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GEOMETRIC AND TOPOLOGICAL SHAPE ANALYSIS: INVESTIGATING AND SUMMARISING THE SHAPE OF DATA

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Tools from geometry and topology can be applied in a wide variety of settings. In particular, they are adept at exploring and summarising the shape of data. By considering the shape of objects, we can answer questions related to object reconstruction and classification problems. These problems often face difficulty distinguishing signals from noise, which can be overcome using ideas from persistent homology and computational geometry.

This thesis can be split into two parts, with Chapters 1 and 2 forming the first, and Chapter 3 the second. We first look at an application of geometry and topology to learning the abstract structure of embedded stratified spaces and then consider an object classification problem relating to human mesenchymal stem cells. Chapter 1 presents an algorithm for learning the abstract structure underlying an embedded graph $|G| \subset \mathbb{R}^n$ given an ε-sample P of $|G|$, as well as a method for modelling the embedding (see [2]). Learning the abstract structure is extended to embedded 2-complexes $|X| \subset \mathbb{R}^n$ from ε-samples P in Chapter 2. Chapter 3 is a change in topic and pace, exploring the use of persistent homology and persistence diagrams to identify abnormal growth patterns in cultured human mesenchymal stem cells.

While these topics seem disconnected, they can be considered two sides of the same dice. In both settings, we are using tools from geometry and topology to understand the *shape* of the data at hand. In Chapters 1 and 2 the end goal is to understand the abstract structure underlying a point cloud, while in Chapter 3 we use shape to identify subpopulations.

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People familiar with topological data analysis, computational topology and computational geometry will notice that Chapters 1 and 2 are centred on results that could be considered purely problems in computational geometry. This is in part because when working with data sets, topological data analysis often becomes computational geometry. In particular, this occurs when we care about the extrinsic structure of the data within the ambient space, rather than just the intrinsic topology and geometry. Exploring the computational geometry aspects of these problems comes with benefits as well as costs. While we are able to develop an algorithm that is more efficient in terms of time complexity, often the associated proofs about its correctness are, to quote a supervisor, 'gross'. This grossness stems from the expressions which bound distances between various objects, as they do not simplify to a 'nice, neat expression'. These expressions could be approximated to become 'nicer', without affecting the proofs, but then the assumptions placed upon our underlying spaces become stricter.

Chapter 3 explores the use of geometric and topological tools to understand growth patterns in human mesenchymal stem cells (see [1]). In particular, we use morphological features of the cells to identify subgroups within experimental conditions. By identifying the subpopulations, we are able to clean the data when investigating the impact of environmental conditions on multi-potent cells in the future, as well as potentially having an unbiased method for identifying unipotent cells. The cells were purchased, cultured and imaged by Dr Florian Rehfeldt (Bayreuth University).

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