

## **Forced convection and some related integral equations**

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A variety of integral equations and partial differential equations which describe two-dimensional and three-dimensional axisymmetric forced convection problems is studied. The correspondence between the integral equation and partial differential equation formulation of these problems is investigated and is used to develop a new technique for solving hitherto intractable Fredholm integral equations of the first kind. Basic properties of the physical models are also established by studying fundamental solutions of the partial differential equations.

The problems are developed by considering the transport of heat in an ideal heat conducting fluid with fixed thermal constants. The transport mechanisms are forced convection and diffusion, and the temperature satisfies a partial differential equation which is a heat conservation equation. These idealised problems have applications to the equations for viscous flow and the integral equations arise in elastic half-space problems.

The new contributions to the two-dimensional problems are included in the more general investigations of the family of axisymmetric problems. In the latter problems, the first order radial derivative term in the partial differential equation has a singular coefficient whose role in the equation is interpreted as that due to a radial component in the forced convection field ('radial' means 'radial, referred to cylindrical polar coordinates'). The crucial problem here is the realisation of a full understanding of the relationship between the partial differential equation and the integral

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equation. This is achieved by rewriting the partial differential equation in the adjoint form for the radial variable. The new equation is interpreted as a conservation equation which describes the transport of a new energy quantity called  $\alpha$ -heat as a diffusion convection process. Physically motivated arguments suggest the relationship between the original partial differential equation and integral equation formulations and this is confirmed by mathematical proofs.

Dependence on Péclet number, uniqueness theorems, and extensions into distribution spaces are problems which are also discussed.