

Linear lattices, by H. Nakano. Wayne State University Press, Detroit, 1966. vi + 157 pages.

This paperback is the collection of the author's work on the Spectral Theory, including specially those author's results which appeared during World War II and after. This monograph is actually the reproduction of his paper titled "Modularized Semiordered Linear Spaces" that was not published before in any mathematics journal and which has been available to mathematicians in the form of manuscripts - copies of which were located at the University of Tokyo and the Institute of Advanced Study.

The monograph (covering 157 pages) has six chapters with the usual front and back material: Preface, Preface to the Second Edition; Chapter I, Semi-ordered Linear Spaces; Chapter II, Spectral Theory; Chapter III, Totally Continuous Spaces; Chapter IV, Continuous Linear Functionals; Chapter V, Reflexive Spaces; Chapter VI, Normed Spaces; Notes, Bibliography and Index.

The author has extended the spectral theory from Hilbert spaces to more general linear spaces, which include some familiar Banach spaces of analysis. Attempts to extend the Spectral Theory in full to more general spaces have not been totally successful. The elegance for which the Spectral Theory on Hilbert spaces is noted remains understandably illusive for general spaces. The author considers modularized semiordered linear spaces and proves a version of the Spectral Theory. One knows that a partial order  $\geq$  for the space of linear operators  $T$  on a Hilbert space  $H$  is provided by its scalar product function  $\langle \cdot, \cdot \rangle$ , viz:  $T \geq 0$  if and only if  $\langle Tx, x \rangle \geq 0$  for all  $x \in H$ . The classical Spectral Theorem was first proved for operators (i.e.  $T \geq 0$ ) on a Hilbert space and then extended to more general operators. For modularized semiordered linear spaces one can define positive operators and the author proves a similar Spectral Theorem for such spaces.

The first chapter is devoted to elementary notions concerning partially ordered linear spaces. In the second, the author introduces the Spectral Theory via projectors, proper values; relative spectra, and integral representations. Chapter 3 deals with totally and super-universally continuous semiordered linear spaces and certain convergence notions of sequences in terms of order. This leads to the set of order-bounded linear functionals and its relation with the set of all linear continuous functionals on a linear topological space in Chapter IV. In Chapter V, reflexive spaces are considered. Finally, Chapter VI deals with normed semiordered linear spaces.

Since the notions used in this monograph were not standardized at the time of writing this monograph, the personal usage of the notions makes the reading of this book rather painful and sometimes exasperating. For example, what we now call "complete linear lattices" are labelled "continuous semiordered linear spaces" by the author. What is generally now called "an extremely disconnected space", the author calls "a universal space"; the space of maximal ideals is called "a proper space", and so on. The excessive over-use of words such as "continuous", "proper", "universal", in different senses for spaces as well as for functions, tells upon the assimilation of the material by a reader. The usage of the notations " $\Sigma$ " for " $\cup$ " reminds one of the forgotten notions put away solemnly in the attics of knowledge.

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