COMPETITION BETWEEN DIFFUSION PROCESSES AND HYDRODYNAMICAL INSTABILITIES IN STELLAR ENVELOPES.

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Since the work of Michaud (1970), the abundance anomalies observed in the peculiar Ap and Am stars are increasingly believed to be a consequence of diffusion processes in stellar atmospheres or stellar envelopes. A number of the problems that seemed at first sight insoluble within the framework of diffusion processes have now been solved by it. Diffusion processes can, for example, account for anomalous helium isotopic ratios (Vauclair et al, 1974 (b)) and mercury isotopic ratios (Michaud et al, 1974). Quantitative results on abundance variations due to diffusion processes have been obtained (Michaud et al, 1976 ; Michaud, this conference ; Alecian, 1976). They show that, in general, the relative abundance anomalies obtained from computation are close to the observed ones. It is now well established that the largest abundance anomalies observed in Ap stars (for rare earths) can be interpreted by diffusion processes with a satisfactory time scale, in a completely stable atmosphere. However, the predicted absolute abundance variations often exceed the observed ones, as in the case of Am stars. This suggests that the assumption of stability is not completely valid for the stellar gas : some kind of macroscopic motion, such as a meridional circulation or turbulence or both, must be at work and slow down the diffusion.

It has so far been generally believed that instabilities in stellar atmospheres, such as convection or mass loss, would inhibit diffusion. This led Strittmatter and Norris (1971) to mark in the HR diagram the region where diffusion could take place (i.e. the region where the stellar atmosphere is stable). However, strong macroscopic motions may carry to the surface some of the anomalies created in the deep interior by the diffusion processes. This is the case for the superficial convection zones. It could also be the case for mass loss, as proposed by Osmer and Peterson (1974) to account for helium rich stars : they suggest that a helium overabundance in the stellar interior could be brought up to the surface by mass flow. (The difficulty of this model is that the radiation force is probably as unable to support the helium inside the star as it is at the surface).

Edith A. Müller (ed.), Highlights of Astronomy, Vol. 4, Part II, 193-203.

However, it is possible that sufficiently slow motions will merely perturb the diffusion without stopping it (time scale not too short compared to the diffusion time scale -cf Schatzman (1969) for the effect of mild turbulent motions).

Let us write the total velocity of elements as :

$$V = V_D + V_M \tag{1}$$

where V_D is the diffusion velocity, and V_M the macroscopic velocity. In the case of laminar movement, it is clear that if the macroscopic velocity V_M is much larger than the diffusion velocity V_D, there will be no diffusion since a concentration gradient cannot arise. In the case of turbulent motions, V_M is a random velocity, whose form may be found by a simple statistical analysis. Consider a turbulent mixture of two gases 1 and 2 with numerical densities n_1, n_2 (gas 2 = test particles), and a composition gradient along the r axis. The flux of particles 2 due to the turbulent velocity V can be written as :

$$n_2 V_M = - \langle l.V \rangle \,.\mathrm{grad}\; n_2 \tag{2}$$

where l is some characteristic mixing length.

We may also write :

$$V_M = -D_T \frac{1}{C}\frac{\partial C}{\partial r} \tag{3}$$

where $D_T = \langle l.V \rangle$ is the turbulent diffusion coefficient, and C the concentration of particles 2. Taking into account the general expression for the diffusion velocity (see for instance Michaud, 1976, this conference) we find for the total velocity :

$$V = -(D+D_T)\frac{1}{C}\frac{\partial C}{\partial r} + D\left(\frac{m_P g}{2kT}(1-2A+Z) + \frac{Am_P}{kT} g_{rad}\right) \tag{4}$$

where D is the "microscopic" diffusion coefficient. We see from expression (4) that the effect of turbulence is to increase the backward force produced by the concentration gradient, i.e. to reduce the concentration gradient built up by diffusion. If the turbulent diffusion coefficient D_T is much larger than the diffusion coefficient D, there is no diffusion since the concentration gradients are smoothed out.

Consequently, we may eliminate from the HR diagram the regions where diffusion cannot take place, draw instead a "Possible diffusion Region" or P.D.R. (section 1). The next step will be to study the hydrodynamical stability of the P.D.R. stars and try to guess which of them will in fact be sufficiently stable to undergo diffusion, at which point in their evolution, and for how long (section 2). If V_M is of the same order as V_D or D_T of the same order as D, macroscopic

https://doi.org/10.1017/S1539299600003282 Published online by Cambridge University Press

motions may interact with diffusion processes ; this can give rise to new effects which it is interesting to study in detail (section 3).

I. THE "POSSIBLE DIFFUSION REGION".

Figure 1 shows the region in the HR diagram where diffusion processes can have observable effects. The region of mass loss is to the left of the blue boundary. Observations made by the Copernicus satellite show that stars hotter than ∿ B_0 type lose mass (Lamers, 1976). Smith and Parsons (1975 and 1976) have observed asymmetries in the absorption lines of some Mn stars. They interpret them as an outward radial motion, with a velocity of several Kms.sec^{-1}. They think that this wind could inhibit diffusion. However, Michaud (1976 - this conference), suggests that the observed line asymmetries would be a simple consequence of diffusion processes.

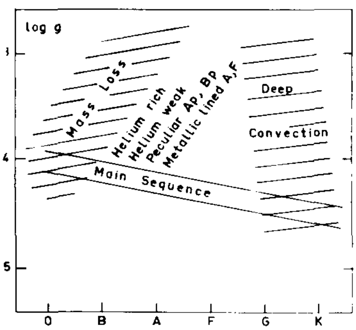

Fig 1.

Fig. 1 : Schematic Possible Diffusion Region.
Diffusion is excluded in the part of the diagram where mass loss or deep convection occur.

Another kind of instability which has not been investigated in detail could be a consequence of the fact that hot stars are rapid rotators. The time scale of the meridional circulation may be shorter than the diffusion time scale.

To the right of the red boundary is the region where the outer convection zones are too deep for diffusion processes to be effective during a stellar lifetime. Michaud et al. (1976) have shown that diffusion below the outer stellar convection zones cannot have observable effects for stars cooler than G. For stars earlier than F2, the diffusion time scales are short compared to the stellar lifetime, and diffusion below the convection zones can lead to large

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abundance anomalies, as observed in Am and Fm stars. For cooler F stars, the diffusion time scales increase and the expected anomalies are much smaller. This effect could be responsible for the metal abundance dispersion observed in F type stars (e.g. Powell, 1972) as well as light element abundance variations. G type stars probably represent the point at which the diffusion time scales exceed the main sequence lifetime. We do not know enough about convection to be able to fix this limit precisely.

II. THE HYDRODYNAMICAL STABILITY OF PDR STARS.

The earlier limiting regions in the HR diagram were based on the reasonable assumption that strong macroscopic motions cancel the effects of small microscopic motions. However, to ensure that diffusion processes will produce observable abundance anomalies in PDR stars, the diffusion zone must remain stable for a sufficiently long time.

In stars with strong magnetic fields, such as Ap stars, the diffusion zone is stable so long as the magnetic energy density exceeds the kinetic energy of the macroscopic motions :

$$\frac{B^2}{8\pi} > \frac{1}{2}\rho V_M^2 \qquad (5)$$

In A stars, a magnetic field stronger than about a hundred Gauss can inhibit convection.

In the case of non-magnetic stars, the stability conditions, even if satisfied on the main sequence, can be destroyed at some later evolutionary phase : mass loss can increase as the star becomes a giant and/or the convection zone can become deeper. Other phenomena, related to the transport of angular momentum through the star could also modify the stability conditions in the diffusion zone : for example, mixing from the deep interior due to meridional circulation on a time scale comparable to the evolutionary time scale. However these large scale motions have not been studied in detail. One case where the transport of angular momentum has been studied is that of stars which achieve a state of uniform rotation at some phase of their evolution (Vauclair, 1976 a, b). This is of course a specialised case, but one for which it is possible to follow the evolution of the velocity field and its stability. Note that the assumption of uniform rotation is not unreasonable for Am type stars, most of which are components of close binary systems. The model can be briefly summarized as follows : in a rotating star, meridional circulation transports angular momentum. Since angular momentum must be conserved, the meridional circulation induces a differential angular velocity which increases with time. In a homogeneous fluid, characterized by its velocity V, its dimension l and its viscosity ν, turbulence occurs if the Reynolds number exceeds some critical value, currently taken in the

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range 10^3 - 10^4. However, since a stellar envelope is stratified in density, this is not a sufficient condition for turbulence to develop. Richardson's criterion must then be applied. In an incompressible fluid, this criterion is simply obtained by comparing the energy stored in the shear $\sim 1/4\ \rho(\delta V)^2$ with the work done by the inertial forces against gravity $\sim -g\delta\rho\delta z$.

The system is stable if :

$$R_i = \frac{-g(1/\rho)(\partial\rho/\partial z)}{(\partial V/\partial z)^2} > \frac{1}{4} \qquad (6)$$

if we take into account the compressibility of the fluid and the radiative losses, we obtain a modified Richardson number which, following Townsend (1958) and Zahn (1974), can be written as :

$$R_i = \frac{\sigma R_e (g/H_P)(\nabla_{ad} - \nabla)}{(dV/dr)^2} \qquad (7)$$

where σ is the Prandtl number (ratio of the viscosity to the thermal conductivity), and R_e is the adopted critical Reynolds number. A fluid becomes turbulent when its modified Richardson number is smaller than one. In this model, the differential velocity and its gradient both increase with time. As a consequence, the associated Reynolds number increases while the Richardson number decreases. Both effects converge towards the onset of turbulent motions. In fact, since the Richardson number drops below one after the Reynolds number has become critical, there is a phase of stability. Diffusion can work freely during this period. When the density stratification is no longer able to damp the perturbations, turbulence develops and smooths out the concentration gradients built up during the stability phase. The mixing will not be complete if the stability phase has lasted sufficiently long for the helium to diffuse ; in this case, the molecular weight barrier built up by the diffusion of the helium will resist the mixing for a longer time, since the effect of the density stratification is reinforced by the variation of molecular weight (Mestel, 1965). The Richardson criterion must be changed to take the μ barrier into account :

$$R_i = \frac{-g(1/\mu)(d\mu/dr)}{(dV/dr)^2} \qquad (8)$$

The "ordinary" stability phase in the homogeneous fluid t_s and the "μ stability" phase in the μ barrier t_μ have been estimated for a representative Am star of 2 $M_\odot$. A number of values are shown in table 1 for various values of the rotational velocity (Vauclair, 1976 a and b). Before the disruption of the μ barrier, the Am character and the pulsations exclude each other as helium has diffused below the region where it could be partially ionized (Vauclair et al., 1974). When the μ barrier disrupt, the mixing of the envelope is complete. Helium is restored to the appropriate temperature domain, where the κ-mechanism, becoming efficient, forces the star to pulsate. Such a model suggests that Am stars may evolve into δ Scuti stars on a time scale comparable to their main sequence lifetime.

TABLE 1

Stability phases of a rotating 2 $M_{\odot}$ star

V_0 km sec^{-1}	t_s years	t_μ years
15	3.8×10^6	4.2×10^8
20	1.6×10^6	2.1×10^8
25	8.0×10^5	1.2×10^8

III. THE EFFECTS OF SMALL MACROSCOPIC MOTIONS ON DIFFUSION.

By small macroscopic motions we mean motions whose time scales are comparable to diffusion time scales. As noted above, we distinguish two kinds of motion : "organized" motion and random motion.

a) "organized motion" : e.g. non-turbulent radial mass loss or laminar meridional circulation. Their main effect is to bring fresh matter into the diffusion zone regularly.

1 - in general, diffusion processes become less efficient and the predicted abundance anomalies decrease. This effect has been used by Kobayashi and Osaki (1973) to interpret the Am phenomenon in terms of diffusion processes superimposed on meridional circulation.

2 - in some cases, diffusion leads to the accumulation (or depletion) of certain elements somewhere in the envelope (clouds or holes). A global motion can shift and deform such regions so as to change the predicted abundance anomalies (Vauclair, 1975 a). Figure 2 illustrates such an effect : the abundance profiles for lithium are shown as a function of depth in the envelope of a 2 $M_{\odot}$ star (after 5×10^5 y) ; one ascending and one descending column of the same size are taken to represent the meridional circulation below the convection zone. The central block in the figure gives the abundance in the convection zone. The left hand block shows the abundance profile in the ascending column ; here, depth increases towards the left. The right hand block shows the abundance profile in the descending column ; here, depth increases towards the right. Going from left to right in the figure, we can get an idea of the mass transport due to meridional circulation. The abundance profile of lithium has bumps - the radiation force is modified by the resonance lines of Li II and Li III. The flux of meridional circulation ($V_c\rho$) is a variable parameter.

3 - the interaction between the global motions and the diffusion processes can also create new clouds and holes, if the total velocity ($V_M + V_D$) changes sign. This has been proposed as an explanation for

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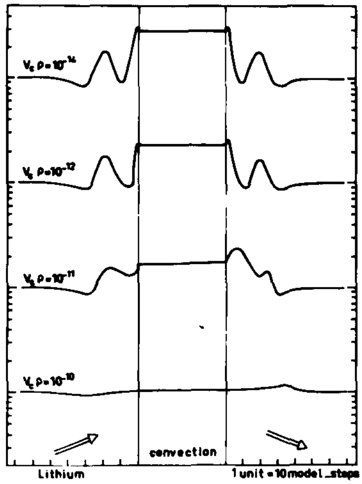

Fig. 2

Fig. 2 : Role of the meridional circulation on the abundance profiles for lithium in a 2 $M_\odot$ envelope.

helium anomalies in helium rich and helium variable stars (Vauclair, 1975 b). This effect is illustrated on figure 3. If somewhere in the atmosphere, the downward diffusion flux of helium is balanced by an opposite global mass loss flux, the helium will accumulate at this level because the diffusion velocity decreases with increasing depth

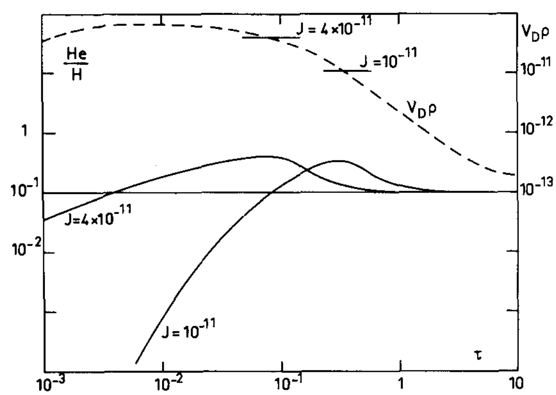

Fig. 3

Fig. 3 : Role of a non turbulent radial mass loss on the helium abundance profile in the atmosphere of a 20,000°K main sequence model. (after Vauclair, 1975 b).

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in the star. Above this level, the downward diffusion flux exceeds the upward global flux : helium sinks by diffusion. Below this level, the upward global flux exceeds the downward helium diffusion flux : helium is transported upwards by the global flow. If the accumulation level falls in the line forming region, we shall have a helium rich star with somewhat special helium line profiles, since the helium abundance is not constant throughout the atmosphere. On the other hand, if the accumulation level is deeper than the line forming region, the star will appear to be helium weak. If mass loss is constrained along magnetic field lines, this process could produce helium rich patches on the stellar surface and account for helium variable stars.

b) "random motion" : we have already noted that turbulent motions partially mix the region where diffusion processes are at work. As a consequence, the concentration gradient built up by diffusion is smoothed out by the turbulent mixing. The important parameter in this problem is the ratio of the turbulent diffusion coefficient D_T to the particle diffusion coefficient D (see eq. 4), so that the effects of turbulence vary from atom to atom. One interesting possibility is that for a given D_T, the downward helium diffusion could be drastically slowed down while the upward diffusion of some other elements could continue normally. An A star with such a turbulent diffusion coefficient would appear as a pulsating star with abundance anomalies. As such stars are not observed on the main sequence but only among giant stars (Kurtz, 1976), this constrains to some extent the value of the turbulence and its time scale.

Another interesting consequence is that "clouds" and "holes" are less pronounced and broader. We show on figure 4 some results on strontium abundance variations in a 2.6 $M_{\odot}$ star (in which the radiation force has been computed according to Michaud et al, 1976). Figure 4a shows profiles of the strontium abundance versus depth through the envelope, for certain time intervals. Turbulence has not been included. The computations were made for the bottom of the hydrogen convection zone in which the abundance is maintained homogeneous. Since the radiation force on strontium is less than gravity at the bottom of the convection zone (where it is mainly Sr III, a noble configuration), the strontium sinks. Below, at a point where the fractional mass exceeds 10^{-10}, the radiation force is larger than gravity and the Sr is pushed upwards. There is an accumulation of Sr in the layers where the radiation force balances gravity. The accumulation peak increases with time while the convection zone progressively loses its strontium. Turbulence is then introduced into the computations. As an example, the turbulent diffusion coefficient is chosen to vary as ρ^{-1} (figure 4b). The value given for D_T in the figure is that fixed at the bottom of the convection zone. For $D_T = 10^5$, the situation is similar to that previously described for $D_T = 0$, except that the profile is smoother. In figure 4c, where the results for $D_T = 10^6$ are presented, the result is qualitatively different : the peak is broadened so much by the turbulence that it enters the convection zone, and produces an overabundance of strontium.

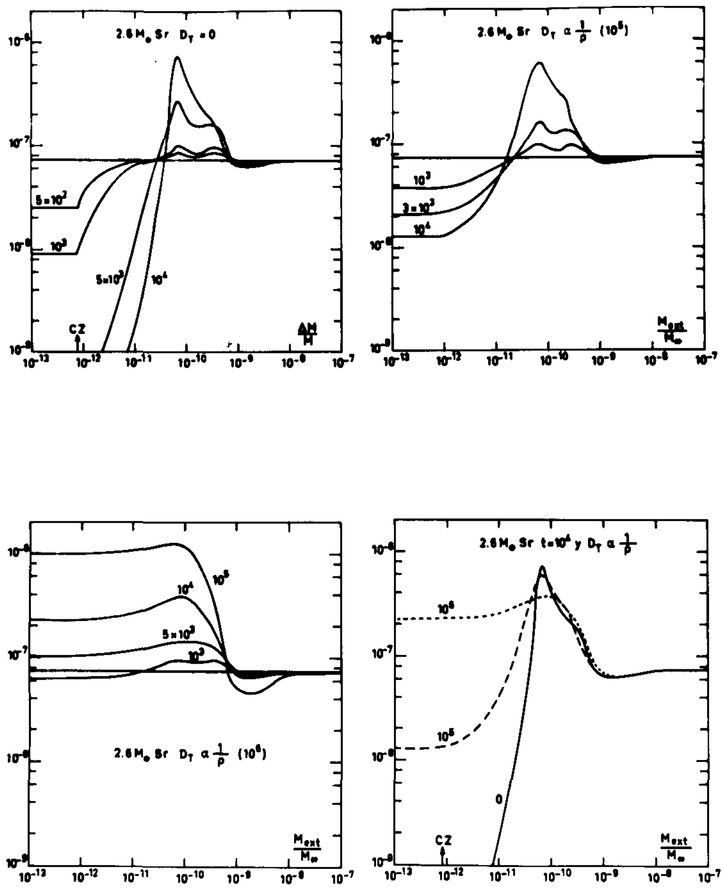

Fig. 4

Fig. 4 : Influence of turbulence on the diffusion of strontium in a $2.6\ M_{\odot}$ star. The diagrams show the abundance profiles (Sr/H) as a function of depth (represented by the external mass fraction). Fig. 4a (up left) : without turbulence. The profiles are shown after 5×10^2, 10^3, 5×10^3 and 10^4 y. Sr sinks in the convection zone. Fig. 4b (up right) : the turbulent diffusion coefficient varies as ρ^{-1}. Its value at the bottom of the convection zone is 10^5. The profiles are shown after 10^3, 3×10^3 and 10^4 y. Fig. 4c (down left) : same as fig. 4b, with $D_T = 10^6$ at the bottom of the convection zone. Here the strontium abundance increases with time in the convection zone. The overabundance is of one order of magnitude after 10^5 y. Fig. 4d (down right) : the abundance profiles are shown for the same epoch (10^4 y) with the three different values of D_T.

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The situation is summarized in figure 4d, where strontium abundance profiles are given for the same epoch (10^4 y) but for different values of the turbulent diffusion coefficient. It is clear from the figure that an underabundance may be transformed into an overabundance just by increasing the turbulence. This effect could have drastic consequences on the predicted abundance anomalies and could help solve some of the difficulties of the diffusion theory raised by Michaud (this conference).

To conclude, this kind of computation applied to abundance anomalies should give us some information about the hydrodynamics of stellar envelopes, about which little is known. The competition betwween diffusion and mixing allows us to study the hydrodynamics of the deep stellar interior : in practice, one needs good spectral observations as well as a sound theoretical analysis of line formation in these kind of stars. Finally, the radiation forces need to be computed accurately, as well as the time dependent abundance variations. Of course, as many elements as possible should be included in the analysis in order to minimize the uncertainty on the results.

ACKNOWLEDGEMENTS-REMERCIEMENTS

C'est un plaisir pour nous de remercier Ludwik Celnikier qui a passé un bon moment à corriger l'anglais de cet article, et Catherine Leguay qui a passé un mauvais moment à le taper.

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