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CHERN CHARACTERS, REDUCED RANKS AND $\mathscr{D}$-MODULES ON THE FLAG VARIETY

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Let D be the factor of the enveloping algebra of a semisimple Lie algebra by its minimal primitive ideal with trival central character. We give a geometric description of the Chern character ch: $K_0(D)\to HC_0(D)$ and the state (of the maximal ideal $\mathbf{m}$) s: $K_0(D)\to K_0(D/\mathbf{m})=\mathbb{Z}$ in terms of the Euler characteristic χ:$K_0(\mathscr{X})\to\mathbb{Z}$, where $\mathscr{X}$ is the associated flag variety.

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1. Introduction

Let G be a connected, semi-simple, complex algebraic group, let $\mathfrak{g}$ be its Lie algebra and let $U(\mathfrak{g})$ be the enveloping algebra of $\mathfrak{g}$. Let P be the minimal primitive deal of $U(\mathfrak{g})$ with trivial central character and set $D=U(\mathfrak{g})/P$. Then D has a unique maximal ideal $\mathbf{m}$ (the annihilator of the trivial representation) and $D/\mathbf{m}\cong\mathbb{C}$. We consider here the relationship between three natural functions on the Grothendieck group $K_0(D)$. First, there is the Chern character, ch: $K_0(D)\to HC_0(D)$ where $HC_0(D)=D/[D,D]=\mathbb{C}$ is the zero-th cyclic homology group. Second, the natural map $D\to D/\mathbf{m}$ induces a map s: $K_0(D)\to K_0(D/\mathbf{m})=\mathbb{Z}$. In Theorem 3.1 we observe that ch and s coincide (via the usual embedding of $\mathbb{Z}$ in $\mathbb{C}$).

The third map is induced from the Euler characteristic on the associated flag variety G/B where B is a Borel subgroup. Let $\mathscr{D}$ denote the sheaf of differential operators on G/B. The Bernstein–Beilinson Theorem establishes an equivalence of categories between the category of D-modules and the category of quasi-coherent $\mathscr{D}$-modules. From this one may deduce that $K_0(D)\cong K_0(G/B)$. Thus the Euler characteristic χ: $K_0(G/B)\to\mathbb{Z}$ induces a function on $K_0(D)$. The relation between this and the above maps is given by Theorem 2.9. Let $\tilde{\chi}$ denote χ composed with the duality automorphism of $K_0(G/B)$; that is, $\tilde{\chi}[\mathscr{E}]=\chi[\mathscr{E}^*]$ for any locally free sheaf $\mathscr{E}$. Then via the above isomorphism, s coincides with $\tilde{\chi}$.

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2. Reduced ranks

2.1. The results announced in the introduction will be proved in a slightly more general situation. Let $\mathscr{X}$ be a complete, homogeneous space of G, that is $\mathscr{X}=G/P$ for some parabolic subgroup of G. Write $\mathcal{O}_{\mathscr{X}}$ for the structure sheaf of $\mathscr{X}$. The sheaf of rings of differential operators on $\mathscr{X}$ is denoted by $\mathscr{D}_{\mathscr{X}}$ and $\mathscr{D}(\mathscr{X})=\Gamma(\mathscr{X},\mathscr{D}_{\mathscr{X}})$, is the ring of globally defined differential operators on $\mathscr{X}$. Then, by [4, Thm. 3.8], $\mathscr{D}(\mathscr{X})$ is isomorphic to a primitive factor of $U(\mathfrak{g})$ with trivial central character. Thus $\mathscr{D}(\mathscr{X})$ has a unique maximal ideal $\mathbf{m}$, the image of the augmentation ideal of $U(\mathfrak{g})$. Note that $\mathbf{m}$ is idempotent and $\mathscr{D}(\mathscr{X})/\mathbf{m}\cong\mathbb{C}$. Let $n=\dim\mathscr{X}$.

2.2. Denote by $\mathscr{D}_{\mathscr{X}}$-**mod** the category of $\mathscr{D}_{\mathscr{X}}$-modules which are quasi-coherent when considered as $\mathcal{O}_{\mathscr{X}}$-modules and by $\mathscr{D}_{\mathscr{X}}$-**mod** the category of $\mathscr{D}(\mathscr{X})$-modules. The main structural result we use is Beilinson and Bernstein's famous theorem [2, 5].

Theorem. *There is an equivalence of categories between $\mathscr{D}_{\mathscr{X}}$-**mod** and $\mathscr{D}(\mathscr{X})$-**mod** given by the mutually inverse functors:*

$$\mathscr{M}\mapsto\Gamma(\mathscr{X},\mathscr{M}) \text{ and } M\mapsto\mathscr{D}_{\mathscr{X}}\otimes_{\mathscr{D}(\mathscr{X})}M,$$

*for $\mathscr{M}\in\mathscr{D}_{\mathscr{X}}$-**mod** and $M\in\mathscr{D}(\mathscr{X})$-**mod**.*

It then follows from [3, VI.1.10], that $\mathscr{D}(\mathscr{X})$ has global dimension less than or equal to $2n$. (In fact, an easy extension of the methods of [10] will give equality here.)

2.3. An important consequence of Beilinson and Bernstein's theorem is that the exact functor $\Gamma\ (\mathscr{X},\mathscr{D}_{\mathscr{X}}\otimes_{\mathcal{O}_{\mathscr{X}}}\_)$ induces an isomorphism in K-theory. Let $K_0(\mathscr{X})$ denote the Grothendieck group of the category of coherent $\mathcal{O}_{\mathscr{X}}$-modules. It is canonically isomorphic to the Grothendieck group of the category of vector bundles on $\mathscr{X}$. Similarly, $K_0(\mathscr{D}(\mathscr{X}))$, the Grothendieck group of the category of finitely generated $\mathscr{D}(\mathscr{X})$-modules, is canonically isomorphic to the Grothendieck group of the category of finitely generated, projective $\mathscr{D}(\mathscr{X})$-modules.

Theorem [1, 7]. *The functor $\Gamma(\mathscr{X},\mathscr{D}_{\mathscr{X}}\otimes_{\mathcal{O}_{\mathscr{X}}}\_)$ induces an isomorphism τ: $K_0(\mathscr{X})\to K_0(\mathscr{D}(\mathscr{X}))$.*

2.4. If $\mathscr{E}$ is a vector bundle on $\mathscr{X}$, denote by $\mathscr{D}_{\mathscr{E}}$ the sheaf of differential operators with coefficients in $\mathscr{E}$, that is:

$$\mathscr{D}_{\mathscr{E}}=\mathscr{E}\otimes_{\mathcal{O}_{\mathscr{X}}}\mathscr{D}_{\mathscr{X}}\otimes_{\mathcal{O}_{\mathscr{X}}}\mathscr{E}^*,$$

where $\mathscr{E}^*=\mathcal{H}om_{\mathcal{O}_{\mathscr{X}}}(\mathscr{E},\mathcal{O}_{\mathscr{X}})$, is the dual bundle. The natural inclusion $\mathcal{O}_{\mathscr{X}}\hookrightarrow\mathscr{D}_{\mathscr{E}}$ makes $\mathscr{D}_{\mathscr{E}}$ a quasi-coherent $\mathcal{O}_{\mathscr{X}}$-module. We denote by $\mathscr{D}_{\mathscr{E}}$-**mod** the category of $\mathscr{D}_{\mathscr{E}}$-modules which are quasi-coherent as $\mathcal{O}_{\mathscr{X}}$-modules. The following result is well known and easy to prove.

Lemma (Geometric Translation). *There is an equivalence of categories between $\mathscr{D}_{\mathscr{E}}$-**mod** and $\mathscr{D}_{\mathscr{X}}$-**mod** given by the mutually inverse functors*

$$\mathscr{M}\mapsto\mathscr{D}_{\mathscr{X}}\otimes_{\mathcal{O}_{\mathscr{X}}}\mathscr{E}^*\otimes_{\mathscr{D}_{\mathscr{E}}}\mathrm{M} \text{ and } \mathscr{N}\mapsto\mathscr{E}\otimes_{\mathcal{O}_{\mathscr{X}}}\mathscr{D}_{\mathscr{X}}\otimes_{\mathscr{D}_{\mathscr{X}}}\mathrm{N},$$

*for $\mathscr{M}\in\mathscr{D}_{\mathscr{E}}$-**mod** and $\mathscr{N}\in\mathscr{D}_{\mathscr{X}}$-**mod**.*

2.5. Let $\mathbb{C}$ be the one-dimensional simple $\mathscr{D}(\mathscr{X})$-module, $\mathscr{D}(\mathscr{X})/\mathbf{m}$. Consider the functors:

$$Hom_{\mathscr{D}(\mathscr{X})}(\_\_, \mathbb{C}): \mathscr{D}(\mathscr{X})\text{-}\mathbf{mod} \to \mathbf{mod}\text{-}\mathbb{C}$$

and

$$\mathbb{C} \bigotimes_{\mathscr{D}(\mathscr{X})} \_\_: \mathscr{D}(\mathscr{X})\text{-}\mathbf{mod} \to \mathbb{C}\text{-}\mathbf{mod}.$$

Since $\mathscr{D}(\mathscr{X})$ has finite global dimension these functors have finite (co)homological dimension. It is easy to see that there is a natural isomorphism:

$$Hom_{\mathscr{D}(\mathscr{X})}(\_\_, \mathbb{C}) \cong Hom_{\mathbb{C}}(\mathbb{C}_{\mathscr{D}(\mathscr{X})}\_\_, \mathbb{C}).$$

In particular, we have the following:

Lemma. *There are natural isomorphisms:*

$$Ext^i_{\mathscr{D}(\mathscr{X})}(\_\_, \mathbb{C}) \cong Hom_{\mathbb{C}}(Tor_i^{\mathscr{D}(\mathscr{X})}(\mathbb{C}, \_\_), \mathbb{C}),$$

for each $i \geqq 0$.

Since the functors $\mathbb{C} \bigotimes_{\mathscr{D}(\mathscr{X})} \_\_$ and $Hom_{\mathscr{D}(\mathscr{X})}(\_\_, \mathbb{C})$ are exact on short exact sequences of finitely generated, projective modules they induce an abelian group map s: $K_0(\mathscr{D}(\mathscr{X})) \to K_0(\mathbb{C}) = \mathbb{Z}$ such that for any finitely generated $\mathscr{D}(\mathscr{X})$-module M,

$$s[M] = \sum_{i=0}^{2n} (-1)^i \dim_{\mathbb{C}} Tor_i^{\mathscr{D}(\mathscr{X})}(\mathbb{C}, M) = \sum_{i=0}^{2n} (-1)^i \dim_{\mathbb{C}} Ext^i_{\mathscr{D}(\mathscr{X})}(M, \mathbb{C}).$$

Theorem 2.6. *Let $\mathscr{E}$ be a vector bundle on $\mathscr{X}$. There are vector space isomorphisms:*

(a) $Ext^i_{\mathscr{D}(\mathscr{X})}(\Gamma(\mathscr{X}, \mathscr{D} \bigotimes_{\mathcal{O}_{\mathscr{X}}} \mathscr{E}), \mathbb{C}) \cong H^i(\mathscr{X}, \mathscr{E}^*)$.

(b) $Tor_i^{\mathscr{D}(\mathscr{X})}(\mathbb{C}, \Gamma(\mathscr{X}, \mathscr{D} \bigotimes_{\mathcal{O}_{\mathscr{X}}} \mathscr{E})) \cong H^{n-i}(\mathscr{X}, \mathscr{E} \bigotimes_{\mathcal{O}_{\mathscr{X}}} \omega)$,

where ω is the canonical sheaf on $\mathscr{X}$.

Proof. Note that (b) follows from (a) using Lemma 2.5 and Serre duality. Let us prove (a). Beilinson and Bernstein's theorem (2.2) implies that there is a vector space isomorphism

$$Ext^i_{\mathscr{D}(\mathscr{X})}(\Gamma(\mathscr{X}, \mathscr{D} \bigotimes_{\mathcal{O}_{\mathscr{X}}} \mathscr{E}), \mathbb{C}) \cong Ext^i_{\mathscr{D}_{\mathscr{X}}\text{-mod}}(\mathscr{D} \bigotimes_{\mathcal{O}_{\mathscr{X}}} \mathscr{E}, \mathcal{O}_{\mathscr{X}}).$$

Geometric translation (2.4) gives another $\mathbb{C}$-linear isomorphism:

$$Ext^i_{\mathscr{D}_{\mathscr{X}}\text{-mod}}(\mathscr{D} \bigotimes_{\mathcal{O}_{\mathscr{X}}} \mathscr{E}, \mathcal{O}_{\mathscr{X}}) \cong Ext^i_{\mathscr{D}_{\mathscr{E}^*}\text{-mod}}(\mathscr{D}_{\mathscr{E}^*}, \mathscr{E}^*),$$

and the next lemma shows that this latter vector space is isomorphic to $H^i(\mathscr{X}, \mathscr{E}^*)$, as required.

Lemma 2.7. *Let $\mathscr{E}$ be a vector bundle on $\mathscr{X}$. If $\mathscr{M} \in \mathscr{D}_{\mathscr{E}}$-**mod** then*

$$Ext^i_{\mathscr{D}_{\mathscr{E}}\text{-mod}}(\mathscr{D}_{\mathscr{E}}, \mathscr{M}) \cong H^i(\mathscr{X}, \mathscr{M}).$$

Proof. It is enough to show that if $\mathscr{I}\in\mathscr{D}_{\mathscr{E}}$-**mod** is an injective object then it is flasque, for then we may compute the cohomology of $\mathscr{M}$ in the category $\mathscr{D}_{\mathscr{E}}$-**mod**. To do this it is in turn sufficient to show that $\mathscr{I}$ is an injective object in the category of quasi-coherent $\mathscr{O}_{\mathscr{X}}$-modules, by [**6**, Exercise III.3.6, page 217].

Suppose then that $0\to\mathscr{F}\to\mathscr{G}$ is an exact sequence of quasi-coherent $\mathscr{O}_{\mathscr{X}}$-modules and that $f\colon \mathscr{F}\to\mathscr{I}$ is an $\mathscr{O}_{\mathscr{X}}$-module morphism. There is an induced morphism $\hat{f}$: $\mathscr{D}_{\mathscr{E}}\bigotimes_{\mathscr{O}_{\mathscr{X}}}\mathscr{F}\to\mathscr{I}$ of quasi-coherent $\mathscr{D}_{\mathscr{E}}$-modules. Note that as $\mathscr{D}_{\mathscr{E}}$ is flat as an $\mathscr{O}_{\mathscr{X}}$-module, $0\to\mathscr{D}_{\mathscr{E}}\bigotimes_{\mathscr{O}_{\mathscr{X}}}\mathscr{F}\to\mathscr{D}_{\mathscr{E}}\bigotimes_{\mathscr{O}_{\mathscr{X}}}\mathscr{G}$ is still exact. The injectivity of $\mathscr{I}$ ensures that $\hat{f}$ extends to a $\mathscr{D}_{\mathscr{E}}$-module morphism h: $\mathscr{D}_{\mathscr{E}}\bigotimes_{\mathscr{O}_{\mathscr{X}}}\mathscr{G}\to\mathscr{I}$. Now the composed morphism $\mathscr{G}\to\mathscr{D}_{\mathscr{E}}\bigotimes_{\mathscr{O}_{\mathscr{X}}}\mathscr{G}\xrightarrow{h}\mathscr{I}$ clearly extends f and so $\mathscr{I}$ is injective in the category of quasi-coherent $\mathscr{O}_{\mathscr{X}}$-modules, as required.

Corollary 2.8. *If $\mathscr{E}$ is a vector bundle on $\mathscr{X}$ and $i>n$ then $Ext^i_{\mathscr{D}(\mathscr{X})}(\Gamma(\mathscr{X},\mathscr{D}\bigotimes_{\mathscr{O}_{\mathscr{X}}}\mathscr{E}),\mathbb{C})=0$.*

2.9. The functor $\Gamma(\mathscr{X},\_\_)$ induces an abelian group map $\chi\colon K_0(\mathscr{X})\to K_0(\mathbb{C})=\mathbb{Z}$ given by

$$\chi[\mathscr{F}]=\sum_{i=0}^{n}(-1)^i\dim_{\mathbb{C}}H^i(\mathscr{X},\mathscr{F}).$$

Note that $\chi[\mathscr{F}]$ is the Euler–Poincaré characteristic of $\mathscr{F}$. Define $\tilde{\chi}\colon K_0(\mathscr{X})\to K_0(\mathbb{C})=\mathbb{Z}$ by $\tilde{\chi}[\mathscr{E}]=\chi[\mathscr{E}^*]$ for any locally free sheaf $\mathscr{E}$.

Theorem. *There is a commutative diagram:*

$$\begin{array}{ccc} K_0(\mathscr{X}) & \xrightarrow{\tilde{\chi}} & \mathbb{Z} \\ \tau\downarrow & & \| \\ K_0(\mathscr{D}(\mathscr{X})) & \xrightarrow{s} & \mathbb{Z}. \end{array}$$

Proof. Let $\mathscr{E}$ be a locally free sheaf. From Theorem 2.6 and Corollary 2.8 we have that

$$\tilde{\chi}[\mathscr{E}]=\sum_{i=0}^{2n}(-1)^iExt^i_{\mathscr{D}(\mathscr{X})}(\Gamma(\mathscr{X},\mathscr{D}_{\mathscr{X}}\bigotimes_{\mathscr{O}_{\mathscr{X}}}\mathscr{E}),\mathbb{C})=s[\Gamma(\mathscr{D}_{\mathscr{X}}\bigotimes_{\mathscr{O}_{\mathscr{X}}}\mathscr{E})]=s\circ\tau[\mathscr{E}].$$

2.10. One can extend the last result to certain other primitive factors of $U(\mathfrak{g})$ using the translation principle. For convenience, and for the remainder of this section only, assume that $\mathscr{X}=G/B$. Our notation is that of [**2**]. In particular $\mathfrak{h}$ is a Cartan subalgebra and if $\lambda\in\mathfrak{h}^*$ is integral, then $\mathscr{O}(\lambda)$ denotes the associated line bundle. We write $\mathscr{D}_\lambda$ for the sheaf of differential operators with coefficients in $\mathscr{O}(\lambda-\rho)$, where ρ denotes the half-sum of the positive roots. The ring of global sections $D_\lambda=\Gamma(\mathscr{X},\mathscr{D}_\lambda)$ is a primitive factor of $U(\mathfrak{g})$ and has a unique maximal ideal $\mathbf{m}_\lambda$. If λ is dominant and regular then [**2**] shows that $\Gamma(\mathscr{X},\_\_)$: $\mathscr{D}_\lambda$-**mod**$\to D_\lambda$-**mod** is an equivalence of categories and, by [**7**], the map $\tau_\lambda\colon K_0(\mathscr{X})\to K_0(D_\lambda)$ induced by $\Gamma(\mathscr{X},\mathscr{D}_\lambda\bigotimes_{\mathscr{O}_{\mathscr{X}}}\_)$ is an isomorphism. For any invertible sheaf $\mathscr{L}$, define $\chi_{\mathscr{L}}\colon K_0(\mathscr{X})\to\mathbb{Z}$ by $\chi_{\mathscr{L}}[\mathscr{F}]=(-1)^n\chi[\mathscr{L}\bigotimes_{\mathscr{O}_{\mathscr{X}}}\mathscr{F}]$. Notice that by Serre duality, if $\mathscr{E}$ is a vector bundle then $\chi_\omega[\mathscr{E}]=\chi[\mathscr{E}^*]$.

Define s_λ to be the natural map $K_0(D_\lambda)\to K_0(D_\lambda/\mathbf{m}_\lambda)$. Since $D_\lambda/\mathbf{m}_\lambda$ is simple artinian, we may identify the latter with $\mathbb{Z}$.

Corollary. *Suppose that $\lambda\in\mathfrak{h}^*$ is integral, dominant and regular. Then there is a commutative diagram:*

$$\begin{array}{ccc} K_0(\mathscr{X}) & \xrightarrow{\mathscr{X}_{\mathscr{O}(-\lambda-\rho)}} & \mathbb{Z} \\ \tau_\lambda\downarrow & & \| \\ K_0(D_\lambda) & \xrightarrow{s_\lambda} & \mathbb{Z}. \end{array}$$

The proof of the Corollary is a routine consequence of Theorem 2.9 and the translation principle. We leave this, and the appropriate generalisation of Theorem 2.6, to the reader.

3. Chern characters

3.1. In this section we show that the Chern character ch: $K_0(\mathscr{D}(\mathscr{X}))\to HC_0(\mathscr{D}(\mathscr{X}))$ essentially coincides with the state s: $K_0(\mathscr{D}(\mathscr{X}))\to\mathbb{Z}$. Recall first the definition of the Chern character. Let R be an algebra over a field k. Then one defines the *zero-th cyclic homology* group of R to be the k-vector space

$$HC_0(R)=R/[R,R],$$

where $[R,R]$ denotes the k-linear span of all $xy-yx$ for $x,y\in R$. There is a natural *trace map* (or Chern character) ch: $K_0(R)\to HC_0(R)$ defined as follows [9]:

If M is a finitely generated, projective R-module then $M\oplus K\cong R^n$, for some n, and so one can associate to M the idempotent matrix $e\in M_n(R)$ given by $e|_M=1_M$ and $e|_K=0$. Now $ch(M)$ is the trace of e modulo $[R,R]$,

$$ch(M)=\sum_{i=1}^{n} e_{ii}+[R,R]\in HC_0(R).$$

3.2. It is clear that $[\mathscr{D}(\mathscr{X}),\mathscr{D}(\mathscr{X})]\subseteq\mathbf{m}$. On the other hand, since $U(\mathfrak{g})=\mathscr{Z}(\mathfrak{g})\oplus[U(\mathfrak{g}),U(\mathfrak{g})]$ it follows that $\mathscr{D}(\mathscr{X})=\mathbb{C}\oplus[\mathscr{D}(\mathscr{X}),\mathscr{D}(\mathscr{X})]$. Hence

$$HC_0(\mathscr{D}(\mathscr{X}))=\mathscr{D}(\mathscr{X})/\mathbf{m}\cong\mathbb{C}.$$

We can thus regard ch as a map to the complex numbers.

Theorem. *The image of ch is $\mathbb{Z}$ and the diagram:*

$$\begin{array}{ccc} K_0(\mathscr{X}) & \xrightarrow{\tilde{\chi}} & \mathbb{Z} \\ \tau\downarrow & & \| \\ K_0(\mathscr{D}(\mathscr{X})) & \xrightarrow{ch} & \mathbb{Z}. \end{array}$$

commutes.

Proof. Let $\rho\colon \mathscr{D}(\mathscr{X})\to\mathscr{D}(\mathscr{X})/\mathbf{m}$ be the canonical homomorphism. It is easy to see that $s=ch\circ K_0(\rho)$. Moreover, the diagram

$$\begin{array}{ccc} K_0(\mathscr{D}(\mathscr{X})) & \xrightarrow{ch} & HC_0(\mathscr{D}(\mathscr{X}))\cong\mathbb{C} \\ {\scriptstyle K_0(\rho)}\downarrow & & {\scriptstyle HC_0(\rho)}\| \\ K_0(\mathscr{D}(\mathscr{X})/\mathbf{m}) & \xrightarrow{ch} & HC_0(\mathscr{D}(\mathscr{X})/\mathbf{m})\cong\mathbb{C} \end{array}$$

commutes and $HC_0(\rho)$ is the identity map. Thus the Chern character $ch\colon K_0(\mathscr{D}(\mathscr{X}))\to HC_0(\mathscr{D}(\mathscr{X}))=\mathbb{C}$ coincides with the map s. The result then follows from Theorem 2.9.

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