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Just-non-Cross varieties of groups

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L.G. Kovács and M.F. Newman have pointed out that a variety (of groups) *cannot* be generated by a single finite group (that is, is non-Cross) if and only if it contains a variety which is minimal with respect to this property: a so-called jnC (just-non-Cross) variety. They noted that the only jnC variety of infinite exponent is the variety $\underline{\underline{A}}$ of all abelian groups, and proved that the decomposable (that is, nontrivially factorisable) jnC varieties are the $\underline{\underline{A}}_p\underline{\underline{A}}_p$, $\underline{\underline{A}}_p\underline{\underline{T}}_q$ and $\underline{\underline{A}}_p\underline{\underline{A}}_q\underline{\underline{A}}_r$ with p, q and r distinct primes. Moreover, they conjectured that all soluble jnC varieties of finite exponent are decomposable.

The present thesis attacks the problem of classifying jnC varieties, and in particular reduces the above conjecture by showing that if there is a soluble jnC variety $\underline{\underline{V}}$ of finite exponent which is not decomposable, then for some pair p, q of (distinct) primes, $\underline{\underline{V}}$ has a nilpotent subvariety $\underline{\underline{N}}$ of class at least three and exponent a power of q, such that $\underline{\underline{V}} \le \underline{\underline{A}}_p\underline{\underline{N}}$. We extend the generality of this result by showing that a jnC variety is soluble of finite exponent if and only if it is reducible (that is, contained in a product of proper subvarieties).

Thus the determination of jnC varieties is reduced to (i) filling the gap remaining in the conjecture of Kovács and Newman, and (ii) finding all irreducible jnC varieties. We observe that a jnC variety is irreducible if and only if either it (a) is not locally finite, or (b) is locally finite and locally nilpotent but insoluble, or (c) is locally finite and contains infinitely many finite simple groups. The determination of irreducible jnC varieties is linked to some difficult

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problems. The existence of an irreducible of type (c) would simultaneously disprove the conjectures that there are only a finite number of finite simple groups of given exponent, and that there is a bound to the number of elements required to generate a finite simple group. The recently announced insolubility of $\underline{\underline{K}}_5$, however, does imply the existence of at least one irreducible of type (b). Irreducibles of type (a) of finite exponent may or may not be generated by their finite groups. If there is one which is not, it can only have finitely many subvarieties, contrary to the conjecture that all varieties with this property are Cross. If, on the other hand, there were an irreducible of type (a) of finite exponent e generated by its finite groups, then the restricted Burnside conjecture for exponent e would be false.