϕ be any class function defined on $\mathbf{S}_l$, then the Schur characteristic function, or S-function, corresponding to ϕ is, by definition, the symmetric function,

$$(1) \quad \Phi = \frac{1}{l!} \sum_{\rho \vdash l} r_\rho \phi_\rho S_\rho$$

where ϕ_ρ is the value of ϕ on the conjugacy class C_ρ of $\mathbf{S}_l$,

$$r_\rho = |C_\rho|$$
$$S_\rho = S_1^{a_1} S_2^{a_2} \ldots \quad \text{for } \rho = (1^{a_1} 2^{a_2} \ldots) \vdash l$$

and with S_k (k a positive integer) the kth power sum, $t_1^k + t_2^k + \ldots$, in the variables $t_1, t_2, \ldots$.

Now if $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_l) \vdash l$ then $\{\lambda\}$ may be defined as the bialternant symmetric function

$$\{\lambda\} = \frac{\sum \pm t_1^{\lambda_1 + l - 1} t_2^{\lambda_2 + l - 2} \ldots t_l^{\lambda_l}}{\sum \pm t_1^{l-1} t_2^{l-2} \ldots t_l^{0}} = \frac{|t^{\lambda_j + l - j}|}{|t_i^{l-j}|} = \frac{\Delta_\lambda}{\Delta},$$

say, where the (i, j) entry of the lth order determinant, Δ_λ, is $t_i^{\lambda_j}$ and where the sums are taken over all permutations of the suffixes of the t's, with $+$ or $-$ sign according as the permutation is even or odd. It follows from the famous

Received June 25, 1975 and in revised form, October 6, 1975.

https://doi.org/10.4153/CJM-1976-044-7 Published online by Cambridge University Press

Frobenius formula for the irreducible characters $\chi^{(\lambda)}$ of $\mathbf{S}_l$, namely,

$$S_\rho \Delta = \sum_{\lambda \vdash l} \chi_\rho^{(\lambda)} \Delta_\lambda$$

that $\{\lambda\}$ is the S-function corresponding to $\chi^{(\lambda)}$ [**2**, §§ 5.2, 6.3]. Thus,

$$(2) \quad \{\lambda\} = \frac{1}{l!} \sum_{\rho \vdash l} r_\rho \chi_\rho^{(\lambda)} S_\rho$$

Let the irreducible rational homogeneous representations of weight l of $\mathbf{GL}_n$ be $\sigma^{(\lambda)}$, $\lambda \vdash l$ into not more than n parts, then the character afforded by $\sigma^{(\lambda)}$ is $\{\lambda\}$, where the variables are now the eigenvalues $t_1, \ldots, t_n$ of $\xi \in \mathbf{GL}_n$. Thus, $\{1\} \equiv S_1 = t_1 + \ldots + t_n = \operatorname{tr} \xi$ and $S_q = t_1^q + \ldots + t_n^q = \operatorname{tr}(\xi^q)$.

If the degree of the $\sigma^{(\mu)}$, $\mu \vdash m$, representation of $\mathbf{GL}_n$ is N then for $\xi \in \mathbf{GL}_n$, the entries of $\sigma^{(\mu)}(\xi) \in \mathbf{GL}_N$ are homogeneous polynomials of degree m in the entries of ξ and $\sigma^{(\mu)}(\mathbf{GL}_n) = \mathbf{R}$, a subgroup of $\mathbf{GL}_N$. Next, consider the $\sigma^{(\lambda)}$ representation of $\mathbf{GL}_N$; the entries of $\sigma^{(\lambda)}(\eta)$, $\eta \in \mathbf{GL}_N$, are homogeneous polynomials of degree l in those of η and

$$\{\lambda\} = \frac{1}{l!} \sum_{\rho \vdash l} r_\rho \chi_\rho^{(\lambda)} Z_\rho$$

where Z_ρ is defined in terms of the eigenvalues $t_1^*, \ldots, t_N^*$ of $\eta \in \mathbf{GL}_N$ in exactly the same way as S_ρ in terms of $t_1, \ldots, t_n$. Now the restriction of $\sigma^{(\lambda)}$ to $\mathbf{R}$, $\sigma^{(\lambda)}|_{\mathbf{R}}$, is a representation of $\mathbf{R}$ and hence of $\mathbf{GL}_n$. In this representation $\xi \in \mathbf{GL}_n$ is mapped onto the matrix $\sigma^{(\lambda)}(\sigma^{(\mu)}(\xi))$, that is, the matrix $\sigma^{(\lambda)}(\eta)$ with $\eta \in \mathbf{R}$ and of form $\sigma^{(\mu)}(\xi)$. The entries of $\sigma^{(\lambda)}(\sigma^{(\mu)}(\xi))$ are, of course, homogeneous polynomials of degree lm in the entries of ξ. The character afforded by $\sigma^{(\lambda)}|_{\mathbf{R}}$ is written $\{\mu\} \otimes \{\lambda\}$. Thus,

$$(3) \quad \{\mu\} \otimes \{\lambda\} = \frac{1}{l!} \sum_{\rho \vdash l} r_\rho \chi_\rho^{(\lambda)} Z_\rho,$$

a symmetric function of weight lm, constructed from the given S-functions $\{\mu\}$, $\{\lambda\}$ of weights m and l respectively.

We require Z_ρ in terms of the eigenvalues t_i, $i = 1, \ldots, n$, of $\xi \in \mathbf{GL}_n$, rather than as a function of the t_j^*, $j = 1, \ldots, N$. Now,

$$\{\mu\} = \frac{1}{m!} \sum_{\rho \vdash m} r_\rho \chi_\rho^{(\mu)} S_\rho$$

where, $S_\rho = S_1^{b_1} S_2^{b_2} \ldots$ for $\rho = (1^{b_1} 2^{b_2} \ldots) \vdash m$ and $r_\rho = |C_\rho|$ of $\mathbf{S}_m$. That is,

$$\operatorname{tr} \sigma^{(\mu)}(\xi) = \frac{1}{m!} \sum_{\rho \vdash m} r_\rho \chi_\rho^{(\mu)} (\operatorname{tr} \xi)^{b_1} (\operatorname{tr} \xi^2)^{b_2} \ldots \quad \text{for all } \xi \in \mathbf{GL}_n.$$

Replace ξ with ξ^q, hence S_k with S_{qk} then, since

$$Z_q = t_1^{*q} + \ldots + t_N^{*q} = \operatorname{tr} \eta^q = \operatorname{tr}(\sigma^{(\mu)}(\xi))^q = \operatorname{tr}(\sigma^{(\mu)}(\xi^q))$$

we have,

$$Z_q = \frac{1}{m!} \sum_{\rho \vdash m} r_\rho \chi_\rho^{(\mu)} (\text{tr } \xi^q)^{b_1} (\text{tr } \xi^{2q})^{b_2} \dots$$

Thus, if we write $\{\mu\}^{(q)}$ for Z_q

(4) $$\{\mu\}^{(q)} = \frac{1}{m!} \sum_{\rho \vdash m} r_\rho \chi_\rho^{(\mu)} S_{q\rho}$$

where $q\rho = (q^{b_1}(2q)^{b_2} \dots) \vdash qm$.
Finally, since $Z_\rho = Z_1^{a_1} Z_2^{a_2} \dots = \{\mu\}(\{\mu\}^{(2)})^{a_2} \dots = \{\mu\}_\rho$, say, then (3) becomes

(5) $$\{\mu\} \otimes \{\lambda\} = \frac{1}{l!} \sum_{\rho \vdash l} r_\rho \chi_\rho^{(\lambda)} \{\mu\}_\rho,$$

a form, used by H. O. Foulkes [**5**, § 5], which invites comparison with a certain irreducible representation of the wreath product $\mathbf{S}_m \wr \mathbf{S}_l$.

3. The character of $\mathbf{S}_{lm}$ corresponding to the S-function $\{\mu\} \otimes \{\lambda\}$. Following the definitions and notation of A. Kerber [**8**, pp. 24–25], we let $(y; x) \equiv (y_1, \dots, y_l; x)$ be a general element of the wreath product $\mathbf{S}_m \wr \mathbf{S}_l$, where y maps the set $\Omega = \{1, \dots, l\}$ into $\mathbf{S}_m$ and $x \in \mathbf{S}_l$. The basis group of $\mathbf{S}_m \wr \mathbf{S}_l$, $\mathbf{S}_m{}^*$, with elements of form $(y; 1_{\mathbf{S}_l})$, $y : \Omega \to \mathbf{S}_m$, is the direct product $\mathbf{S}_{m_1} \times \dots \times \mathbf{S}_{m_l}$ of l copies of $\mathbf{S}_m$. The complement $\mathbf{S}_l{}'$ of $\mathbf{S}_m{}^*$ is isomorphic to $\mathbf{S}_l$ and its elements are of the form $(e; x)$, $x \in \mathbf{S}_l$, e the identity of $\mathbf{S}_m{}^*$. Thus, the factor group $(\mathbf{S}_m \wr \mathbf{S}_l)/\mathbf{S}_m{}^* = \mathbf{S}_l{}'$ and if x is a given element of $\mathbf{S}_l$ then the set of elements $\{(y; x)\}$ constitute a coset of $\mathbf{S}_m{}^*$ in $\mathbf{S}_m \wr \mathbf{S}_l$.

From the definition of $\mathbf{S}_m \wr \mathbf{S}_l$ it is easily seen that the cycle decomposition of elements $(y; x)$, $x \in C_\rho$ of $\mathbf{S}_l$ and $\rho = (1^{a_1} 2^{a_2} \dots s^{a_s} \dots)$ a partition of l into r parts, is of the form

(6) $$\nu_\rho = \nu_1 \oplus \dots \oplus \nu_{a_1} \oplus 2\nu_{a_1+1} \oplus \dots \oplus 2\nu_{a_1+a_2} \oplus \dots$$

a direct sum of r partitions, where the first a_1 terms are of form ν_i, the next a_2 of form $2\,\nu_i, \dots$, the next a_s of form $s\nu_i, \dots$ with $s\nu_i = (s^{b_1}(2s)^{b_2} \dots) \vdash sm$ for $\nu_i = (1^{b_1} 2^{b_2} \dots) \vdash m$.

Now, Kerber shows [**8**, §§ 5, 6] that certain irreducible representations of $\mathbf{S}_m \wr \mathbf{S}_l$ are of the form $(\mu; \lambda) \equiv (\tilde{\sigma} \otimes \rho^{(\lambda)\prime})$ where, $\rho^{(\lambda)\prime}$ is the (irreducible) representation of $\mathbf{S}_m \wr \mathbf{S}_l$ derived from the irreducible representation $\rho^{(\lambda)}$ of the factor group $\mathbf{S}_l{}'$, σ is the (irreducible) Kronecker product representation $\rho^{(\mu)} \otimes \dots \otimes \rho^{(\mu)}$ (l factors) of $\mathbf{S}_m{}^*$, with $\rho^{(\mu)}$ the irreducible representation (of degree n_μ) of $\mathbf{S}_m$, and $\tilde{\sigma}$ is the (irreducible) representation, derived from σ by permuting the columns of the matrices $\sigma((y; 1_{\mathbf{S}_l}))$. The representation $\tilde{\sigma}$ is given by $\tilde{\sigma}((y; x))$ with $(i_1, \dots, i_l; j_1, \dots, j_l)$ entry equal to

$$\rho_{i_1 j_{x^{-1}(1)}}^{(\mu)}(y_1) \rho_{i_2 j_{x^{-1}(2)}}^{(\mu)}(y_2) \dots \rho_{i_l j_{x^{-1}(l)}}^{(\mu)}(y_l), \ (1 \leqq i_k, j_k \leqq n_\mu).$$

Therefore the $(i_1, \dots, i_l; i_1, \dots, i_l)$ entry of $\tilde{\sigma}((y; x))$, if $x \in C_\rho$ with

$\rho = (1^{a_1}2^{a_2}\ldots)$, is equal to

$$\rho_{i_1 i_1}{}^{(\mu)}(y_1)\ldots\rho_{i_{a_1} i_{a_1}}{}^{(\mu)}(y_{a_1})\cdot\rho_{i_{a_1+1} i_{a_1+2}}{}^{(\mu)}(y_{a_1+1})\rho_{i_{a_1+2} i_{a_1+1}}{}^{(\mu)}(y_{a_1+2})\ldots$$

which includes, corresponding to an s-cycle (say the first) in the kth to $(k+s-1)$th parts of ρ, the product of factors

$$\rho_{i_k i_{k+1}}{}^{(\mu)}(y_k)\cdot\rho_{i_{k+1} i_{k+2}}{}^{(\mu)}(y_{k+1})\ldots\rho_{i_{k+s-1} i_k}{}^{(\mu)}(y_{k+s-1}).$$

Hence,

$$\operatorname{tr}\tilde{\sigma}((y;x)) = \operatorname{tr}\rho^{(\mu)}(y_1)\ldots\operatorname{tr}\rho^{(\mu)}(y_{a_1})\operatorname{tr}\rho^{(\mu)}(y_{a_1+1}y_{a_1+2})\ldots$$
$$\operatorname{tr}\rho^{(\mu)}(y_k y_{k+1}\ldots y_{k+s-1}) = \chi_{\nu_1}{}^{(\mu)}\chi_{\nu_2}{}^{(\mu)}\ldots\chi_{\nu_r}{}^{(\mu)}\ (r\text{ factors})$$

where $y_1 \in C_{\nu_1}, \ldots, y_{a_1} \in C_{\nu_{a_1}}, y_{a_1+1}y_{a_1+2} \in C_{\nu_{a_1+1}}, \ldots, y_k y_{k+1}\ldots y_{k+s-1} \in C_{\nu_{a_1+\ldots+a_{s-1}+1}}, \ldots$ of $\mathbf{S}_m$ and here all the y_i in $\rho^{(\mu)}(y_i)$ are considered as elements of a single $\mathbf{S}_m$, since the factors of σ are all equivalent and so may be made equal. Thus, the value of the character afforded by the irreducible representation $(\mu;\lambda) \equiv (\tilde{\sigma}\otimes\rho^{(\lambda')})$ of $\mathbf{S}_m \wr \mathbf{S}_l$ on $(y;x)$ with $x \in C_\rho$, $\rho = (1^{a_1}2^{a_2}\ldots) \vdash l$ into r parts is equal to $\prod_{i=1}^r \chi_{\nu_i}{}^{(\mu)}\,\chi^{(\lambda)}$.

Finally, we show that the S-function corresponding to the character ϕ, say, afforded by the induced representation, $(\mu;\lambda)\uparrow\mathbf{S}_{lm}$, is $\{\mu\}\otimes\{\lambda\}$. Now, the element $(y;x)\in\mathbf{S}_m\wr\mathbf{S}_l$ with $x \in C_\rho$, from (6), corresponds to a partition of lm of the form ν_ρ and therefore belongs to the conjugacy class C_{ν_ρ} of $\mathbf{S}_{lm}$. Thus [**6**, Theorem 16.7.2], the value of the character ϕ on $(y;x)$ is

$$\phi((y;x)) \equiv \phi_{\nu_\rho} = \frac{(lm)!}{(m!)^l l!\, r_{\nu_\rho}} \sum_{(y;x)\in C_{\nu_\rho}\cap\mathbf{S}_m\wr\mathbf{S}_l} \chi_\rho{}^{(\lambda)}\left(\prod_{i=1}^r \chi_{\nu_i}{}^{(\mu)}\right)$$

the sum being over all $(y;x)$ of $\mathbf{S}_m\wr\mathbf{S}_l$ of the form ν_ρ. But the number of cosets of $\mathbf{S}_m{}^*$ in $\mathbf{S}_m\wr\mathbf{S}_l$ corresponding to a particular $\rho\vdash l$ is r_ρ, the number of ways of building ν_ρ in each of these cosets is

$$\sum_{\bigoplus_i \nu_i=\nu_\rho}\left(\prod_{i=1}^r r_{\nu_i}\right)$$

and every one of these occurs $(m!)^{l-r}$ times in each such coset. Hence,

$$\phi_{\nu_\rho} = \frac{(lm)!}{l!\,r_{\nu_\rho}} \sum_{\rho\vdash l\text{ into }r\text{ parts}} \frac{r_\rho}{(m!)^r}\chi_\rho{}^{(\lambda)}\left(\sum_{\bigoplus_i \nu_i=\nu_\rho}\prod_{i=1}^r r_{\nu_i}\chi_{\nu_i}{}^{(\mu)}\right),$$

for given ν_ρ. Now, the corresponding S-function,

$$\Phi = \frac{1}{(lm)!}\sum_{\zeta\vdash lm} r_\zeta\phi_\zeta S_\zeta = \frac{1}{(lm)!}\sum_{\nu_\rho,\rho\vdash l} r_{\nu_\rho}\phi_{\nu_\rho}S_{\nu_\rho}$$

since $\phi_\zeta = 0$ unless $\zeta = \nu_\rho$ for some $\rho\vdash 1$. Thus,

$$\Phi = \frac{1}{l!}\sum_{\rho\vdash l} r_\rho\chi_\rho{}^{(\lambda)}\left[\frac{1}{(m!)^r}\sum_{\nu_\rho=\bigoplus_i \nu_i\vdash rm}\left(\prod_{i=1}^r r_{\nu_i}\chi_{\nu_i}{}^{(\mu)}\right)S_{\nu_\rho}\right],$$

now summed over all ν_ρ.

https://doi.org/10.4153/CJM-1976-044-7 Published online by Cambridge University Press

But $S_{\nu_\rho} = S_{\nu_1} \ldots S_{\nu_{a_1}} S_{2\nu_{a_1+1}} \ldots S_{2\nu_{a_1+a_2}} \ldots$ (r factors) for $\rho = (1^{a_1}2^{a_2}\ldots) \vdash l$ into r parts. Therefore,

$$\Phi = \frac{1}{l!} \sum_{\rho \vdash l} r_\rho \chi_\rho^{(\lambda)} \left[\left(\frac{1}{m!} \sum_{\nu_1} r_{\nu_1} \chi_{\nu_1}^{(\mu)} S_{\nu_1} \right) \ldots \left(\frac{1}{m!} \sum_{\nu_{a_1}} r_{\nu_{a_1}} \chi_{\nu_{a_1}}^{(\mu)} S_{\nu_{a_1}} \right) \right.$$
$$\left. \times \left(\frac{1}{m!} \sum_{\nu_{a_1+1}} r_{\nu_{a_1+1}} \chi_{\nu_{a_1+1}}^{(\mu)} S_{2\nu_{a_1+1}} \right) \ldots \right] = \frac{1}{l!} \sum_{\rho \vdash l} r_\rho \chi_\rho^{(\lambda)} \{\mu\}_\rho,$$

as required.

4. The reduction of $\{\mu\} \otimes \{\lambda\}$. We conclude with a brief reference to the problem of reducing the S-function $\{\mu\} \otimes \{\lambda\}$ to a sum of S-functions, that is, to the decomposition of the character ϕ of $\mathbf{S}_{lm}$ with corresponding S-function $\{\mu\} \otimes \{\lambda\}$ to a sum of irreducible characters of $\mathbf{S}_{lm}$. Many methods (e.g. [**1**], also [**4**, p. 166] for more references) have been devised for this reduction; we consider $\{\mu\} \otimes \{\lambda\}$ in the form (5).

The $\chi_\rho^{(\lambda)}$ may be found from the character table of $\mathbf{S}_l$, or by applying the Littlewood-Richardson recurrence rule [**2**, § 5.3, Theorem II] and the order of C_ρ is

$$r_\rho = \frac{l!}{1^{a_1}\, a_1!\, 2^{a_2}\, a_2! \ldots}$$

for $\rho = (1^{a_1}2^{a_2}\ldots) \vdash l$. The differential operator method of H. O. Foulkes [**5**] gives a simple determinantal procedure for the coefficient of $\{\nu\}$, $\nu \vdash lm$, in $\{\mu\}_\rho$; it is also very useful in conjunction with other methods which may determine the coefficients of S-functions $\{\nu\}$ in $\{\mu\} \otimes \{\lambda\} = \sum_{\nu \vdash lm} c_{\mu\lambda\nu}\{\nu\}$ corresponding to certain - but not all - forms of the partition ν of lm.

If, however, each $\{\mu\}^{(q)}$ in $\{\mu\}_\rho$ were expressed as a sum of S-functions, the problem would then reduce to that of the ordinary multiplication of S-functions [**2**, § 6.3, Theorem V]. We have,

$$\{\mu\}^{(q)} = \frac{1}{m!} \sum_{\rho \vdash m} r_\rho \chi_\rho^{(\mu)} S_{q\rho}$$

from (4). But

$$S_\rho = \sum_{\mu \vdash m} \chi_\rho^{(\mu)} \{\mu\}$$

for each $\rho \vdash m$. In particular for $q\rho \vdash qm$,

$$S_{q\rho} = \sum_{\sigma \vdash qm} \chi_{q\rho}^{(\sigma)} \{\sigma\}$$

Thus,

$$\{\mu\}^{(q)} = \frac{1}{m!} \sum_{\substack{\rho \vdash m \\ \sigma \vdash qm}} r_\rho \chi_\rho^{(\mu)} \chi_{q\rho}^{(\sigma)} \{\sigma\}$$

https://doi.org/10.4153/CJM-1976-044-7 Published online by Cambridge University Press

where, $\chi^{(\mu)}$, $\chi^{(\sigma)}$ are irreducible characters of $\mathbf{S}_m$ and $\mathbf{S}_{qm}$ respectively. Hence $\{\mu\} \otimes \{\lambda\}$ becomes a sum of products of S-functions, the coefficients in which are integral multiples of products of characters of $\mathbf{S}_l$, $\mathbf{S}_m$ and $\mathbf{S}_{qm}$ (q a divisor of lm). Now we require the values of $\chi^{(\sigma)}$, $\sigma \vdash qm$, on classes of form $C_{q\rho}$, $\rho \vdash m$, only. But D. E. Littlewood [**2**, § 8.1] has shown that $\chi_{q\rho}{}^{(\sigma)}$ may be expressed in terms of the irreducible characters $\chi^{(\mu)}$ of $\mathbf{S}_m$; we therefore require the irreducible characters of only $\mathbf{S}_l$ and $\mathbf{S}_m$.

References

1. D. E. Littlewood, *Invariant theory, tensors and group characters*, Trans. Royal Phil. Soc. *239* (1944), 305–365.

2. ——— *The theory of group characters* (Oxford, 1940).

3. G. de B. Robinson, *On the disjoint product of irreducible representations of the symmetric group*, Can. J. Math. *1*, (1949), 166–175.

4. ——— *Representation theory of the symmetric group* (Edinburgh, 1961).

5. H. O. Foulkes, *Differential operators associated with S-functions*, J. London Math. Soc. *24* (1949), 136–143.

6. M. Hall, *The theory of groups* (Macmillan, 1959).

7. C. W. Curtis and I. Reiner, *Representation theory of finite groups and associative algebras* (Wiley, 1962).

8. A. Kerber, *Representations of permutation groups*, Lecture notes in Math. vol. 240 (Springer-Verlag, 1971).

9. R. C. Read, *The use of S-functions in combinatorial analysis*, Can. J. Math. *20* (1968), 808–841.

10. D. Knutson, *Lecture notes on λ-rings and the theory of the symmetric group*, Lecture notes in Math. vol. 308 (Springer-Verlag, 1973)

Royal Holloway College (University of London),
Englefield Green, Surrey

https://doi.org/10.4153/CJM-1976-044-7 Published online by Cambridge University Press