Note on the preceding Locus.

By A. J. Pressland, M.A.

The perpendicular from A on BC = AB·AC/2R, which is constant.

Describe a circle with centre A and radius $m^2/2R$. This, which may be called the perpendicular circle, will touch each position of the base, and therefore will touch FG.

Now AD is the axis of the inscribed and the perpendicular circles, and BDC is a common tangent.

Therefore D is their external centre of similitude.

But FG touches the perpendicular circle.

Therefore it touches the inscribed circle.

We have now to find the locus of a point whose distance from a fixed line $= r$ and from a fixed point (the circumcentre) $= \sqrt{R^2 - 2Rr}$, which can easily be proved to be a circle.

The triangle and its escribed parabolas.

By A. J. Pressland, M.A.

In what follows, a triangle ABC is taken, from it another triangle A′B′C′ is formed so that

A is the mid point of B′C′,
B is the mid point of C′A′,
and C is the mid point of A′B′.

From A′B′C′ another triangle A″B″C″ is similarly derived.

If a parabola escribed to ABC touch BC at P, CA at Q, and AB at R, we have shown that AP, BQ and CR intersect in a point O, the locus of which is the minimum ellipse circumscribing ABC.

The equation of PQ is

$$n(x-a) = l(y-b)$$

whence PQ passes through C′.
Similarly QR passes through A′,
and RP passes through B′.

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If we join QR and produce it to meet CB in K, and also join PR and produce it to meet CA in L, we find as equation of LK

$$l\{2xb/a - b + y\} = n\{2ya/b - a + x\}$$

an equation which is satisfied by the co-ordinates of the centroid and of the point T where PQ cuts AB.

It is to be noted that O is the pole of LK with respect to the parabola.

The locus of the centroid of PQR will be found to be

$$(3x - 2a)(3y - 2b)\{3ay + 3bx - ab\} = a^2b^2$$

the asymptotes of which are the lines joining the centroids of A′BC, AB′C, and ABC′.

Taking as the equation of the parabola

$$(nx + ly)^2 - 2(nx - ly)(an - lb) + (lb - an)^2 = 0$$

we get for the envelope of the polar of $(x'y')$

$$\begin{aligned} &x^2(y' + b)^2 \quad - 2xy(x'y' - 3ay' \quad - 3bx' \quad + ab) \\ &+ y^2(x' + a)^2 \quad + 2y\ (bx'^2 + 3ax'y' - 3abx' - a^2y') \\ &+ (bx' \ - ay')^2 + 2x\ (ay'^2 + 3bx'y' - 3aby' - b^2x') = 0 \end{aligned}$$

If we put $x' = a/3, y' = b/3$, we get

$$\frac{x^2}{a^2} + \frac{xy}{ab} + \frac{y^2}{b^2} - \frac{x}{a} - \frac{y}{b} = 0$$

the equation of the minimum ellipse.

Hence the polar of the centroid with respect to the parabola is the tangent at O to the minimum ellipse.

Discussing the envelope equation obtained above we shall find that it represents an ellipse if the point $(x'y')$ be within the triangle A′B′C′, or within any division corresponding to the space between A′C′ produced and B′C′ produced. If the locus be an hyperbola the point $(x'y')$ is in one of the three spaces bounded by A′B′, C′A′ produced, and C′B′ produced, or other similar lines.

If the point $(x'y')$ be on A′B′, or A′C′, or B′C′, the equation from its form represents a parabola, but is really two straight lines which are coincident and pass through one of the points A′,B′,C′.

In other words, A′B′C′ is a self-conjugate triangle with respect to any of the parabolas. This property would seem to point to an easy treatment of the parabola and its self-conjugate triangle, for the median triangle of any self-conjugate can be proved to be circumscribed to the parabola.

If the envelope be a rectangular hyperbola, $(x'y')$ must be on the circle whose equation is

$$x^2 + 2xy\cos C + y^2 + 2(c\cos B - 2b\cos C)x$$
$$+ 2(c\cos A - 2a\cos C)y + 2a^2 + 2b^2 - c^2 = 0.$$

If $(x'y')$ be on the median through A, the envelope passes through A. Hence the envelope is only a circumscribed conic when $(x'y')$ is the centroid.

If $(x'y')$ be on CA, CB or BA, the envelope passes through $(x'y')$.

If $(x'y')$ be A, B, or C, the envelope is two coincident lines, the corresponding medians.

If the envelope be a circle, the point is one of the in-centres or ex-centres of A″B″C″.

If $(x'y')$ be the centre of the envelope, it must be either A, B, C, or the centroid.

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J. S. MACKAY, Esq., M.A., LL.D., ex-President, in the Chair.

On the Use of Dimensional Equations in Physics.

By WILLIAM PEDDIE, D.Sc.

Though every quantity, whatever be its nature, has magnitude, no quantity can be said to be large or small *absolutely*. When we speak of the size of any body we mean its size relatively to the size of some other body with which we compare it. A yard is large if we compare it with an inch; it is small when compared with a mile. In the former case the number which represents it is more than 60,000 times larger than the number by which it is represented in the latter case. A mere number is therefore useless as regards the statement of magnitude, except when accompanied by a clear indication of what the thing measured is compared with. The quantity