Planetary Nebulae: A Universal Toolbox in the Era of Precision Astrophysics
Proceedings IAU Symposium No. 384, 2025
O. De Marco, A. Zijlstra & R. Szczerba, eds.
doi:10.1017/S1743921323005215

Precise Determination of Extinction Corrections and Plasma Diagnostics

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Abstract. Extinction correction is the quintessence of astronomy. To achieve precision astrophysics in plasma diagnostics as in the theme of the present Proceedings, one must perform extinction correction properly before executing any line diagnostics of line-emitting objects including planetary nebulae. By making use of the inseparable relationship between extinction correction and plasma diagnostics, we establish a novel method to determine the physical conditions of a line-emitting target and the extinction characteristics along the line of sight toward the target simultaneously and self-consistently. This approach is made possible by the exact analytical expressions for the extinction parameters in terms of the emission properties of the target and by statistical optimization of the extinction parameters to find the robust physical conditions of the target.

Keywords. plasmas, methods: analytical, methods: statistical, techniques: spectroscopic, dust, extinction

1. Introduction

Plasma diagnostics are the gold standard to understand the physical conditions of line-emitting gaseous systems (e.g., Osterbrock & Ferland 2006; Pradhan & Nahar 2011). Given the observed relative strengths of diagnostic emission lines from various astrophysical targets, it is possible to determine the excitation states of specific gaseous species in terms of the electron density (n_e) and temperature (T_e), from which ionic and elemental abundances can also be derived (e.g., Peimbert, Peimbert, & Delgado-Inglada 2017; Kewley, Nicholls, & Sutherland 2019). Thus, measuring line strengths at high accuracy and precision is of the utmost importance to succeed.

Meanwhile, it must also be recognized that one's success in plasma diagnostics hinges on the quality of extinction correction, because observed strengths of diagnostic emission lines are all subject to the circumsource and interstellar extinction (e.g., Savage & Mathis 1979; Salim & Narayanan 2020; Gordon et al. 2023). No matter how precise and accurate line measurements are, inadequate extinction correction could potentially ruin the rest of the data analysis and compromise all the subsequent derivatives and implications. In the era of precision astrophysics in the 21st century as in the theme of the present Proceedings, one should really refrain from making a naive assumption such as extinction being benign or negligible. It is imperative that extinction is rigorously accounted for and that observed line strengths are properly corrected for extinction *before any subsequent analysis is performed.*

Hence, it is not too hard to see that extinction correction and plasma diagnostics are to be performed simultaneously and self-consistently for the best results. This is simply because the degree of extinction is usually determined by comparing observed line strength ratios with their unattenuated theoretical counterparts, provided that the physical conditions of the target are known *a priori*. In this contribution, we outline a robust update on the "Direct Method" to streamline both extinction correction and plasma diagnostics so that the physical conditions of the line emitting target and the extinction toward the target are determined *all at once* based exclusively on spectroscopic data of the most abundant and prevalent gas species, hydrogen.

2. Extinction Correction of Line-Emitting Gaseous Nebulae

According to the theory of radiative transfer, the amount of radiation from a target source $I_0(\lambda)$ is reduced to the observed value $I(\lambda)$ as

$$I(\lambda) = I_0(\lambda)\exp(-\tau_\lambda) \tag{1}$$

upon passing through the circumsource and interstellar media, which as a whole amount to the optical depth τ_λ along the line of sight. Observationally, this reduction of the intrinsic signal is often expressed by means of the wavelength-dependent base-10 power index $c(\lambda)$ as

$$I(\lambda) = I_0(\lambda)\cdot 10^{-c(\lambda)} = I_0(\lambda)\cdot 10^{-\tau_\lambda/\ln 10}. \tag{2}$$

This base-10 power index c is usually referred to as "the extinction c" where $c(\lambda) \approx 0.434\tau_\lambda$.

Meanwhile, the magnitude-based total extinction A_λ is defined to be

$$A_\lambda = -2.5\log_{10}\frac{I(\lambda)}{I_0(\lambda)}, \tag{3}$$

for which we immediately see the relationship between A_λ and $c(\lambda)$ as $A_\lambda = 2.5c(\lambda)$. The total extinction A_λ is often normalized by A_V, the total extinction at the V band, to define an extinction law of the form

$$\left\langle \frac{A_\lambda}{A_V} \right\rangle = a_\lambda + b_\lambda \cdot R_V^{-1}. \tag{4}$$

Here, R_V is the total-to-selective extinction, $A_V/E(B-V) = A_V/(A_B - A_V)$, which is meant to capture the wavelength-dependent nature of extinction along a line of sight (e.g., Cardelli, Clayton, & Mathis 1989). It can alternatively be understood as the color excess at the V band with respect to an infinitely red band (i.e., $A_\infty \equiv 0$) normalized by the $B-V$ color excess. More broadly, R_V is often considered to represent the average dust size along a particular line of sight: the larger the dust size is, the greater R_V value becomes. Mathematicaly, the a_λ and b_λ parameters in Eq. 4 are the y-intercept and slope, respectively, of the fit of the extinction curve A_λ/A_V to R_V^{-1} averaged over many lines of sight in the Milky Way or toward a specific galaxy (hence the angle brackets $\langle\rangle$ are used to mean it is the "average"). These parameters are usually expressed as a combination of polynomials representing the continuum and Drude profiles describing significant spectral features (e.g., Gordon et al. 2023).

When a gaseous nebula of the electron density n_e and electron temperature T_e radiates emission lines, the ratio of intrinsic line strengths of a particular ionic species X^{i+} can be expressed as a ratio of volume emissivities or emission coefficients $\epsilon_{X^{i+}}$ between two lines as

$$\frac{I_0(\lambda_2)}{I_0(\lambda_1)} = \frac{n_\mathrm{e} n_{i+} \epsilon_{X^{i+}}(\lambda_2; n_\mathrm{e}, T_\mathrm{e})/4\pi}{n_\mathrm{e} n_{i+} \epsilon_{X^{i+}}(\lambda_1; n_\mathrm{e}, T_\mathrm{e})/4\pi} = \frac{\epsilon_{X^{i+}}(\lambda_2; n_\mathrm{e}, T_\mathrm{e})}{\epsilon_{X^{i+}}(\lambda_1; n_\mathrm{e}, T_\mathrm{e})} \tag{5}$$

where n_{i+} is the number density of the ionic species X^{i+} given that emission lines arise from the same part of the target nebula. This means that the intrinsic line ratios are practically functions of n_e and T_e of the line emitting region of the target (e.g., Pradhan & Nahar 2011).

Then, based on the base-10 power-law formalism (Eq. 2), the modification of the intrinsic line ratio $I_0(\lambda_2)/I_0(\lambda_1)$ into the observed line strength ratio $I(\lambda_2)/I(\lambda_1)$ is expressed as

$$\frac{I(\lambda_2)}{I(\lambda_1)} = \frac{I_0(\lambda_2) \cdot 10^{-c(\lambda_2)}}{I_0(\lambda_1) \cdot 10^{-c(\lambda_1)}} = \frac{I_0(\lambda_2)}{I_0(\lambda_1)} \cdot 10^{-c(\lambda_1)\left(\frac{c(\lambda_2)}{c(\lambda_1)}-1\right)} = \frac{I_0(\lambda_2)}{I_0(\lambda_1)} \cdot 10^{-c(\lambda_1)\left(\frac{A_{\lambda_2}}{A_{\lambda_1}}-1\right)} \quad (6)$$

where the relationship between $c(\lambda)$ and A_λ introduced above is used at the last step. This equation gives us the basis for the determination of the absolute amount of extinction c at some reference wavelength λ_1 as

$$c(\lambda_1) = \left(1 - \frac{A_{\lambda_2}}{A_{\lambda_1}}\right)^{-1} \log_{10}\left(\frac{\frac{I(\lambda_2)}{I(\lambda_1)}}{\frac{I_0(\lambda_2)}{I_0(\lambda_1)}}\right) \quad (7)$$

in which the coefficient involving the ratio of total extinction A at λ_1 and λ_2 can be replaced by adopting an R_V-dependent extinction law (e.g. Eq. 4) as

$$\left(1 - \frac{A_{\lambda_2}}{A_{\lambda_1}}\right) \approx \left(1 - \frac{\left\langle \frac{A_{\lambda_2}}{A_V} \right\rangle}{\left\langle \frac{A_{\lambda_1}}{A_V} \right\rangle}\right) = \left(1 - \frac{a_{\lambda_2} + b_{\lambda_2} \cdot R_V^{-1}}{a_{\lambda_1} + b_{\lambda_1} \cdot R_V^{-1}}\right). \quad (8)$$

Here, we emphasize that the inseparable relationship between extinction correction and plasma diagnostics arises from the dependence of the intrinsic line ratio $I_0(\lambda_2)/I_0(\lambda_1)$ on n_e and T_e (Eq. 7) via the line emissivities (Eq. 5). In other words, the extinction c at a reference wavelength λ_1 cannot be determined unless the physical conditions of the line emitting gas, n_e and T_e, are known *a priori*. Thus, extinction correction and plasma diagnostics must be performed together – iteratively or otherwise – for the maximum consistency. To restate it further, *extinction can be, and hence, should be, determined as part of plasma diagnostics.*

3. Analytical Solutions for the Relative Visibility and Extinction Index

In the literature, there is a long history of plasma diagnostics (e.g., Aller 1999). Naturally, diagnostics have long been performed most often in the optical using the H I Balmer lines with the Hβ line as a reference. The initial choice of the Hβ line as a reference was most likely motivated because this line happens to be relatively isolated from other major lines in the optical. However, there is no particular theoretical reason why the Hβ line must be used as a reference. For that matter, there is no particular theoretical reason why the Hα-to-Hβ ratio must be treated as a canon. As long as measurements are accurate and precise enough, one can adopt other H I recombination lines or even lines of other species.

Theoretically, line strength ratios are adopted as proxies for the ratio of emissivities of the corresponding lines (Eq. 5 above; see Hummer & Storey 1987; Storey & Hummer 1995; Storey & Sochi 2014 for cases of hydrogenic ions). In fact, Osterbrock (1989) originally suggested the use of a Paschen line and a Balmer line from the same upper set of levels, because then line intensity ratios would depend mostly on the ratio of their transition probabilities. However, they hesitated to go with the Paschen-to-Balmer ratio method stating that "the relative insensitivity of photomultipliers and of CCDs at wavelengths longward of 1 μm." This statement reflected the state of IR detectors when the first edition of the book was published (Osterbrock 1989) and was somehow kept in the second edition even when sensitive IR spectroscopic measurements were enabled

https://doi.org/10.1017/S1743921323005215 Published online by Cambridge University Press

routinely (Osterbrock & Ferland 2006). Of course, such is no longer a source of concern now in the 21st century.

At any event, we continue our discussion below using λ_1 as the wavelength of a reference line to keep the generality of the discussion. Provided that a specific R_V-dependent extinction law is adopted, by combining Eqns. 7 and 8, we can determine the extinction $c(\lambda_1)$ as

$$c(\lambda_1) = \frac{a_{\lambda_1} \cdot R_V + b_{\lambda_1}}{(a_{\lambda_1} - a_{\lambda_2}) \cdot R_V + (b_{\lambda_1} - b_{\lambda_2})} \log_{10} \left(\frac{\frac{I(\lambda_2)}{I(\lambda_1)}}{\frac{I_0(\lambda_2)}{I_0(\lambda_1)}} \right), \tag{9}$$

and similarly the relative visibility R_V as

$$R_V = \frac{b_{\lambda_1} \cdot \log_{10} \left(\frac{\frac{I(\lambda_2)}{I(\lambda_1)}}{\frac{I_0(\lambda_2)}{I_0(\lambda_1)}} \right) - (b_{\lambda_1} - b_{\lambda_2}) \cdot c(\lambda_1)}{(a_{\lambda_1} - a_{\lambda_2}) \cdot c(\lambda_1) - a_{\lambda_1} \cdot \log_{10} \left(\frac{\frac{I(\lambda_2)}{I(\lambda_1)}}{\frac{I_0(\lambda_2)}{I_0(\lambda_1)}} \right)} \tag{10}$$

for a particular line at λ_2 with respect to a reference line at λ_1. The above two equations form a system of equations for unknowns, R_V and $c(\lambda_1)$, given the observed line ratio $I(\lambda_2)/I(\lambda_1)$ and its unattenuated theoretical counterpart $I_0(\lambda_2)/I_0(\lambda_1)$, which is a function of n_e and T_e via Eq. 5. Because both $c(\lambda_1)$ and R_V characterize extinction along the line of sight to a specific target source, we should find the same $c(\lambda_1)$ and R_V irrespective of the choice of the line at λ_2. Therefore, we can analytically solve this system of equations for $c(\lambda_1)$ and R_V, if we have at least another set of equations with yet another line at λ_3, i.e., Eqns. 9 and 10 in terms of another line ratio, $I(\lambda_3)/I(\lambda_1)$. Then, with the $c(\lambda_1)$ and R_V values determined, we can also fix the n_e and T_e values as those that yield $I_0(\lambda_2)/I_0(\lambda_1)$ and $I_0(\lambda_3)/I_0(\lambda_1)$ consistently. Thus, we have established *a new method to determine the extinction parameters toward the source from line measurements alone simultaneously in a self-consistent and self-contained manner.*

4. The Optimum Solutions for the Electron Density and Temperature

In practice, however, this is not the end of the analysis. We must now find the optimum solutions for the physical conditions of the target, (n_e, T_e). Based on the above system of equations, when we secure measurements of two line ratios, e.g., $I(\lambda_2)/I(\lambda_1)$ and $I(\lambda_3)/I(\lambda_1)$, or measurements of three lines at λ_1, λ_2, and λ_3, the R_V and $c(\lambda_1)$ surfaces can be defined over a certain range of the (n_e, T_e) space. Then, we can make use of the same expectation that R_V and $c(\lambda_1)$ represent extinction along the same line of sight to the target and conclude that the same R_V and $c(\lambda_1)$ values must always result irrespective of the choice of diagnostic line pairs. That is, we can determine the optimum (n_e, T_e) values as the location in the (n_e, T_e) space at which the same R_V and $c(\lambda_1)$ always result for any diagnostic line ratio pairs $I(\lambda_i)/I(\lambda_1)$ and $I(\lambda_j)/I(\lambda_1)$ where $i \neq j > 1$.

In reality, however, because of observational uncertainties, these R_V and $c(\lambda_1)$ surfaces would vary for each pair of diagnostic line ratios or each trio of diagnostic line measurements. In fact, when we have more than three line strengths measured so that more than two line ratios can be defined (as is fairly routinely done nowadays), we can establish multiple R_V and $c(\lambda_1)$ surfaces to compare against each other. To be more precise, there are ${}_nP_2$ distinct surfaces (where n is the number of line ratios) for each of R_V and $c(\lambda_1)$. Then, it is expected that the R_V and $c(\lambda_1)$ values among all the R_V and $c(\lambda_1)$ surfaces formed with distinct line ratio pairs would vary the least amount at the optimum (n_e, T_e) location. Such a point in the (n_e, T_e) space can easily be determined via some statistical method.

For example, if we obtain four line strength measurements ($I(\lambda_i)$ for $i = 1$ to 4 and λ_1 is used as a reference), we can form three line ratios, $I(\lambda_2)/I(\lambda_1)$, $I(\lambda_3)/I(\lambda_1)$, and $I(\lambda_4)/I(\lambda_1)$, to establish the R_V and $c(\lambda_1)$ surfaces for each pair of line ratios among the ${}_3P_2 = 3$ permutations over a certain (n_e, T_e) range. Thus, we can find the optimum (n_e, T_e) solutions as the location where both of the R_V and $c(\lambda_1)$ values remain the most consistent (i.e., the least amount of variation). Then, the R_V and $c(\lambda_1)$ values at the convergence point in the (n_e, T_e) space would of course be the corresponding optimum solutions for R_V and $c(\lambda_1)$. When a greater number of line measurements, and hence, a greater number of line ratio pairs are available, we can form more R_V and $c(\lambda_2)$ surfaces with every single permutation of line ratio pairs. Then, the optimum values of n_e, T_e, $c(\lambda_1)$, and R_V are determined by the location in the (n_e, T_e) space where $c(\lambda_1)$ and R_V vary the least amount.

In this way, we can seek out the optimum solution for $(R_V, c(\lambda_1))$ and (n_e, T_e) simultaneously and self-consistently if we have the line strength measured at more than three H I recombination lines. Now, we have implemented *an updated method to determine the physical conditions of a line emitting source and the extinction parameters toward the source simultaneously and self-consistently based exclusively on one's own line measurements.*

5. Conclusion and Implications

Following the procedure outlined above, one can simultaneously determine the physical conditions of line emitting plasma (n_e, T_e) together with the base-10 power-law extinction index c at some reference wavelength and the relative visibility R_V along the line of sight to the line emitting plasma, provided that (i) we have measurements of at the very least four H I recombination lines, and (ii) a certain extinction law $\langle A_\lambda/A_V \rangle$ of the R_V-dependent form covering the wavelength range of the measured lines is adopted (e.g., Cardelli, Clayton, & Mathis 1989; Gordon et al. 2023). This is a remarkable improvement on the presently widely-exercised Direct Method of plasma diagnostics, because the proposed method yields (i) all the pertinent quantities simultaneously and self-consistently and (ii) based solely on one's own spectroscopic measurements of H I recombination lines. In other words, the proposed method is *completely data-driven and self-contained*, and *no need to make unnecessary assumptions and rely on separate extinction studies*. In addition, the method can be used effectively for *fainter line emission objects in which only H I recombination lines are readily available*, significantly lowering the requirement threshold for the applicability of the analysis.

Lines to be used in the proposed procedure do not have to be exclusively H I recombination lines. As originally suggested by (Osterbrock 1989), comparing the [S II] lines at λλ 4069 and 4076 (^{4}S–^{2}P) with those at λλ 10287, 10320, 10336, and 10370 (^{2}D–^{2}P) is a viable option. The He II lines are also possible alternatives, while these lines probe higher excitation regions of the target. At any rate, it would make the most sense to adopt H I recombination lines because hydrogen is obviously the most abundant and prevalent species and these lines are readily available in various wavelength ranges, especially in the optical and infrared that are routinely observed nowadays. For that matter, higher order H I recombination lines within the JWST/MIRI range alone will suffice for the proposed method to yield results.

However, some caution needs to be exercised when practicing the proposed analysis. Obviously, the precision and accuracy of line measurements must be sufficiently high. Among all the sources of uncertainty in the analysis, the measurement uncertainties of the input line measurements impose the largest influence on the precision and accuracy of the derived values. The H I recombination line emissivities show relatively weak n_e dependence over a relevant n_e range. Therefore, it is comparatively more difficult to pinpoint the optimum n_e value than the optimum T_e value. Moreover, line ratio measurements that are offset too much by measurement uncertainties may not yield valid solutions for R_V and $c(\lambda_1)$ (i.e., unphysical negative values may result).

In our preliminary assessment of the validity and effectiveness of the proposed method with a large number of randomly generated models with simulated measurement uncertainties, we have found that (i) T_e, R_V, and $c(\mathrm{H}\beta)$ can be recovered with precision and accuracy within 1 % and n_e can be recovered with precision and accuracy within around 10 % as long as the (n_e, T_e) space is searched at the 0.001 dex resolution in the absence of measurement uncertainties, while (ii) assumed 1 % uncertainties in typical line measurements would translate to 0.1 ± 2.0, 20 ± 50, 5 ± 12, and 6 ± 8 % deviations in the resulting $\log T_e$, $\log n_e$, R_V, and $c(\mathrm{H}\beta)$, respectively. The resulting T_e is always fairly precise and accurate thanks to the relatively large gradient in the line emissivity along the T_e dimension. Therefore, we can in practice fix T_e, R_V and $c(\mathrm{H}\beta)$ first, and further optimize n_e (and R_V and $c(\mathrm{H}\beta)$ again) using other n_e-diagnostic lines already recognized in the traditional Direct Method or any line ratio pairs that would yield a large gradient in the n_e dimension.

If we were to rely on the single fixed line ratio of $I_0(\mathrm{H}\alpha)/I_0(\mathrm{H}\beta) = 2.85$ (corresponding exclusively to gas at $T_e = 10^4$ K and $n_e = 10^3\,\mathrm{cm}^{-3}$) as has been traditionally practiced, the extinction correction would immediately be inconsistent unless the target object is of exactly $T_e = 10^4$ K and $n_e = 10^3\,\mathrm{cm}^{-3}$ (via Eqns. 9 and 10), which of course is never guaranteed. Hence, we must be wary of unnecessary assumptions that can lead to imprecise and inaccurate results by way of inconsistent extinction correction, which in turn can invite unwarranted discussions to mend the incidental inconsistencies (e.g. Ueta & Otsuka 2021, 2022).

Similarly, assuming the "standard" MW value of $R_V = 3.1$ for *any* Galactic object may introduce some unwanted consequence. For example, Moraga Baez et al. (2023) have reported a discrepancy in the resulting $c(\mathrm{H}\beta)$ value for NGC 7027 based on the $\mathrm{H}\alpha/\mathrm{H}\beta$ and $\mathrm{Pa}\beta/\mathrm{H}\beta$ ratios according to the "canonical" Direct Method (i.e., assuming $T_e = 10^4$ K and $n_e = 10^3\,\mathrm{cm}^{-3}$ as well as $R_V = 3.1$). By a quick search for the optimum $c(\mathrm{H}\beta)$ and R_V values by minimizing the discrepancy among the $c(\mathrm{H}\beta)$ values based on seven Balmer and Paschen line ratios for the target observed by us (using the KOOLS-IFU instrument on the Seimei 4 m telescope; e.g. Otsuka 2022), we obtain the optimum value of $R_V = 3.39$, which is consistent with the previous result toward the direction of the target (Seaton 1979). This is potentially a significant issue because the R_V value greatly influences extinction in the UV and can affect the resulting elemental abundances based on UV lines (such as C III 1909 Å). For this particular case, $R_V = 3.1$ yields the C/O abundance ratio of 1.37, while $R_V = 3.4$ gives 1.04, suggesting that the stellar chemistry for NGC 7027 is not as C-rich as previously thought.

In the 21$^{\mathrm{st}}$ century, we can *start performing precision astrophysics* in plasma diagnostics with all associated uncertainties quantified. There is no reason not to invest in a method that is clear and straightforward as well as self-consistent and self-contained, leaving no room for any simplifying assumptions that are totally unnecessary. Those who are interested in the present method are encouraged to refer to our forthcoming set of publications, in which more detailed formulation of this solution method will be presented (Ueta & Otsuka *in preparation*) and various case studies in which the method is applied to various types of astrophysical targets are introduced (Otsuka & Ueta *in preparation*).

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