VIBRATIONAL INSTABILITIES AND PULSATIONAL PROPERTIES OF COOL WHITE DWARFS

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Abstract

Attention is focused on those aspects of the theory that may be relevant in understanding the nature of ZZ Ceti-type variable white dwarfs. Recent calculations show that the opacity mechanism can drive a large variety of oscillation modes, including the ones that fit observed periods. An estimate of nonlinear effects shows that resonant mode coupling plays a dominant role in determining the finite amplitude behaviour of oscillations and is also probably responsible for rapid amplitude changes observed in these variables.

1. Introduction

Pulsation properties of the white dwarfs will be reviewed with the special emphasis on those which are or may be relevant in understanding ZZ Ceti variables. Consequently, I will have in mind rather cool stars with effective temperatures ranging from 10,000 to 15,000 K.

The nature of the variability of these stars is still a matter of debate. In keeping with the title of this review, only those interpretations that postulate excitation of oscillations of some sort will be discussed. The possibilities for using the observed periods as a diagnostic tool for white dwarf structure will also be explored.

Pulsational properties of white dwarfs are quite different from those of the Cepheids. Thus, although the tendency exists to consider ZZ Ceti variables as an extension of the usual pulsation instability strip toward the low luminosity range, a simple extrapolation of results obtained for Cepheids may be quite misleading. Numerical models of finite amplitude development of the oscillations are probably needed to explain phenomena that are seen in ZZ Ceti stars. These, however, are not and probably will not be soon available. Among the problems are lack of sufficiently detailed knowledge of the equilibrium structure of

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white dwarf envelopes, appropriate opacity data and, primarily, difficulties in dealing with nonspherically symmetric motion of finite amplitude.

In this situation it appears safer to rely on those conclusions that may be reached without reference to very specific models. Thus, this paper will address types of white dwarf oscillations and driving mechanisms mostly utilizing asymptotic formulae and qualitative arguments. These will be followed by presenting for the sake of illustration results of a numerical analysis of the linear nonadiabatic properties of a white dwarf model. Nonlinear effects leading to pulsation amplitude limitation will then be discussed. This again will be largely on a qualitative basis.

2. White dwarf modes of oscillation

By assuming that the star is sufficiently slowly rotating and that the oscillations are of low amplitude, the angular dependence of perturbations may be separated in terms of spherical harmonics,

$$Y_{l,j}\ (\theta,\phi) = P_{l,j}\ (\cos\theta)\ \exp(\pm ij\phi)/I_{l,j} \qquad (1)$$

where $P_{l,j}$ is a Legendre function and $I_{l,j}$ is the normalization factor. Pulsation amplitude for nonradial oscillations ($l > 0$) are therefore understood as r.m.s. values calculated over the surface of a sphere at a given distance, r, from the star center. It is clear that only modes associated with low order l harmonics can give rise to observable luminosity variations. With increasing l, cancellation causes a drastic reduction of the net luminosity amplitude. The factor is ~ 10^{-2} for l = 5 and ~ 10^{-3} for l = 10.

The temporal dependence of perturbations is given by exp ($i\omega t$), where, in the case of adiabatic oscillations, ω is real. Nonadiabatic effects give rise to a complex part in ω; the growth rate is given by $-Im(\omega)$. The equations for the r-dependent amplitudes, supplemented by the appropriate boundary conditions, constitute a fourth order real eigenfrequency problem in the adiabatic case and a sixth order complex eigenfrequency problem in the nonadiabatic case.

For realistic stellar models, one needs to employ numerical methods to obtain solutions to these problems. However, considerable insight into the properties of various modes can be achieved by means of a qualitative discussion of propagation properties of various waves in the stellar interior. Examples of discussions of this type can be found in the following papers: Scuflaire (1974), Unno (1975), Osaki (1975) and Dziembowski (1975). The waves that give rise to stellar oscillations propagate in some portion of the stellar interior which is called a propagation zone; these zones are surrounded by areas called evanescent zones where propagation is impossible. The propagation zone is recognized by oscillatory dependence of the pulsation amplitude on r while the evanescent zones may be identified by its monotonic decrease

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or increase. The location of these zones depends on the values of ω and l.

By arguing, for instance, that the eigenfunctions corresponding to various frequencies are mutually orthogonal, it may be seen that almost all of the energy of oscillation is confined to the propagation zone. The situation is quite analogous to that considered in quantum mechanics, with propagation zones corresponding to allowed regions and evanescent zones corresponding to prohibited regions for a given energy level.

2.1. Gravity modes

It was suggested by Warner and Robinson (1972) that gravity modes are responsible for the ZZ Ceti phenomenon and this still seems the most plausible interpretation. These modes are trapped internal gravity waves and their principal restoring force comes from buoyancy. Consecutive modes are denoted as $g_1, g_2, \ldots g_n$, according to the number of nodes in the eigenfunction describing the radial displacement.

Asymptotically for short radial wavelengths, the following dispersion relation is valid locally

$$\kappa^2 = l(l+1)\,(N^2/\omega^2 - 1) \qquad (2)$$

where κ is the radial component of the wave vector multiplied by r and N is the Brunt-Vaisala frequency. We have

$$N^2 = \frac{g}{r}\left(\frac{1}{\Gamma}\frac{d\ln P}{d\ln r} - \frac{d\ln \rho}{d\ln r}\right) \qquad (3)$$

where g is the local gravitational acceleration, P is pressure, ρ is density and Γ is the adiabatic exponent. Although approximation of the actual eigenfunctions by trigonometric functions is valid only for high order gravity modes, the dispersion relation is always useful in determining the approximate location of the propagation zones. It is seen from Eq. (2) that propagation is possible only if the oscillation frequency is less than the local Brunt-Vaisala frequency. But another condition which must be satisfied states that the estimated wavelength must be shorter than the distance at which the mean properties of the star change significantly. That distance can be estimated as $|dr/d\ln P|$, leading to the following two conditions which must be satisfied in the propagation zone

$$\omega < N \qquad \omega \lesssim lNP/g\rho\ . \qquad (4)$$

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The second condition is a rough approximation and for its derivation, $N^2/\omega^2 >> 1$ was assumed.

Due to electron degeneracy, $\frac{d\ln P}{d\ln \rho} \approx \Gamma$ throughout most of white dwarf interior. Thus N is small and the low order gravity modes do not penetrate the interior. This is in sharp contrast to other types of stars where most of the gravity modes tend to be confined to the deep interior. This property makes gravity modes in the white dwarfs particularly easy to excite by any mechanism operating in the outer layers.

The fact that g-modes tend to be trapped in the outer layers means that their frequencies should be sensitive to the envelope structure and particularly to the distribution of major elements. The gradient of the mean molecular weight, μ, directed inward increases N. Consequently ω for a given mode order will increase. This effect is quite dramatic, as illustrated in Table 1 where periods of gravity modes are compared for models with a deep H + He-rich envelope (about 10^{-8} $M_\odot$) and a shallow envelope (about 10^{-10} $M_\odot$). In the former case, the μ-gradient occurs in the propagating zone, while in the latter it appears in the evanescent zone.

Table 1. Periods of g-modes corresponding to l = 1 in three white dwarf models with M = 0.6 $M_\odot$.

Mode	Deep H + He with $\log T_e = 4.04$	Deep H + He with $\log T_e = 4.08$	Shallow H + He with $\log T_e = 4.08$
g1	126.5	124.0	317.2
g2	242.4	232.8	354.6
g3	259.5	245.6	410.6
g4	325.8	316.6	472.9

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It is clear that at present the theory cannot provide the relation between period and luminosity for white dwarfs. The behaviour of the pulsation amplitude for gravity modes inside white dwarfs is somewhat similar to that of radial pulsation in a giant. There is a large inward amplitude drop in the outer evanescent zone which increases with larger ratios of $l(l + 1)/ \omega^2$. The outer edge of the propagation zone is determined by the second of the conditions given in Eq. (4). Finally, the location of the inner evanescent zone, where the ultimate decline in amplitude takes place, may be determined by either of those conditions, depending on value of l. Pulsation energy is almost entirely confined to the propagation zone; however, most driving or damping originates within the outer evanescent zone.

This picture may be complicated by the existence of local μ-gradient related maxima of N, which may give rise to local trapping of the modes. If occurring close to the surface, the decrease in pulsation energy may promote mode driving. The onset of crystallization in the white dwarf core may introduce further complexities by producing new restoring force for gravity-modes related to shear (Hansen and Van Horn, 1979). However, this phenomenon should not affect modes trapped in the outer layers but rather would give rise to a new set of modes trapped in the deep interior.

Because their frequencies are low, gravity modes are expected to be strongly affected by rotation even if it produces a negligible distortion of the star from spherical shape. The lowest order effect which is due to the Coriolis force removes frequency degeneracy with respect to j leading to splitting of each frequency into $2l + 1$ component distributed at equal distances. These distances are proportional to the angular velocity of rotation and depend somewhat on the stellar model. Equidistant patterns such as this have been seen in the periodograms of several ZZ Ceti stars. Estimates of rotational velocity of the stars were drawn from this which turned out to be in the range from a few to about 30 km s^{-1} (McGraw and Robinson, 1975; McGraw, 1977; Chlebowski, 1978). In Fig. 1, which is taken from Chlebowski's paper, the relationship between increasing rotational velocity and the behaviour of gravity mode frequencies is shown. Terms up to the second order in the square of rotational frequency have been included. From this, it may be seen that, starting from a rotational velocity of about 30 km s^{-1}, there is an appreciable deviation from equidistant distribution of the frequencies.

The figure shows also that interpretation of equidistant patterns in the periodograms in terms of rotational splitting is self-consistent. The estimate of the equatorial velocity of rotation made in this way is so far the only attempt to diagnose white dwarf properties with the use of periodograms.

2.2. Acoustic modes

These modes have periods shorter by at least two orders of magnitude than the ones that are observed in ZZ Ceti stars. However, they deserve special attention because the mechanism that can drive the

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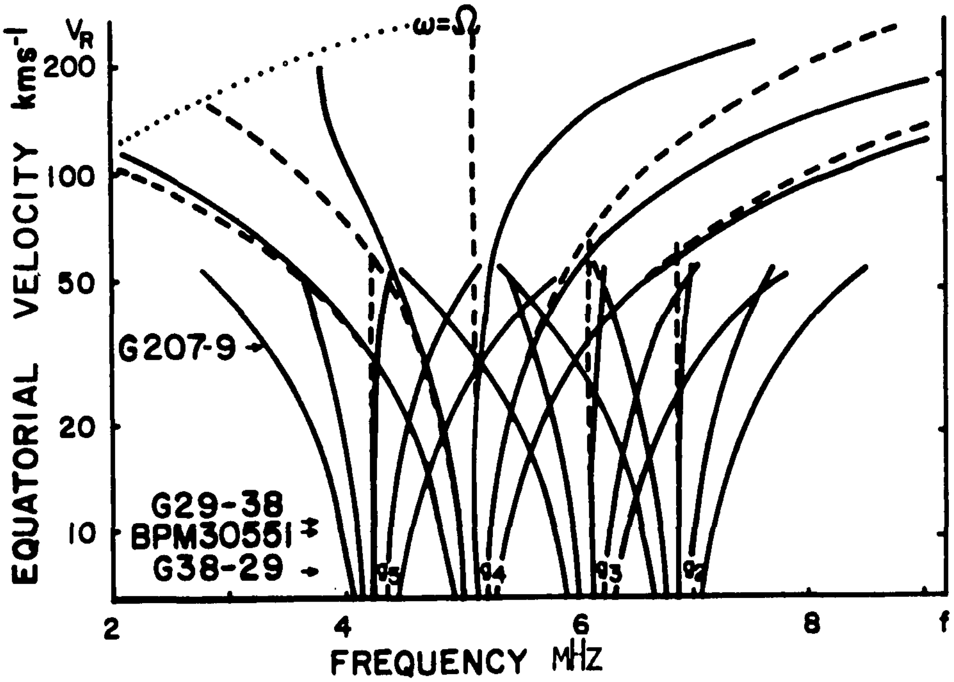

Figure 1. Splitting of gravity mode frequencies by rotation for a M = $0.6M_\odot$ white dwarf model. Dotted lines represent the inclusion of only linear effects in rotation frequency, Ω. Positions corresponding to some ZZ Ceti stars with observed frequency splitting indicated.

gravity modes is likely to drive these modes too. Moreover, it has been suggested by Cox, Hodson and Starrfield (1979) that "long periods observed can be aliases due to undersampling short period variations."

These modes are denoted p_1, p_2, ..., p_n according to the number of nodes in the radial displacement eigenfunction. The mode that has no nodes is called the fundamental (f-) mode. The principal restoring force in this case comes from pressure perturbations.

The dispersion relation in the present case is

$$\kappa^2 = \omega^2 r^2/c^2 - l(l + 1), \qquad (5)$$

where c is the velocity of sound. Arguments similar to those presented in the previous subsection lead to the following criteria for propagation

$$\omega \gtrsim (l + 1)\ c/r \qquad\qquad \omega \gtrsim g/c \qquad (6)$$

In the case of acoustic modes, as in those previously discussed, there are important differences between white dwarfs and giants like Cepheids. In the latter case there is a pronounced maximum of g/c in the interior that prevents radial pulsations ($l = 0$) from propagating into the stellar core. In the case of white dwarfs, g/c decreases monotonically inward and the only modes that can be trapped in the envelope are the ones with high l as then the first of the conditions given in Eq. (6) is more restrictive.

On the other hand, higher surface gravity causes a much larger number of modes in white dwarfs that are evanescent in the outer layers. This is so because the frequency of a given acoustic mode is proportional to $R^{-3/2}$ while g is proportional to R^2. White dwarfs are indeed almost perfect acoustic systems.

Low order p-modes corresponding to low values of l show a very small inward decline in pulsation amplitudes. This however is not true for high order modes, as demonstrated by A. Cox et al. (1979) for radial pulsation and as will be shown further in the present paper for nonradial oscillations. Such modes are again good candidates for excitation by Cepheid-like mechanisms.

2.3. Toroidal modes

The restoring force for toroidal perturbations which are completely horizontal and divergence free may come from Coriolis, Lorentz, or shear forces.

Modes resulting from the Coriolis force are related to Rossby waves. Their frequencies are approximately given by j Ω. Papaloizou

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and Pringle (1978) suggest that their excitation through Kelvin-Helmholtz instability may be responsible for variability of the white dwarfs in cataclysmic binaries. The instability mechanism is strictly connected with the accretion process; thus it is not likely to be of relevance for single white dwarfs.

Hansen and Van Horn (1979) calculated toroidal mode frequencies related to the shear in crystalline cores. The frequencies, though lower than those of acoustic modes, are higher than those observed in ZZ Ceti stars. No mechanism for the excitation of these modes was suggested.

3. Vibrational instabilities

There is a well-known formula relating the growth rate of the mode to the rate of heat gain perturbation, ΔQ,

$$\omega_I = - \frac{\int \Delta Q \left(\frac{\Delta T}{T}\right)^* d^3\vec{x}}{2\omega_R^2 \int |\vec{\xi}|^2 \rho d^3\vec{x}} \qquad (7)$$

where $\Delta T/T$ is the relative temperature perturbation (Lagrangian), ω_R, ω_I are real and imaginary parts of eigenfrequency, $Q = \epsilon\rho - \mathrm{div}\vec{F}$ where ϵ is rate of nuclear energy generation, $\vec{F}$ is the radiative flux, and $\vec{\xi}$ is the displacement.

This formula shows that a layer contributes to mode driving if it receives heat in the phase of high temperature and loses in the phase of low temperature. It shows also that driving is more rapid if, with a given amplitude in the driving zone, the pulsation energy is possibly low.

3.1. The opacity mechanism

This is the mechanism that causes Cepheid pulsation. Location of ZZ Ceti stars on H-R diagrams suggests that it may be operating in these stars too. There are three conditions for local driving by this mechanism.

1. Opacity perturbation should be dominant in $\Delta \mathrm{div}\vec{F}$. This may happen only in an evanescent zone. In a propagation zone the dominant term is always term proportional to the second derivative of the temperature perturbation.

2. Opacity perturbation has to increase (algebraically) outward in the high temperature phase.

3. The heat diffusion time from the zone should not be much shorter than the pulsation period, as in such a case $\Delta \mathrm{div}\vec{F}$ tends to

zero and the layer exerts a neutral influence on stability.

The role of this mechanism in causing vibrational instability in white dwarfs was first recognized by Vauclair (1971). However, because he considered only low order modes, the instabilities he found were extremely slow. Much faster instabilities were found in the survey by Dziembowski (1977) but the modes with observed long periods were all found to be stable. This was so because the deepest lying driving zone was the He II ionization zone; temperatures here ranged from 4-5 x 10^4 and the heat diffusion time from this zone is of the order of 10 s. More recent opacity data reveal that there is a driving zone at the temperature range 1-2 x 10^5; as it was noted by Stellingwerf (1978, 1979), this zone plays an important role in destabilizing δ Scuti and possibly β Cep-type variables. The importance of this zone was also noted by A. Cox et al. (1979) in their survey of radial modes in white dwarfs. Recently, J. P. Cox and Hansen (1979) have pointed out that the opacity mechanism operating in this zone is likely to drive long period modes because its heat diffusion time is considerably longer than that for the He II ionization zone. The calculations to be discussed in the next section confirm this.

3.2. Other driving mechanisms

It has been known for some time that driving due to the perturbation of nuclear energy generation rates may cause vibrational instability in white dwarfs. In the effective temperature range typical for ZZ Ceti stars, however, nuclear burning most likely does not occur. Even if it does occur it will result in thermal rather than vibrational instabilities (Sienkiewicz, 1979).

A mechanism which specifically drives g-modes has been considered in application to white dwarfs by Baglin (1971). It has been found by Kato (1966) that gravity modes may be driven in a layer where the temperature gradient exceeds the adiabatic value but where convective instability is prevented by the existence of a μ-gradient. Baglin found that such a situation may occur in the white dwarf envelope as a result of gravitational settling of helium. This mechanism, however, is not directly relevant to ZZ Ceti stars as it operates only if the layer is in the propagation zone which implies that gravity modes have to be associated with extremely high order spherical harmonics.

4. Gravity and pressure modes in a white dwarf model -- numerical results of the linear nonadiabatic analysis

The model is characterized by the following parameters: $M = 0.6\ M_\odot$ $\log T_e = 4.08$, surface composition: $X = 0.7$, $Z = 0.03$. Smooth transition to a pure carbon-oxygen interior was assumed to occur at the depth between $10^{-10}\ M_\odot$ and $2 \times 10^{-9}\ M_\odot$. Stellingwerf's (1975) opacity formula was used up to $T = 2 \times 10^5$K. The results are shown in Figure 2.

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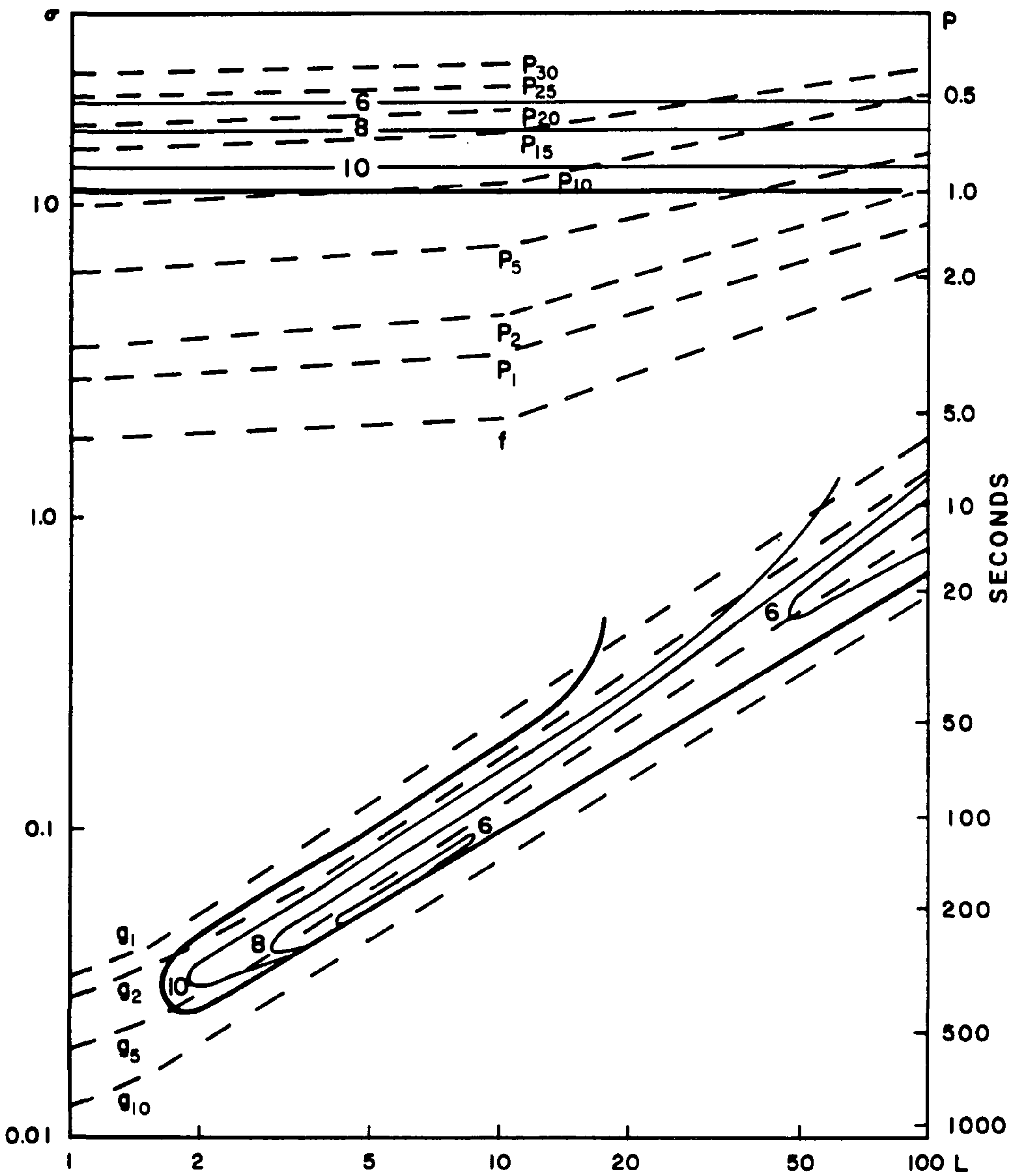

Figure 2. Linear oscillation modes in a white dwarf model. Frequency $\sigma = \omega(4\pi G\langle\rho\rangle)^{-1/2}$. Heavy lines show the locus of the modes with zero growth rates. Thin lines labeled with 10, 8 and 6 show the locus of the modes with growth rates 10^{-10}, 10^{-8} and 10^{-6} respectively in the same units as the frequency.

There is a local maximum of growth rates at $\sigma \simeq 24$ which amounts to 5×10^{-6} for $l = 1$ and 7×10^{-6} for $l = 100$ but there is no significant decrease for higher frequencies. Qualitatively, driving of these high order p-modes is similar to that found for radial modes by A. Cox et al. (1979), as the properties of high-order acoustic modes depend almost exclusively on ω and not on l. Similar results are expected to be valid until the optimum frequency $\sigma \simeq 24$ approaches the value corresponding to the fundamental mode, i.e., $l = 2000$.

Growth rates for gravity modes depend primarily on mode order, but there is also dependence on frequency. The maximum of 1.2×10^{-6} for g_6-modes in l range of 4 - 7 is due almost exclusively to the driving region at a temperature of $1\text{-}2 \times 10^5$ K. Decline of the growth rate at higher frequencies is due to the increased role of the damping layer located above. At still higher frequencies, driving is dominated by the He II ionization zone. Maximum growth rates for these modes are $\sim 1.7 \times 10^{-6}$. Partial trapping of g_6-modes in the μ-gradient zone contributes to their rather high growth rate values.

There is considerable uncertainty involved in estimating growth rates, but several points are significant.

1. Modes with periods in the range observed in ZZ Ceti are among those with the highest growth rates.

2. Gravitational settling of helium will preferentially enhance the growth rates of these modes.

3. Lowering effective temperatures will shift the maximum of instability toward still longer periods and lower l-values.

4. The instability clearly selects some intermediate order of the mode.

5. Nonlinear effects

Only nonlinear theory can be used to determine which of the linearly driven modes will reach observable amplitudes. There are two types of nonlinear effects which must be considered when there are many modes driven with similar rates:

1. Collective saturation of the driving mechanism such as that suggested by Hill (1978) for solar 5 min. modes.

2. Direct interaction between modes.

The amplitude level at which saturation may be important can be estimated by noting that in the zone where driving occurs, part of the radiative flux is absorbed and carried by oscillations to the layers where damping takes place. The amount of flux carried in this way cannnot be more than the total luminosity, L. Actually, in Cepheids, it is about 0.1 L. If we adopt this estimate and make some assumptions about the number of modes involved, the conclusion is that saturation is

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probably not important until $\Delta L/L$ (ΔL is r.m.s. luminosity fluctuation) is of the order of 10^{-2}.

Direct interaction between modes occurs through quadratic terms in the equations for oscillations; these are "collision terms" in our problem. If there are such modes involved which satisfy the resonance condition

$$\omega_3 \simeq \omega_1 \pm \omega_2$$

then there is an energy exchange taking place on the time scale, τ_C, related to the amplitudes of the modes involved.

The estimate of τ_C will be provided for two limiting cases: interaction of three high order p-modes and interaction of three high order g-modes. In each case explicit formulae for coupling can be obtained (Dziembowski, in preparation). It should be noted that interaction is possible only if l and j numbers of the modes satisfy the following conditions:

$$l_3 = l_1 + l_2 + 2n \qquad \text{where } n = 1, 2, \ldots$$

$$j_3 = j_1 \pm j_2$$

Moreover, since we are concerned with high order modes, the eigenfunctions are locally approximated by the trigonometric functions:

$$\xi_r/r = f \cos \Phi$$

where $\Phi = \int \kappa d \ln r$. Thus, for strong interaction, we must have $\kappa_3 \simeq \kappa_2 \pm \kappa_1$.

In such a case, we have the following estimate:

$$\tau_C \sim \frac{1}{\sqrt{4\pi G\langle\rho\rangle}} \frac{1}{\sigma_R\langle\kappa f\rangle}$$

for p-modes, and

$$\tau_C \sim \frac{1}{\sqrt{4\pi G\langle\rho\rangle}} \frac{1}{\langle\kappa f\rangle}$$

for g-modes, where κf is averaged over the volume of the star weighted with pulsational energy.

Inspection of the eigenfunctions obtained for the model discussed in the previous section gives

$$\langle\kappa f\rangle\sigma_R \sim 10^{-3}\Delta L/L$$

for p-modes and also

$$\langle\kappa f\rangle \sim 10^{-3}\Delta L/L$$

for g-modes.

Mode coupling becomes important when $\sqrt{4\pi G\langle\rho\rangle}\,\tau_C \simeq \sigma_I$; therefore, in both p-modes and g-modes, $\Delta L/L \sim 10^{-3}$. It appears therefore likely that this is the dominant nonlinear effect determining the amplitude of the modes.

This conclusion is supported by the fact that many of the ZZ Ceti variables exhibit amplitude changes occurring on the time scale of days. With $\sqrt{4\pi G\langle\rho\rangle} \simeq 0.6\ s^{-1}$, $\tau_C \sim 1$ day, using $\Delta L/L \sim 10^{-2}$, a typical luminosity amplitude for these variables.

More reliable estimates of linear growth rates and nonlinear coupling are needed to find conditions under which gravity modes may reach higher amplitudes than the acoustic modes.

6. Conclusions

In spite of many uncertainties exposed in this review, it seems justified to say that recent theoretical studies give support to the following assessments made at the McDonald Observatory:

1. Variability of ZZ Ceti stars is caused by gravity-mode excitation (Warner and Robinson, 1972).

2. Excitation is caused by the opacity mechanism (McGraw and Robinson, 1976).

3. Equidistant patterns seen in the periodogram are a manifestation of mode splitting by rotation (McGraw and Robinson, 1975).

It is perhaps somewhat premature to attempt to fit observed periods to theoretical models, but if the presented picture is correct then the prospect for using observed periods as a diagnostic tool are good. This is so because the modes that are likely to be excited should be g_n-modes of the same order, imposing a very restrictive condition on possible choices.

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