Can. J. Math., Vol. XXXIV, No. 6, 1982, pp. 1396–1397

ERRATA: ON THE CONJUGACY PROBLEM IN THE BRAID GROUP

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The classical braid group B_n, the fundamental word Δ, positive words in B_n, the diagram $D(P)$ and cyclic diagram $C(P)$ of a positive word, the power, tail, summit power and summit tail of an element $\beta \in B_n$, initial and final segments of Δ, are all as defined in [**1**].

In Theorem 2.7 of [**1**] the author claimed to have proved that the summit power of an element $\beta \in B_n$ of even power exceeds its power only if the diagram $D(P)$ of the tail P of β contains a word of the form FQI, where F is a final segment and I is an initial segment of Δ. An analogous statement is given for words of odd power.

There is, however, a gap in the proof of Theorem 2.7 and also in the statement of Lemma 2.7.5, which preceded the proof. A weaker version of each hold:

LEMMA 2.7.5 (corrected). *Suppose that $\beta = \Delta^m P$ is in standard form. Let Z be a positive word in B_n. Let $Z \doteq IY$, where I is an initial route of maximal length in Z. Suppose that $Z^{-1}\beta Z$ has power m. Then $I^{-1}\beta I$ has power $\geqq m$.*

THEOREM 2.7 (corrected). *Let $\beta = \Delta^m P$ be in standard form in B_n. Then β has summit power $> m$ if and only if its cyclic diagram contains Δ.*

A counterexample to Lemma 2.7.5 as originally stated is given by

$$\beta = \sigma_3\sigma_2\sigma_1\sigma_2\sigma_1\sigma_2\sigma_1, \; Z = \sigma_3\sigma_2\sigma_1\sigma_2\sigma_1, \; I = \sigma_3\sigma_2\sigma_1\sigma_2,$$

all in B_4. Both β and $Z^{-1}\beta Z$ have power 0, but $I^{-1}\beta I$ has power 1.

A gap in the proof of Theorem 2.7 occurs on page 91, at lines 5^- to 1^-. A counterexample to the assertion given there is

$$P = \sigma_1\sigma_2\sigma_2\sigma_3\sigma_1\sigma_2\sigma_2 \in B_4.$$

Its diagram $D(P)$ contains two words: P and $P' = \sigma_1\sigma_2\sigma_2\sigma_1\sigma_3\sigma_2\sigma_2$. Neither is of the form FQI, where F is a final segment and I a related initial segment. However, its cyclic diagram $C(P)$ contains the sequence of related words

$$P = P_0 = (\sigma_1\sigma_2\sigma_2\sigma_3)(\sigma_1\sigma_2\sigma_2),$$
$$P_1 = (\sigma_1\sigma_2\sigma_2)(\sigma_1\sigma_2\sigma_2\sigma_3) = (\sigma_1\sigma_2)(\sigma_2\sigma_1\sigma_2)(\sigma_2\sigma_3),$$
$$P_2 = (\sigma_1\sigma_2)(\sigma_1\sigma_2\sigma_1)(\sigma_2\sigma_3) = (\sigma_1\sigma_2\sigma_1\sigma_2)(\sigma_1\sigma_2\sigma_3)$$

and

$$P_3 = (\sigma_1\sigma_2\sigma_3)(\sigma_1\sigma_2\sigma_1\sigma_2) = (\sigma_1\sigma_2\sigma_3\sigma_1\sigma_2\sigma_1)(\sigma_2) = \Delta\sigma_2.$$

The proofs of the corrected versions of the lemma and the theorem are given by the alleged proofs of the stronger statements claimed in [**1**].

The author is grateful to Professor G. S. Makanin and to Professor J. McCool for pointing out the errors, and to the Canadian Journal for publishing this errata.

References

1. J. S. Birman, *Braids, links and mapping class groups*, Annals of Math Studies *82* (Princeton University Press, 1974).

2. F. A. Garside, *The braid groups and other groups*, Quarterly J. Math. Oxford *20*, No. 78, 235–254.

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