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ON PRIME ONE-SIDED IDEALS, BI-IDEALS AND QUASI-IDEALS OF A GAMMA RING

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Abstract

Let M be a Γ-ring with right operator ring R. We define one-sided ideals of M and show that there is a one-to-one correspondence between the prime left ideals of M and R and hence that the prime radical of M is the intersection of its prime left ideals. It is shown that if M has left and right unities, then M is left Noetherian if and only if every prime left ideal of M is finitely generated, thus extending a result of Michler for rings to Γ-rings.

Bi-ideals and quasi-ideals of M are defined, and their relationships with corresponding structures in R are established. Analogies of various results for rings are obtained for Γ-rings. In particular we show that M is regular if and only if every bi-ideal of M is semi-prime.

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1. Introduction

All definitions and fundamental concepts concerning Γ-rings and their operator rings can be found in [5]. Throughout this paper M will denote an arbitrary Γ-ring (which does not necessarily possess unities, except for part of Section 2), and L and R will denote its left and right operator rings, respectively. We note that an ideal, one-sided ideal or other substructure I of M is called *finitely generated* if there exists a finite subset X of I such that I is the intersection of all such substructures of M which contain X. We call M *left* (*right*) *noetherian* if M satisfies the ascending chain condition for left (right) ideals. The following characterization of noetherian Γ-rings is proved as for rings.

PROPOSITION 1.1. *A Γ-ring M is left (right) noetherian if and only if every left (right) ideal of M is finitely generated.*

Let A be a ring. If B is an additive subgroup of A such that $BAB \subseteq B$, then B is called a *bi-ideal* of A. If C is an additive subgroup of A such that $(CA) \cap (AC) \subseteq C$, then C is called a *quasi-ideal* of A. A bi-ideal, quasi-ideal or one-sided ideal B of A is called *prime* (*semiprime*) if $a, b \in A$, $aAb \subseteq B$ implies $a \in B$ or $b \in B$ ($a \in A$, $aAa \subseteq B$ implies $a \in B$). For further details concerning bi-ideals and quasi-ideals of a ring, we refer to Steinfeld [7] and Van der Walt [8].

2. Prime one-sided ideals

A one-sided ideal P of M is called *prime* if $x, y \in M$, $x\Gamma M\Gamma y \subseteq P$ implies $x \in P$ or $y \in P$. As for rings, we have

PROPOSITION 2.1. *Let P be a left ideal of M. Then the following are equivalent:*
(a) *P is prime;*
(b) *I, J left ideals of M, $I\Gamma J \subseteq P$ imply $I \subseteq P$ or $J \subseteq P$.*

PROOF. (a) $\Rightarrow$ (b) Let I, J be left ideals of M such that $I, J \nsubseteq P$. Let $x \in I$, $y \in J$ with $x, y \notin P$. Then there exist $m \in M$, $\gamma, \mu \in \Gamma$ such that $x\gamma m\mu y \notin P$. Since $x\gamma m\mu y \in I\Gamma J$, $I\Gamma J \nsubseteq P$.

(b) $\Rightarrow$ (a) Let $x, y \in M$ be such that $x\Gamma M\Gamma y \subseteq P$. Then $(M\Gamma x)\Gamma(M\Gamma y) \subseteq P$. Since $M\Gamma x$ and $M\Gamma y$ are left ideals of M, we have that either $M\Gamma x \subseteq P$ or $M\Gamma y \subseteq P$. Suppose $M\Gamma x \subseteq P$. Let I be the left ideal of M generated by x. Then $I\Gamma I \subseteq M\Gamma x \subseteq P$, whence $I \subseteq P$. Hence, $x \in P$. Similarly, $M\Gamma y \subseteq P$ implies $y \in P$.

We now establish the relationships between prime one-sided ideals of M and R.

PROPOSITION 2.2. *Let P be a prime left (right) ideal of R. Then P^* is a prime left (right) ideal of M.*

PROOF. Since P is a left (right) ideal of R, P^* is a left (right) ideal of M. Let $x, y \in M\backslash P^*$. Then there exist $\gamma, \mu \in \Gamma$ such that $[\gamma, x], [\mu, y] \notin P$. Since P is prime there exists $r \in R$ such that $[\gamma, x]r[\mu, y] \notin P$. It follows that there exist $\nu \in \Gamma$, $m \in M$ such that $[\gamma, x][\nu, m][\mu, y] \notin P$, that is, $[\gamma, x\nu m\mu y] \notin P$, whence $x\nu m\mu y \notin P^*$. Hence, P is prime in M.

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Proposition 2.3. *Let Q be a prime left (right) ideal of M. Then $Q^{*\prime}$ is a prime left (right) ideal of R.*

Proof. Since Q is a left (right) ideal of M, $Q^{*\prime}$ is a left (right) ideal of R. Suppose $a, b \in R \backslash Q^{*\prime}$. Then there exist $x, y \in M$ such that $xa, yb \notin Q$. Since Q is a prime one-sided ideal of M, there exist $m \in M$, $\gamma, \mu \in \Gamma$ such that $(xa)\gamma m\mu(yb) \notin Q$, i.e. $x(a[\gamma, m][\mu, y]b) \notin Q$. It follows that $a[\gamma, m][\mu, y]b \notin Q^{*\prime}$, whence $aRb \not\subseteq Q^{*\prime}$. Hence $Q^{*\prime}$ is prime in R.

Theorem 2.4. *The mapping $P \to P^*$ defines a one-to-one correspondence between the sets of prime left ideals of R and M.*

Proof. Let P be a prime left left ideal of R. By Proposition 2.2, P^* is a prime left ideal of M. It is easily verified that $(P^*)^{*\prime} = \{r \in R: Rr \subseteq P\}$. Since P is a left ideal of R, $P \subseteq (P^*)^{*\prime}$. If $a \in (P^*)^{*\prime}$, $Ra \subseteq P$, and hence $aRa \subseteq P$. Since P is prime $a \in P$, and so $P = (P^*)^{*\prime}$.

Suppose now that Q is a prime left ideal of M. By Proposition 2.3, $Q^{*\prime}$ is a prime left ideal of R. Moreover, $(Q^{*\prime})^* = \{x \in M: M\Gamma x \subseteq Q\}$. Since Q is a left ideal of M, $Q \subseteq (Q^{*\prime})^*$. If $x \in (Q^{*\prime})^*$, then $M\Gamma x \subseteq Q$, whence $x\Gamma M\Gamma x \subseteq Q$. Since Q is prime, $x \in Q$, and so $(Q^{*\prime})^* = Q$. This completes the proof.

Corollary 2.5. *Let $\mathscr{P}(M)$ be the prime radical of M. Then $\mathscr{P}(M)$ is the intersection of the prime left ideals of M.*

Proof. Let $\mathscr{P}(R)$ denote the prime radical of R. Then $\mathscr{P}(R)$ is the intersection of the prime left ideals of R [1, Proposition 2.1]. Moreover, $\mathscr{P}(R)^* = \mathscr{P}(M)$ [3, Theorem 4.1]. Hence

$$\begin{aligned}\mathscr{P}(M) &= \left(\bigcap\{I: I \text{ is a prime left ideal of } R\}\right)^* \\ &= \bigcap\{I: I \text{ is a prime left ideal of } R\}^* \\ &= \bigcap\{J: J \text{ is a prime left ideal of } M\} \qquad \text{(by Theorem 2.4).}\end{aligned}$$

Michler [6, Theorem 6] showed that if A is a ring with unity, then A is left noetherian if and only every prime left ideal of A is finitely generated. We prove an analogue of this result for Γ-rings.

We recall that a left (right) *unity* for M is an element $\sum_{i=1}^m [d_i\delta_i]$ of $L(\sum_{j=1}^n [\varepsilon_j, e_j]$ of $R)$ such that, for all $x \in M$

$$\sum_{i=1}^m d_i\delta_i x = x \left(\sum_{j=1}^n x\varepsilon_j e_j = x\right).$$

Note that in this case $\sum_{i=1}^{m}[d_i, \delta_i]$ $(\sum_{j=1}^{n}[\varepsilon_j, e_j])$ is the (two-sided) unity for $L(R)$. If M has a left unity, and I is a left ideal of M which is finitely generated by the subset $\{a_1, \dots, a_n\}$ of I, it may be shown that

$$I = M\Gamma a_1 + \cdots + M\Gamma a_n.$$

For the remainder of this section, M will be assumed to have left and right unities $d = \sum_{i=1}^{m}[d_i, \delta_i]$ and $e = \sum_{j=1}^{n}[\varepsilon_j, e_j]$, respectively.

LEMMA 2.6. *If A is a left ideal of M, then $A^{*\prime} = \{\sum_{j=1}^{n}[\varepsilon_j, a_j]: a_j \in A\}$.*

PROOF. If $a_1, \dots, a_n \in A$, then for all $x \in M$, $x\varepsilon_j a_j \in A$, whence $\sum_{j=1}^{n}[\varepsilon_j, a_j] \in A^{*\prime}$. Conversely, if $a \in A^{*\prime}$, then $a = (\sum_{j=1}^{n}[\varepsilon_j, e_j])a = \sum_{j=1}^{n}[\varepsilon_j, e_j a]$. Since $a \in A^{*\prime}$, $e_j a \in A$ for $1 \le j \le n$ and the result follows.

LEMMA 2.7. *Let A be a finitely generated left ideal of M. Then $A^{*\prime}$ is a finitely generated left ideal of R.*

PROOF. Suppose that A is finitely generated by the set $\{a_1, \dots, a_r\} \subseteq A$. Let $a \in A^{*\prime}$. By Lemma 2.6, there exists $x_1, \dots, x_n \in A$ such that $a = \sum_{j=1}^{n}[\varepsilon_j, x_j]$. Now $A = M\Gamma a_1 + \cdots + M\Gamma a_r$ whence there exist $l_{jk} \in [M, \Gamma]$ $(1 \le j \le n,\ 1 \le k \le r)$ such that

$$x_j = \sum_{k=1}^{r} l_{jk} a_k.$$

Hence

$$\begin{aligned} a &= \sum_{j=1}^{n}[\varepsilon_j, x_j] = \sum_{j=1}^{n}[\varepsilon_j, \sum_{k=1}^{r} l_{jk} a_k] \\ &= \sum_{j=1}^{n}\sum_{k=1}^{r}[\varepsilon_j, l_{jk} a_k] = \sum_{j=1}^{n}\sum_{k=1}^{r}\left[\varepsilon_j, l_{jk}\left(\sum_{i=1}^{m}[d_i, \delta_i]\right) a_k\right] \\ &= \sum_{j=1}^{n}\sum_{k=1}^{r}\sum_{i=1}^{m}[\varepsilon_j, l_{jk} d_i][\delta_i, a_k] \\ &= \sum_{k=1}^{r}\sum_{i=1}^{m}\left(\sum_{j=1}^{m}[\varepsilon_j, l_{jk} d_i]\right)[\delta_i, a_k]. \end{aligned}$$

Hence, $A^{*\prime}$ is finitely generated by the set $\{[\delta_k a_k]: 1 \le i \le m,\ 1 \le k \le r\}$.

LEMMA 2.8. *Let A be a finitely generated left ideal of R. Then A^{*} is a finitely generated left ideal of M.*

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PROOF. Suppose A is generated by the set $\{a_1, \dots, a_k\} \subseteq A$. Let $a \in A^*$. Then $a = \sum_{i=1}^{m} d_i\delta_i a$, and $[\delta_i, a] \in A$ for $1 \le i \le m$. Hence there exists $x_{ik} \in R$, $1 \le i \le m$, $1 \le k \le r$ such that

$$[\delta_i, a] = \sum_{k=1}^{r} x_{ik}a_k = \sum_{k=1}^{r}\left(x_{ik}\sum_{j=1}^{n}[\varepsilon_j, e_j]\right) a_k.$$

Hence

$$a = \sum_{i=1}^{m} d_i\delta_i a =]\sum_{i=1}^{m}\sum_{k=1}^{r}\sum_{j=1}^{n}(d_ix_{ik})\varepsilon_j(e_ja_k)$$
$$= \sum_{k=1}^{r}\sum_{j=1}^{n}\left(\sum_{i=1}^{m}(d_ix_{ik})\right)\varepsilon_j(e_ja_k).$$

So A^* is finitely generated by the set $\{e_ja_k : 1 \le j \le n,\ 1 \le k \le r\}$.

THEOREM 2.9. *A Γ-ring M is left noetherian if and only if every prime left ideal of M is finitely generated.*

PROOF. Now M is left noetherian if and only if R is left noetherian [4, Corollary 1]. Hence, in view of Michler's result, we need only show that every prime left ideal of M is finitely generated if and only if every prime left ideal of R is finitely generated.

Suppose every prime left ideal of R is finitely generated. Let P be a prime left ideal of M. Then $P^{*\prime}$ is a prime left ideal of R by Proposition 2.3, and is therefore finitely generated. By Lemma 2.8, $(P^{*\prime})^*$ is finitely generated. By [4, Theorem 1], $P = (P^{*\prime})^*$, and so P is finitely generated. Conversely, suppose that every prime left ideal of M is finitely generated. Let Q be a prime left ideal of R. By Proposition 2.2, Q^* is a prime left of M, and is thus finitely generated. By Lemma 2.7, $(Q^*)^{*\prime}$ is finitely generated in R. Again by [4, Theorem 1], $Q = (Q^*)^{*\prime}$, and so Q is finitely generated. This completes the proof.

Analogues of all the results in this section may of course be obtained by substituting L for R and "right ideal" for "left ideal" wherever these occur.

3. Bi-ideals and quasi-ideals

An additive subgroup A of M such that $A\Gamma M\Gamma A \subset A$ is called a *bi-ideal* of M. If B is an additive subgroup of M such that $(B\Gamma M)\cap(M\Gamma B) \subseteq B$, then B is called a *quasi-ideal* of M. It is easily seen that one-sided ideal

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$\Rightarrow$ quasi-ideal $\Rightarrow$ bi-ideal. We now establish some relationships between the bi-ideals and quasi-ideals of M and of R.

PROPOSITION 3.1. *If A is a bi-ideal of R, then A^* is a bi-ideal of M. If M has a right unity and B is a bi-ideal of M, then $B^{*\prime}$ is a bi-ideal of R.*

PROOF. Since A is an additive subgroup of R, it is easily verified that A^* is an additive subgroup of M. Let $a, b \in A^*$, $\gamma, \mu, \nu \in \Gamma$, $m \in M$. Then $[\gamma, a], [\nu, b] \in A$. Since A is a bi-ideal of R, $[\gamma, a][\mu, m][\nu, b] \in A$, i.e. $[\gamma, a\mu m\nu b] \in A$. Hence $a\mu m\nu b \in A^*$, whence $A^*\Gamma M\Gamma A^* \subseteq A^*$. Hence A^* is a bi-ideal of M.

Suppose that M has a right unity $\sum_{i=1}^{n}[\varepsilon_i, e_i]$, and that B is a bi-ideal of M. Let $a, b \in B^{*\prime}$, $r \in R$. Let $r = \sum_{j=1}^{r}[\gamma_j y_j]$ and let $x \in M$. Then $xa \in B$ and $e_i b \in B$ for $1 \le i \le n$. Hence

$$(xa)\gamma_j y_j \varepsilon_i (e_i b) \in B, \qquad 1 \le i \le n,\ 1 \le j \le r.$$

Hence

$$\sum_{i=1}^{n}\sum_{j=1}^{r}(xa)\gamma_j y_j \varepsilon_i (e_i b) \in B,$$

that is,

$$xa\left(\sum_{j=1}^{r}[\gamma_j, y_j]\right)\left(\sum[\varepsilon_i, e_i]\right) b \in B,$$

that is, $x(arb) \in B$, whence $arb \in B^{*\prime}$, and so $A^{*\prime}RB^{*\prime} \subseteq B^{*\prime}$. Hence $B^{*\prime}$ is a bi-ideal of R.

LEMMA 3.2.
(a) *Let $A, B \subseteq R$. Then $A^*\Gamma B^* \subseteq (AB)^*$.*
(b) *Let $A \subseteq M$. Then $A^{*\prime}R \subseteq (A\Gamma M)^{*\prime}$ and $RA^{*\prime} \subseteq (M\Gamma A)^{*\prime}$.*

PROOF.
(a) Let $a \in A^*$, $b \in B^*$, $\gamma, \mu \in \Gamma$. Then $[\gamma, a\mu b] = [\gamma, a][\mu, b] \in AB$. Hence $a\mu b \in (AB)^*$ and so $A^*\Gamma B^* \subseteq (AB)^*$.
(b) Let $a \in A^{*\prime}$, $r = \sum_i[\gamma_i, x_i] \in R$, $x \in M$. Then $xa \in A$, whence $x(ar) = (xa)\sum_i[\gamma_i x_i] = \sum_i(xa)\gamma_i x_i \in A\Gamma M$. Hence $ar \in (A\Gamma M)^{*\prime}$ and so $A^{*\prime}R \subseteq (A\Gamma M)^{*\prime}$. Similarly, $RA^{*\prime} \subseteq (M\Gamma A)^{*\prime}$.

PROPOSITION 3.3.
(a) *If A is a quasi-ideal of R, then A^* is a quasi-ideal of M.*

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(b) *If* B *is a quasi-ideal of* M, *then* $B^{*\prime}$ *is a quasi-ideal of* R.

PROOF.

(a) Clearly, A^* is an additive subgroup of M. Moreover, $A^*\Gamma M = A^*\Gamma R^* \subseteq (AR)^*$ by Lemma 3.2(a). Similarly, $M\Gamma A^* \subseteq (RA)^*$, whence $(A^*\Gamma M) \cap (M\Gamma A^*) \subseteq (AR)^* \cap (RA)^* = ((AR) \cap (RA))^* \subseteq A^*$, since A is a quasi-ideal of R. Hence A^* is a quasi-ideal of M.

(b) $B^{*\prime}$ is an additive subgroup of R. By Lemma 3.2(b), we have that $B^{*\prime}R \subseteq (B\Gamma M)^{*\prime}$ and that $RB^{*\prime} \subseteq (M\Gamma B)^{*\prime}$. Hence

$$(B^{*\prime}R) \cap (RB^{*\prime}) \subseteq (B\Gamma M)^{*\prime} \cap (M\Gamma B)^{*\prime} = ((M\Gamma B) \cap (B\Gamma M))^{*\prime} \subseteq B^{*\prime},$$

since B is a quasi-ideal of M. Hence $B^{*\prime}$ is a quasi-ideal of R.

A bi-ideal or quasi-ideal P of M is called *prime* if $x, y \in M$, $x\Gamma M\Gamma y \subseteq P$ imply $x \in P$ or $y \in P$.

PROPOSITION 3.4 (cf. [8, Proposition 2.2]). *A prime bi-ideal* P *of* M *is a prime one-sided ideal of* M.

PROOF. Suppose that P is not a one-sided ideal of M. Then $P\Gamma M \not\subseteq P$ and $M\Gamma P \not\subseteq P$. Since P is prime, $(P\Gamma M)\Gamma M\Gamma(M\Gamma P) \not\subseteq P$. Since $(P\Gamma M)\Gamma M\Gamma(M\Gamma P) \subseteq P\Gamma M\Gamma P$, and since P is a bi-ideal of M, $P\Gamma M\Gamma P \subseteq P$, then $(P\Gamma M)\Gamma M\Gamma(M\Gamma P) \subseteq P$, a contradiction. Hence, $P\Gamma M \subseteq P$ or $M\Gamma P \subseteq P$, that is, P is a one-sided ideal of M.

As an immediate consequence of this result, Corollary 2.5 and its right analogue, we obtain

COROLLARY 3.5. $\mathscr{P}(M)$ *is the intersection of the prime bi-ideals of* M.

PROPOSITION 3.6 (cf. [8, Proposition 2.4]). *A bi-ideal* P *of* M *is prime if and only if* I *a right ideal of* M, J *a left ideal of* M, $I\Gamma J \subseteq P$ *imply* $I \subseteq P$ *or* $J \subseteq P$.

The proof is similar to that for the ring case, and will be omitted. We remark that although van der Walt considers only a ring with identity in [8], the analogues for rings of Propositions 3.4 and 3.6 are valid for arbitrary rings. In view of Proposition 1.1 and Theorem 2.9, we obtain the following characterization of Γ-rings which are both left and right noetherian. The proof is again similar to the ring case [8, Proposition 2.7].

Proposition 3.7. *Suppose M has both left and right unities. Then M is both left and right noetherian if and only if every prime quasi-ideal of M is finitely generated (as a quasi-ideal).*

4. Semi-prime bi-ideals and regular Γ-rings

A bi-ideal or quasi-ideal Q of M is called *semiprime* if $x \in M$, $x\Gamma M\Gamma x \subseteq Q$ implies $x \in Q$.

Proposition 4.1. *Let Q be a semiprime bi-ideal of M. Then Q is a semiprime quasi-ideal of M.*

Proof. Let $x \in (Q\Gamma M) \cap (M\Gamma Q)$. Then $x\Gamma M\Gamma x \subseteq Q\Gamma M\Gamma M\Gamma M\Gamma Q \subseteq Q\Gamma M\Gamma Q \subseteq Q$ since Q is a bi-ideal of M. Since Q is semiprime, $x \in Q$, and hence $(Q\Gamma M) \cap (M\Gamma Q) \subseteq Q$.

We now establish some relationships between semiprime quasi-ideals of M and R.

Proposition 4.2.

(a) *Let P be a semiprime quasi-ideal of R. Then P^* is a semiprime quasi-ideal of M.*

(b) *Let Q be a semiprime quasi-ideal of M. Then $Q^{*\prime}$ is a semiprime quasi-ideal of R.*

Proof.

(a) By Proposition 3.3(a), P^* is a quasi-ideal of M. Let $a \in M \backslash P^*$. Then there exists $\gamma \in \Gamma$ such that $[\gamma, a] \notin P$. Since P is semiprime, there exists $r \in R$ such that $[\gamma, a]r[\gamma, a] \notin P$. Put $r = \sum_i [\gamma_i, x_i]$. Then $\sum_i [\gamma, a\gamma_i x_i \gamma a] \notin P$, whence $a\gamma_i x_i \gamma a \notin P^*$ for some i. Hence $a\Gamma M\Gamma a \nsubseteq P^*$, so P^* is semiprime.

(b) By Proposition 3.3(b), $A^{*\prime}$ is a quasi-ideal of R. Let $a \in R \backslash Q^{*\prime}$. Then $xa \notin Q$ for some $x \in M$. Since Q is semiprime, there exist $\gamma, \mu \in \Gamma$, $m \in M$ such that $(xa)\gamma m\mu(xa) \notin Q$ whence $a[\gamma, m][\mu, x]a \notin Q^{*\prime}$. Thus $aRa \nsubseteq Q^{*\prime}$, and so $Q^{*\prime}$ is semiprime.

From Chen [2], M is regular if for all $a \in M$, $a \in a\Gamma M\Gamma a$.

Proposition 4.3. *M is regular if and only if every bi-ideal of M is semiprime. The proof is similar to that for the corresponding result for rings [8, Proposition 3.3].*

Finally, we remark that it is easily shown that if A and B are bi-ideals of M, then $A\Gamma B$ is a bi-ideal of M. Van der Walt [8, Corollary 3.5] has given an example of a ring A with two quasi-ideals P and Q such that PQ is not a quasi-ideal of A. A is a $\mathbb{Z}$-ring with the normal addition operation and $xny = n(xy)$ for all $n \in \mathbb{Z}$, $x, y \in A$. Furthermore, it is easily seen that P and Q are quasi-ideals of A considered as a $\mathbb{Z}$-ring and $P\mathbb{Z}Q = PQ$ is not a quasi-ideal of the $\mathbb{Z}$-ring.

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