Ground-based gravimetry for measuring small spatial-scale mass changes on glaciers

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ABSTRACT. Gravity change on a glacier surface is a composite of several effects (e.g. melting and accumulation of snow and ice, redistribution of mass with depth by refreezing of meltwater and height and thickness changes of the snow and ice layers). Models and equations necessary to estimate the measured gravity change due to different effects are presented, and the propagation of observational errors is evaluated. The paper presents experiences with ground-based gravity measurements carried out on Hardangerjøkulen, Norway, in spring and autumn 2007. It was found that the vertical gradient of gravity contributes most to the uncertainty in the determined mass change. With present instrumentation, gravity can be measured with the required accuracy to determine the mass loss to $\sim$10% of the loss determined by conventional mass-balance measurements. Improvements in field procedures to achieve the required accuracy for measuring the mass/density changes directly, combining gravity measurements and GNSS (Global Navigation Satellite Systems), are discussed.

INTRODUCTION

The gravity field at the Earth's surface is determined by its own internal distribution of mass, by the mass distribution at or near the surface and by other nearby masses such as the sun, moon and planets. The field is not static, but varies continuously with time because of the movement of these masses. The principal sources of gravity variation are tides, hydrology, land uplift/subsidence, ocean tide loading, atmospheric loading and changes in the Earth's cryosphere.

It has recently been shown that mass changes of ice sheets with sufficiently large spatial coverage can be measured from satellites. The Gravity Recovery and Climate Experiment (GRACE) has measured gravity fields at latitudes above 60°, providing monthly estimates of mass changes with accuracies of 10 mm in equivalent water thickness when averaged over discs of radius 600–700 km and larger (Velicogna and Wahr, 2006b). GRACE data have revealed a mass loss of $248 \pm 36\,km^3\,a^{-1}$ in the period 2002–06 of the Greenland ice sheet (Velicogna and Wahr, 2006b), which is equivalent to $0.5 \pm 0.1\,mm\,a^{-1}$ increase in global sea level. GRACE data also show that in the period 2002–05 the volume of the Antarctic ice sheet decreased by $152 \pm 80\,km^3\,a^{-1}$, equivalent to a $0.4 \pm 0.2\,mm\,a^{-1}$ contribution to global sea level (Velicogna and Wahr, 2006b). As discussed by Velicogna and Wahr and by Murray (2006), uncertainties in these results stem from tidal and non-tidal changes in the oceans, changes in the atmosphere and from rebound of the Earth's mantle since the last ice age, where the latter contributes most to the uncertainty.

Ground-based gravimeters operate at the surface of the Earth and are integrating sensors that observe the vertical component of Earth's gravitational acceleration. Today, ground-based gravimeters observe gravity with a repeatability of some microgals ($1\,\mu gal = 10^{-8}\,m\,s^{-2}$) and are widely used for geoid determination and observation of gravitational effects of geophysical phenomena such as postglacial rebound (Larson and Dam, 2000), solid Earth tides (Baker and Bos, 2003), ocean tide loading ((Dittfeld and others, 1997; Lysaker and others, 2007)), density anomalies in the Earth's lithosphere and mass changes (e.g. due to mining).

It is important to be aware of the conceptual differences between space-borne and ground-based gravimeters. First of all, the spatial resolution is different. The short distance to the attracting masses means that ground-based gravimeters have a much finer spatial resolution compared to space-borne gravimeters such as GRACE, which observes the gravitational signal from a ground footprint with a radius of 500–700 km. This means that space-borne gravitational sensors are only suitable to observe mass changes from large glacier systems such as Antarctica and Greenland. For ground-based gravimetry, the glacier's size has no restrictions. In addition, ground-based gravimeters are coupled to the Earth's surface and are sensitive to both a change in gravitational potential and a height change of the instrument. In contrast, space-borne gravimeters sense only changes in the Earth's gravitational potential.

Ground-based gravimetry is not a well-established method used to study glaciers, but some work exists. Klingelé and Kahle (1977) used ground-based gravimetry as a technique for determining the thickness of the ice cap of Gornergletscher, Switzerland. In Fukuda and others (2003, 2007), ground-based gravimetry is described as a method to detect the ice-sheet thinning rate of the Shirase Glacier drainage basin in Antarctica with a view to calibration and validation of GRACE data. They focus mainly on fieldwork procedures and present preliminary results.

Ground-based gravimetric measurements are sensitive to changes in height (there is a strong gravitational gradient at the Earth's surface) and to nearby changes in mass. They can therefore be used as an alternative method to observe height changes on glaciers, but also to observe mass changes which are not connected to height changes such as changes in internal density due to the effect of refreezing of meltwater.

As already mentioned, Fukuda and others (2003) proposed ground-based gravimetry as a method to calibrate and validate satellite data from GRACE. Ground-based gravimetry can also be useful for validation of data from GOCE (Gravity

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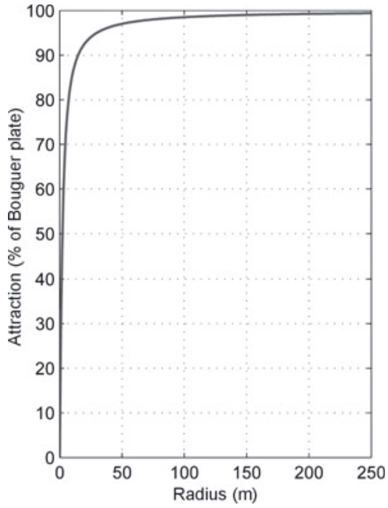

Fig. 1. The relative error in using the Bouguer plate approximation as a function of cylinder radius. The major part of the attraction from a Bouguer plate is formed by masses close to the observation point. A snow layer of density $600\,\mathrm{kg\,m^{-3}}$ and depth 3 m was assumed.

Field and Steady-State Ocean Circulation Explorer). GOCE is expected to be launched during 2008. The main goal of the mission is to observe gravity anomalies with an accuracy of 1 mgal at a spatial resolution of 70 km or better (Seeber, 2003). However, at present, GOCE is only expected to provide one or two gravity fields. In order to derive mass changes, a gravity satellite mission is required.

The aim of this study is to evaluate the use of gravimetric measurements for determining the local mass balance of glaciers. We describe a simple gravitational model of a glacier and its surface mass balance and review the propagation of measurement errors through this model. To gain practical experience in carrying out the required gravimetric and field measurements in order to implement this model, a field experiment was carried out on Hardangerjøkulen, Norway, during the spring and autumn of 2007. The results are compared to GNSS (global navigation satellite systems) measurements, carried out as part of the study, and to annual mass-balance measurements made by the Norwegian Water Resources and Energy Directorate (NVE). The results are further discussed in relation to the estimated error, the practical application of the method and to future methodological and accuracy requirements for the use of gravimetric measurements to determine glacier mass balance. It is found that the gravitational measurements are dominated by the vertical gradient of gravity, rather than the actual change in mass of the glacier.

GRAVIMETRIC METHODS AND GLACIER MODEL

The problem of determining mass changes from gravity observations is, in geodesy, a classical inverse problem, i.e. there exists an infinite number of mass distributions which make the same gravitational signal. The crux of the problem is that the gravitational attraction from a mass is determined by both the size of the mass and the distance to the mass, i.e. a small mass close to the observer has the same gravitational attraction as a more distant larger mass. This makes it difficult to use gravity observations to distinguish between different sources of gravity change on a glacier. In order to distinguish the different gravitational sources, additional observations are necessary. In the model presented below, snow-probe measurements are included to isolate the mass change of the snowpack. The need for solving the inverse problem for gravity observations is only relevant for ground-based gravimetry close to the gravitational masses. From space, all glacial mass changes are of virtually the same distance.

In order to observe mass changes with a gravimeter, the gravity in a study point should be observed at least twice. It is the observed gravity *difference* which is related to mass changes, not the individual gravity observations.

Basic gravitational modelling

A layer of ice or snow could be modelled as a circular cylinder with defined thickness, radius and density. The attraction of such a layer in a study point is found in Hofmann-Wellenhof and Moritz (2005), for example. Simplified formulae are achieved by modelling the layer as a Bouguer plate which is a circular cylinder with infinite radius. For an observation point located on or above the Bouguer plate, the gravitational attraction is calculated by:

$$A_{\mathrm{B}} = 2\pi\,\mathrm{G}\,\rho\,b, \qquad (1)$$

where $\mathrm{G} = 6.6742 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$ is the gravitational constant, ρ is the cylinder density and b is the cylinder thickness.

The attraction from masses located on the sides of the observation point attenuates quite quickly, which is why the Bouguer plate approximation can be readily applied. Figure 1 illustrates this: for example, the contribution from the mass within 100 m comprises 98% of the attraction from a Bouguer plate with thickness 3 m. This illustrates two aspects. Firstly, the major part of observed gravity change can focus on local mass changes. Secondly, the use of a Bouguer plate and Equation (1) in preference to a cylinder with a defined radius is a good approximation. A numerical example also illustrates this. The attraction, at an observational point localized 4 m above the ground, of a cylinder with thickness 1 m, radius 200 m and density $900\,\mathrm{kg\,m^{-3}}$ is 37.08 μgal. If we replace the cylinder with a Bouguer plate, Equation (1) yields an attraction of 37.74 μgal. The error using approximate formulae is 1.8% of the total gravitational effect. In the present analysis, we model the glacier with Bouguer plates and benefit from the simplified formulae.

Gravity changes on a glacier include the effect of any height change the instrument may experience. If the vertical gradient of gravity ($\partial g/\partial H$) and the height change (Δh) are known quantities, the gravitational effect ($\Delta g_{\mathrm{free\ air}}$) is calculated:

$$\Delta g_{\mathrm{free\ air}} = \frac{\partial g}{\partial H}\Delta h. \qquad (2)$$

The gradient should be observed locally at the study site by successive measurements of gravity over a representative vertical distance of known length. Typical values are about $300\,\mu\mathrm{gal\,m^{-1}}$. In the present analysis, we use a sign convention that implies positive height changes corresponding to height reductions. Equation (2) implies that gravity increases towards the Earth's centre.

Modelling gravity change on a glacier

We model the glacier as a homogeneous layer of ice with density ρ_{ice} covered by a homogeneous top layer of snow with density ρ_{snow}. The model presented allows both layers to change between two periods of observation. By combining gravity observations and snow-probe measurements, the presented model resolves both the total height change of the glacier surface and the isolated gravitational effect of a change in the thickness of the snow and ice.

Mass changes can be determined in two ways: by multiplying the calculated height changes with the corresponding densities or by the actual change in the gravitational mass (when an alternative measurement of the height change (e.g. from GNSS) is available).

The gravity change between two separate periods is the combined gravitational effect of accumulation/ablation of snow (Δg_{snow}) and ice (Δg_{ice}) and the height change of the observation point ($\Delta g_{\text{free air}}$):

$$\Delta g = g_{t2} - g_{t1} = \Delta g_{\text{free air}} + \Delta g_{\text{snow}} + \Delta g_{\text{ice}}. \quad (3)$$

The effect of ($\Delta g_{\text{free air}}$) in Equation (3) is estimated by Equation (2). Gravity changes due to melted or accumulated snow and ice are determined by the latter two terms, defined:

$$\begin{aligned}\Delta g_{\text{snow}} &= -2\pi\, G\, \rho_{\text{snow}}(h_1 - h_2)\\ &= -2\pi\, G\, \rho_{\text{snow}} \Delta h_{\text{snow}} \quad (4)\\ \Delta g_{\text{ice}} &= -2\pi\, G\, \rho_{\text{ice}} \Delta h_{\text{ice}}\\ &= -2\pi\, G\, \rho_{\text{ice}}(\Delta h - \Delta h_{\text{snow}}), \quad (5)\end{aligned}$$

where Δh is the total height change of the observation point and h_1 and h_2 are the depth of the snow layers found by snow-probe measurements at the first and the last observation, respectively. They are combined into Δh_{snow} which is the depth change of the snow layer. The change of the ice thickness is represented by Δh_{ice}. The minus signs are added because of the chosen sign convention, i.e. a positive height change implies ablation and a corresponding gravitational reduction. When all terms are combined, Equation (3) can be used to solve the total height change of the glacier's surface:

$$\Delta h = \frac{\Delta g - 2\pi\, G\, \Delta h_{\text{snow}}(\rho_{\text{ice}} - \rho_{\text{snow}})}{\frac{\partial g}{\partial h} - 2\pi\, G\, \rho_{\text{ice}}}. \quad (6)$$

The change in the ice thickness Δh_{ice} is found by subtracting the change in the snow layer's depth from the total height change using

$$\Delta h_{\text{ice}} = \Delta h - \Delta h_{\text{snow}}. \quad (7)$$

The estimated height changes can then be used to determine mass changes per square metre if the snow and ice densities are known. Equations (8–10) give the mass changes due to accumulation/ablation of snow, ice and the total mass change, respectively:

$$\Delta m_{\text{snow}} = \Delta h_{\text{snow}} \times \rho_{\text{snow}} \quad (8)$$

$$\Delta m_{\text{ice}} = \Delta h_{\text{ice}} \times \rho_{\text{ice}} \quad (9)$$

$$\Delta m = \Delta h_{\text{snow}} \times \rho_{\text{snow}} + \Delta h_{\text{ice}} \times \rho_{\text{ice}}. \quad (10)$$

The error propagation through the model is presented in Equations (A1–A6) in the Appendix.

FIELDWORK AND RESULTS

Experience in operating a ground-based gravimeter on a glacier was gathered during two field campaigns carried out at Hardangerjøkulen, Norway. Hardangerjøkulen is a mountainous glacier covering an area of 73 km^2. It is situated at 60°32′ N, 7°22′ E and reaches an elevation of 1860 m a.s.l. (Fig. 2). The winter mass balance was observed on 3 May 2007 and the summer mass balance on 3 October 2007. Both field campaigns involved gravimetry and snow-probe measurements at a collocated study site in the glacier's accumulation area. GNSS observations were also carried out for the vertical positioning of the measurements, in order to validate and assess the results. Measurements were carried out at only one study point.

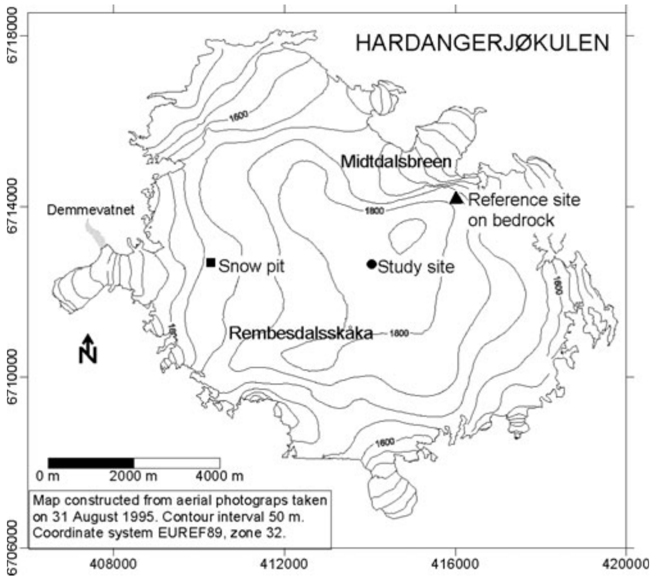

Fig. 2. Map of Hardangerjøkulen with the study site, the reference site and the snow pit for density measurements.

Gravity observations

The gravity observations were collected with a LaCoste & Romberg Model G relative gravimeter (instrument G-761; Fig. 3). A relative gravimeter observes spatial or temporal gravity differences with respect to an arbitrary reference. In order to find absolute gravity differences from May to October, it was important to use the same reference for both campaigns. With the same reference, absolute gravity change between different periods was found by differencing the spatial gravity differences.

The gravity change from May to October at the glacier was determined as follows. At each site, the gravimeter was allowed to settle for some minutes before gravity was observed and recorded for about 15 min with a sampling interval of 10 s. The observations were corrected for Earth tides using the software provided by the instrument manufacturer. Final gravity differences of –21 547 μgal and –20 386 μgal between the reference and the study site were observed in May and October, respectively. The difference between these two spatial gravity differences gives an absolute gravity change of 1161 μgal at the study site from May to October. For the May observations, only one gravity measurement was obtained on the glacier; for October, two measurements were made. A hand-held GPS (global positioning system) receiver was used in October to relocate the horizontal position of the May measurements on the glacier.

Most spring gravimeters have a sensitivity of 1 μgal and an accuracy in the field which depends on the size of the gravity difference, weather conditions, field procedures and the stability of the platform where the observations are made. Rymer (1989) has investigated the effect of noise, field procedures and instrumental effects on LaCoste & Romberg gravimeters. Rymer quantifies the total error for one single gravity difference measurement to be at maximum ~33 μgal and at minimum ~10 μgal. The two gravity measurements on the glacier for October differ by 22 μgal. This is within the range

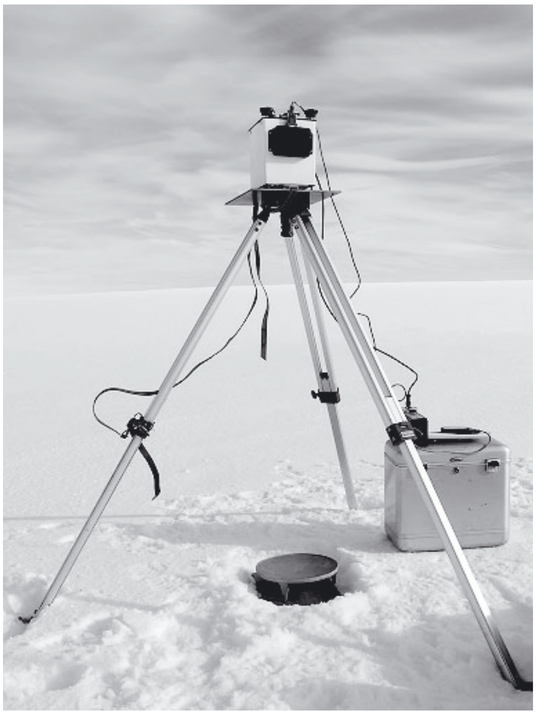

Fig. 3. The gravimeter in the upper position of a gravity gradient survey. For gravity gradient observations, the instrument is moved between upper and lower positions on the baseplate at the glacier's surface. For mass-change observations, the gravimeter is placed on its baseplate at the glacier's surface.

suggested by Rymer, and we adopt it as the uncertainty of the gravity measurements of the present analysis.

The uncertainty of the absolute gravity change (dg) is found by calculating the square root of the sum of each individual gravity difference's squared error (dg_1 and dg_2):

$$\mathrm{d}g = \sqrt{\mathrm{d}g_1^2 + \mathrm{d}g_2^2}. \quad (11)$$

For our gravity measurements, this yields an absolute gravity difference with an error of 31 μgal. This error substituted into Equation (A1) results in an uncertainty of 0.11 m for the total/ice height change.

The vertical gradient of gravity

The vertical gradient of gravity was observed at the glacier study site in October only. The gravity change over a vertical distance of 1.20 m was observed five times successively and the gradient was found by dividing the gravity change by the distance. Mean vertical gravity gradient was calculated to be 312 ± 8 μgal m^{-1} and was, in the model calculations, assumed to be constant from May to October. The tripod was placed directly on snow. We experienced no problems with the tripod sinking into the snow during the measurements.

The uncertainty of the end result due to gravity gradient measurement errors depends on the total height change of the glacier surface. Equation (A2) shows that the uncertainty may grow considerably for large height changes. It is therefore important to use accurate gradients. The gradient was observed with a standard deviation of 8 μgal under calm weather conditions. Substitution into Equation (A2) yields an uncertainty of 0.12 m.

Probing and density observations

Traditional probe observations were carried out. The depth of the snowpack in May was found to be 6.65 m. In October, the snowpack was probed to be 4.05 m of which 0.75 m was formed by fresh snow.

The accuracy of the snow-probe measurements is difficult to quantify. Each single reading is accurate to a few centimetres. However, the depth of the snowpack may vary considerably over short distances. We estimate the error to be 0.20 m here. From Equation (A3) this results in a total height-change uncertainty of only 0.01 m. This is a very small error when determining the change in height and indicates how insensitive the gravitational model and measurements are to uncertainties in the actual change of mass. The probing error propagates directly to the estimated ice-thickness change, i.e. according to Equation (A4), a probing error of 0.20 m results in an ice-thickness uncertainty of 0.19 m.

The mean density through the snowpack was measured in a snow pit marked on the map in Figure 2. It was determined to be 540 kg m^{-3} and 580 kg m^{-3} in May and October, respectively. All density measurements were provided by NVE (personal communication from H. Elvehøy, 2007). Equation (6) includes only one term that represents the snow layer's density. The effects of depth-dependent snow density or a density change between the observation periods are not included. The mean density of the two epochs was therefore calculated to be 560 kg m^{-3} and used in Equation (6).

Equation (A5) shows that the density error theoretically leads to a small height-change uncertainty: for example, a density error of 50 kg m^{-3} results in a height change uncertainty of 0.02 m for a height change of ~3.00 m. The density of ice is easier to quantify. Usually a density of 917 kg m^{-3} is adopted and is assumed to be constant. In this analysis, we adopt an ice density error of 20 kg m^{-3} which results in a height change uncertainty of 0.002 m.

In principle, it is possible to include more layers with different densities in the model. It is necessary to extract the thickness of each layer (e.g. from a snow-core sample). For the measurements on Hardangerjøkulen, including more layers changes the end result by only 5–10 cm and is consequently omitted from the calculations. Note that it is of vital importance to use accurate densities when mass change is calculated from the height changes. In such calculations, the uncertainty of the mass change is proportional to the height change.

GNSS observations

GPS and GLONASS (Global Navigation Satellite System) data were collected with a Topcon Legacy GNSS receiver. The raw GNSS observations were processed with TerraPos which represents a state-of-the-art solution to Precise Point Positioning (PPP) (Kjørsvik and others, 2008). Precise satellite ephemerides, satellite clock corrections and Earth orientation parameters were downloaded from the Center for Orbit Determination in Europe (CODE). From the GNSS observations, the glacier surface was found to be lowered by 3.00 m during the summer. Unfortunately, because of instrumental failure, the duration of the GNSS campaign in October was limited to less than 3 hours. A considerable degradation of the accuracy of the height estimate therefore follows. Following TerraTec (2007), the accuracy is expected to be about 0.20 m. Final gravity, GNSS, probes and density observations are tabulated in Table 1.

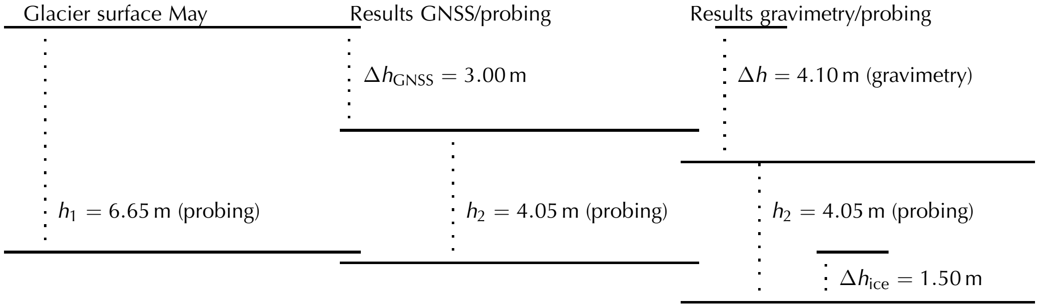

Fig. 4. Schematic drawing of the results and the modelled upper part of the glacier. h_1 and h_2 are the depths of the snow layer measured in May and October, respectively. Δh_{GNSS} and Δh are the height change of the glacier's surface observed with GNSS and estimated from gravity observations. Δh_{ice} is the change in the ice layer's thickness.

Calculated height change from May to October using the gravimetric measurements

Gravity observations, snow-probing measurements and the mean of the density observations were inserted into Equation (6) to calculate the total height change at the observation point. The total height change was calculated to 4.10 m and the ice height change was found to be 1.50 m by Equation (7). This differs significantly from the GNSS height-change measurements of 3.0 m. From Equation (10), the corresponding total mass change was calculated to be 2803 kg m^{-2} or 2.80 m in equivalent water thickness. The results are schematically illustrated in Figure 4.

It is fair to assume that all estimated uncertainties in Table 2 are independent. Hence, the total uncertainty ($\mathrm{d}h_{\mathrm{total}}$) is the calculated square root of the sum of each individual squared uncertainty:

$$\mathrm{d}h_{\mathrm{total}} = \sqrt{\mathrm{d}h_g^2 + \mathrm{d}h_\gamma^2 + vh_{h_{\mathrm{snow}}}^2 + \mathrm{d}h_{\rho_{\mathrm{snow}}}^2 + \mathrm{d}h_{\rho_{\mathrm{ice}}}^2}. \quad (12)$$

For this analysis, the total uncertainty in the estimated change of height and ice thickness is found to be 0.22 m and 0.29 m, respectively. The contribution from each individual error is summarized in Table 2.

The total uncertainty depends on the size of the height change. In particular, for small height changes, the estimated ice-thickness change is more sensitive to observational errors compared to the total height change. This is explained by the direct propagation of probing errors to the estimated ice-thickness change.

DISCUSSION

The uncertainty of the glacier's height change was estimated to be 0.22 m. Compared to the height change calculated from conventional methods such as probing and GNSS, the uncertainty forms ~10% of the total height change. This illustrates the potential accuracy of the presented methodology with present instrumentation. However, the total height change estimated from gravity observations differs by 1.10 m in comparison to the GNSS observations. Based on the error estimates, the deviations are larger than expected. The deviation between the gravimetrically determined and the observed GNSS height change indicates the existence of gross errors in either the measurements or the assumptions used in the calculation. The most likely error of this type comes from the methodology employed in the fieldwork.

Gravity was observed only once in May and is consequently not possible to verify. Gross errors may occur in gravity observations. During the May fieldwork, repeated gravity observations between markers localized on bedrock revealed unprovoked jumps of more than 200 µgal. At present, we are not able to explain the origin of these jumps. This indicates the importance of carrying out fieldwork in such a way that gross errors are detected when operating a spring gravimeter. Unfortunately, this general principle was not fulfilled during the May field campaign. Repetition of measurements, at both the glacier and the reference site, is a recommended methodology for future observational campaigns that will provide both verifiable measurements and uncertainty estimates.

Table 1. Summary of the observations

	May	October	Change
Ellipsoidal GNSS height (m)	1874.39	1871.38	3.00
Gravity difference (µgal)	–21 547	–20 397	1150
		–20 375	1172
Depth of snow (m)	6.65	4.05	2.60
Density of snow (kg m^{-3})	540	580	40
Gradient (µgal m^{-1})	–	312	–
Fresh snow (m)	0	0.75	0.75

Table 2. Measurement errors and their impact on the total change in height

Source of error	Size of error	dh m	dh_{ice} m
Gravity change	31 µgal	0.11	0.11
Gradient	8 µgal	0.12	0.12
Probing	0.20 m	0.01	0.19
Density snow	50 kg	0.02	0.02
Density ice	20 kg	0.002	0.002
Total error		0.22	0.29

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It is important to ensure that gravity observations from different periods are carried out at the same horizontal position before mass changes are derived. This comes from the sensitivity of ground-based gravimeters to small-scale gravity anomalies. The typical size of such anomalies on glaciers is currently a topic of further investigation. An alternative solution to this problem is found in Fukuda and others (2007) who suggest observing gravity and GNSS positions in a grid covering an area of 40 × 40 m. The grid observations are then used to predict, through interpolation, the gravity value of a virtual reference point. Accurate ties between the GNSS antenna phase centre and the gravity observation platform should also be established with care.

A number of assumptions are made in the calculations that are also open to errors. The two most likely sources of error are the constancy of both the gravitational field at the reference point and the vertical gravitational gradient, which were assumed to experience no changes from May to August. It should be ensured that the reference point is connected to absolute gravity measurements.

In the present analysis, Equation (6) was solved for the total height change. The estimated height changes were compared to GNSS height observations. It is possible to include the GNSS observations in the model and to solve for other parameters (e.g. the mean density of the snow layer). However, it is necessary to include snow-probing measurements to distinguish between depth changes of the snowpack and a change in the thickness of the ice. This approach was not tested in the present analysis due to the dominance of the vertical gravitational gradient in Equation (3). With a 3 m vertical displacement of the glacier surface, the contribution to the change of gravity due to the mass change is ~7% of the total, or the equivalent of an estimated height change of ~22 cm. This is close to the current estimated uncertainty, which would therefore have to be reduced significantly for any meaningful conclusions to be made concerning changes in density or pointing out the effect of superimposed ice. This reduction could, to a large degree, be obtained if the three-dimensional (3-D) spatial location of the gravimetric measurements were coincident. Given the uncertainty in the gravitational gradient determined in this study, the gravimeter would need to be positioned to within a few centimetres of the previous measurement.

Traditional mass-balance observations assume that all meltwater from the snow layer is completely removed from the glacier. The calculations made here do not take into account the possibility of superimposed ice. Superimposed ice is formed by refreezing of meltwater when the meltwater encounters a cold surface such as ice or firn (Wright and others, 2005). This results in mass losses that are less than the apparent observed mass loss when using stakes. The model presented in the present paper includes a change in the ice volume height. Such height changes could be the combined result of firn transformed into ice, glacial dynamics and superimposed ice. The problem of using a gravimeter to quantify one of these components is an inverse problem impossible to solve by gravimetry alone. On the other hand, prediction of gravity change due to superimposed ice is a direct problem. In this way, ground-based gravimetry is a potential method to validate independent observations or models of superimposed ice and ice dynamics. This kind of analysis requires high-quality gravity observations and careful modelling and observation of the accumulation and ablation of snow. For most glaciers, the change in gravity due to superimposed ice only represents a few microgals from one year to another. Ground-based gravimetry, therefore, is likely to be most useful for studying long-term mass changes due to the effect of superimposed ice.

Finally, further improvement in the uncertainty of the measurements is also obtainable by utilizing more precise gravimeters, such as a Scintrex CG-5 (Scintrex Ltd., 2006) or an A10 absolute gravimeter. These new generation gravimeters allow observations with a field repeatability of 5–10 μgal, improving the accuracy of the measurements by a factor of five.

CONCLUSION

We have discussed ground-based gravimetry as an alternative method to observe mass changes on glaciers. A model was established and error propagation was assessed.

It was shown that the presented methodology is capable of resolving the height change within ~10% of the total height change observed by conventional methods. However, practical tests at Hardangerjøkulen demonstrated that carrying out gravitational measurements on a glacier is a challenging task. The height change of the glacier surface was measured to be 4.10 m which differs by 1.10 m from the GNSS measurements. This difference is larger than expected and indicates the existence of gross errors in the gravity observations that are most likely the result of procedural and methodological errors in the measurements. Improved accuracy in the results is expected to be obtained through improved fieldwork procedures. Repeated observations to identify gross errors in the measurements are most important.

The current application using gravimetric measurements provides, in essence, an alternative method for determining the change in height of the surface of the glacier. This does not realize the full potential of the methodology and is chiefly due to the dominance of the vertical gravitational gradient in the gravitational budget. By measuring the change in gravity at coincident heights, the effect of the vertical gradient can be eliminated and a more accurate assessment of the change in mass can be derived.

Although there are a number of uncertainties presented in the current study, experience gained through this work will lead to a significant improvement in the methodology used for future applications of gravimetric measurements. The recommended improvements in methodology should, in combination with improved instrumentation, make it possible to realize the potential of ground-based gravimetry for observing mass changes on glaciers.

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REFERENCES

Baker, T.F. and M.S. Bos. 2003. Validating Earth and ocean tide models using tidal gravity measurements. *Geophys. J. Int.*, **152**(2), 468–485.

Dittfeld, H.J. *and 7 others*. 1997. Tidal gravity measurements within the MOTIVE project. *Bulletin d'Information des Marées Terrestres*, **127**, 9843–9850.

Fukuda, Y., K. Shibuya, K. Doi and S. Aoki. 2003. A challenge to the detection of regional to local scale ice sheet movements in Antarctica by the combination of in-situ gravity measurements and gravity satellite data. *In* Tziavos, I.N., *ed. Gravity and Geoid 2002: 3rd Meeting of the International Gravity and Geoid Commission*. Thessaloniki, Ziti Editions.

Fukuda, Y., Y. Hiraoka and K. Doi. 2007. An experiment of precise gravity measurements on ice sheet, Antarctica. *In* Tregoning, P. and C. Rizos, *eds. Dynamic planet: monitoring and understanding a dynamic planet with geodetic and oceanographic tools*. Berlin, etc., Springer-Verlag. (International Association of Geodesy Symposia 130.)

Hofmann-Wellenhof, B. and H. Moritz. 2005. *Physical geodesy*. Vienna, etc., Springer-Verlag.

Kjørsvik, N.S., O. Øvstedal and J.G.O. Gjevestad. In press. Kinematic precise point positioning during marginal satellite availability. *In* Ubertini, L., P. Maniciola, S. Casadei and S. Grimaldi, *comps*. In press. *Earth: Our Changing Planet. Proceedings of IUGG XXIV General Assembly, Perugia, Italy 2007*. Perugia, Umbria Scientific Meeting Association.

Klingelé, E. and H.-G. Kahle. 1977. Gravity profiling as a technique for determining the thickness of glacier ice. *Pure Appl. Geophys.*, **115**(4), 989–998.

Larson, K.M. and T. van Dam. 2000. Measuring postglacial rebound with GPS and absolute gravity. *Geophys. Res. Lett.*, **27**(23), 3925–3928.

Lysaker, D.I., K. Breili and B.R. Pettersen. 2007. The gravitational effect of ocean tide loading at high latitude coastal stations in Norway. *J. Geod.*, **82**(9), 569–583.

Murray, T. 2006. Climate change: Greenland's ice on the scales. *Nature*, **443**(7109), 277–278.

Rymer, H. 1989. A contribution to precision microgravity data analysis using Lacoste and Romberg gravity meters. *Geophys. J. Int.*, **97**(2), 311–322.

Scintrex Ltd. 2006. *CG-5 Autograv system operation manual*. Concord, Ont., Scintrex.

Seeber, G. 2003. *Satellite geodesy. Second edition*. Berlin, Walter de Gruyter.

TerraTec AS 2007. *TerraPos user's manual. Version 1.2*. Oslo, TerraTec AS.

Velicogna, I. and J. Wahr. 2006a. Acceleration of Greenland ice mass loss in spring 2004. *Nature*, **443**(7109), 329–331.

Velicogna, I. and J. Wahr. 2006a. Measurements of time-variable gravity show mass loss in Antarctica. *Science*, **311**(5768), 1754–1756.

Wright, A., J. Wadham, M. Siegert, A. Luckman and J. Kohler. 2005. Modelling the impact of superimposed ice on the mass balance of an Arctic glacier under scenarios of future climate change. *Ann. Glaciol.*, **42**, 277–283.

APPENDIX: ERROR PROPAGATION

Formulae for calculating the *height-change* uncertainties are listed below. They are all found by studying the absolute value of Equations (6) and (7) differentiated with respect to the variable to be studied. All errors, except the snow-probe measurement error, propagate with equal size to the estimated total height change and the ice-thickness change. Hence, two equations are presented for the snow-probe measurement error. Equation (A3) should be used to calculate the uncertainty of the total height change and Equation (A4) (marked with an asterisk) for the uncertainty of the ice-thickness change.

The *mass-change* uncertainties due to gravity, gradient and probing measurement errors are found by multiplying the height uncertainties with the densities. When it comes to the mass-change uncertainty due to density measurement errors, the procedure is different. The uncertainties are found by multiplying the density error with the height change. Here, the second-order effect resulting from height-change uncertainties due to density errors is neglected.

In Equations (A1–A6), $\mathrm{d}g$, $\mathrm{d}\gamma$, $\mathrm{d}h_{\mathrm{snow}}$, $\mathrm{d}\rho_{\mathrm{snow}}$ and $\mathrm{d}\rho_{\mathrm{ice}}$ are gravity error, gradient error, probing error, snow density error and ice density error, respectively. The corresponding uncertainties in the observed height change are denoted by $\mathrm{d}h_x$ where x represents the error.

1. Gravity measurement error $\mathrm{d}g$:

$$\mathrm{d}h_{\mathrm{g}} = \frac{\mathrm{d}g}{\frac{\partial g}{\partial h} - 2\pi\,\mathrm{G}\,\rho_{\mathrm{ice}}}. \tag{A1}$$

2. Gravity gradient measurement error $\mathrm{d}\gamma$:

$$\mathrm{d}h_{\gamma} = \left[\frac{\Delta g - 2\pi\,\mathrm{G}\,\Delta h_{\mathrm{snow}}(\rho_{\mathrm{ice}} - \rho_{\mathrm{snow}})}{(\frac{\partial g}{\partial h} - 2\pi\,\mathrm{G}\,\rho_{\mathrm{ice}})^2}\right]\mathrm{d}\gamma. \tag{A2}$$

3. Snow-probe measurement error $\mathrm{d}h_{\mathrm{snow}}$ and the corresponding uncertainty on the estimated total height change $\mathrm{d}h_{h_{\mathrm{snow}}}$:

$$\mathrm{d}h_{h_{\mathrm{snow}}} = \left[-\frac{2\pi\,\mathrm{G}(\rho_{\mathrm{ice}} - \rho_{\mathrm{snow}})}{\frac{\partial g}{\partial h} - 2\,\pi\,\mathrm{G}\,\rho_{\mathrm{ice}}}\right]\mathrm{d}h_{\mathrm{snow}}. \tag{A3}$$

4. Snow-probe measurement error $\mathrm{d}h_{\mathrm{snow}}$ and the corresponding uncertainty on the estimated ice-height change $\mathrm{d}h^*_{h_{\mathrm{snow}}}$:

$$\mathrm{d}h^*_{h_{\mathrm{snow}}} = \mathrm{d}h_{h_{\mathrm{snow}}} + \mathrm{d}h_{\mathrm{snow}}. \tag{A4}$$

5. Snow density measurement error $\mathrm{d}\rho_{\mathrm{snow}}$:

$$\mathrm{d}h_{\rho_{\mathrm{snow}}} = \left[\frac{2\pi\,\mathrm{G}\Delta h_{\mathrm{snow}}}{\frac{\partial g}{\partial h} - 2\pi\,\mathrm{G}\,\rho_{\mathrm{ice}}}\right]\mathrm{d}\rho_{\mathrm{snow}}. \tag{A5}$$

6. Ice density measurement error:

$$\mathrm{d}h_{\rho_{\mathrm{ice}}} = \left[\frac{2\,\pi\,\mathrm{G}\left(\Delta g - \Delta h_{\mathrm{snow}} \times \frac{\partial g}{\partial h}\right)}{\left(\frac{\partial g}{\partial h} - 2\pi\,\mathrm{G}\,\rho_{\mathrm{ice}}\right)^2}\right]\mathrm{d}\rho_{\mathrm{ice}}. \tag{A6}$$