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Comment

Comment on 'Effect of ionization on ion acoustic solitary waves in a collisional dusty plasma' (*J. Plasma Phys.* 71, 519 (2005))

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In a recent study [1] the effect of ionization and losses on the propagation of ion acoustic solitary waves is investigated. For that purpose the corresponding source and sink terms of the form

$$Q_{\rm i} - \nu_{\rm L} n_{\rm i}, \tag{1}$$

are included in the right-hand side of the ion continuity equation. The related contribution of the source–sink effects in the ion momentum equation is then given as the above source–sink term multiplied by the ion velocity, so that the ion momentum equation used in [1] becomes

$$n_{\rm i}\left(\frac{\partial v_{\rm i}}{\partial t} + v_{\rm i}\frac{\partial v_{\rm i}}{\partial x}\right) = -n_{\rm i}\frac{\partial \phi}{\partial x} - \sigma_{\rm i}\frac{\partial n_{\rm i}}{\partial x} - \nu_{\rm i} n_{\rm i} v_{\rm i} - (Q_{\rm i} - \nu_{\rm L} n_{\rm i}) v_{\rm i}. \tag{2}$$

The objective of this Comment is to show that the last term in (2) cancels out exactly, as will be shown in the further text, so that any effect originating from this term is spurious. In addition, this term implies that the velocity of the neutrals that become ions, before the impact ionization (or whatever the cause of its ionization may be), is equal to the velocity of ions that become neutrals, since this source–sink term is multiplied by the same velocity. A similar improper model has been used previously in the literature (see the references cited in [1]). It is remarked that the proper inclusion of the source–sink effects has been described in the literature before, although it seems to remain unnoticed. Therefore, it is timely to address this issue in order to provide a formally correct model for future studies.

Normally, in deriving the fluid equations that describe waves in multi-component plasmas which involve elastic as well as inelastic collisions, one follows a well known procedure. One starts from the corresponding kinetic equation which can be written in the form

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}\cdot\frac{\partial f_\alpha}{\partial \mathbf{r}} + \frac{\mathbf{F}_\alpha}{m_\alpha}\cdot\frac{\partial f_\alpha}{\partial \mathbf{v}} = \left(\frac{\partial f_\alpha}{\partial t}\right)_{\rm el} + \left(\frac{\partial f_\alpha}{\partial t}\right)_{\rm inel}. \tag{3}$$

Here, on the right-hand side $(\partial f_\alpha/\partial t)_{\rm el}$ is the change in the unit time of the number of particles α in the six-dimensional phase space $\mathbf{r}, \mathbf{v}$ due to elastic collisions, and the second term is the corresponding change due to inelastic collisions (in principle any of the aforesaid). $\mathbf{F}_\alpha$ is the total average force in the phase space acting on particles of the species α.

Following the standard procedure, the macroscopic continuity equation for the plasma species α is obtained after an appropriate integration of (3) over all velocities yielding

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{v}_\alpha) = \int d^3\mathbf{v} \left(\frac{\partial f_\alpha}{\partial t}\right)_{\text{inel}} = \frac{\partial}{\partial t}\left(\int f_\alpha d^3\mathbf{v}\right)_{\text{inel}} = \left(\frac{\partial n_\alpha}{\partial t}\right)_{\text{inel}} \equiv S_\alpha. \quad (4)$$

Here, $S_\alpha \equiv S_{\alpha,\text{source}} + S_{\alpha,\text{sink}}$ is the appropriate sum of all source/sink terms due to the inelastic collisions. The elastic collisions do not cause a change of the mass in a plasma unit volume. In a stationary equilibrium (the subscript 0 will be used to describe equilibrium quantities) plasma we have

$$S_{\alpha,\text{source},0} + S_{\alpha,\text{sink},0} = 0. \quad (5)$$

From the same kinetic equation, after multiplication with $\mathbf{v}d^3\mathbf{v}$ and integration over all velocities, one obtains the momentum (motion) equation

$$\frac{\partial(n_\alpha \mathbf{v}_\alpha)}{\partial t} + \nabla \cdot \{n_\alpha \mathbf{v}_\alpha, \mathbf{v}_\alpha\} = \mathbf{\Sigma}_\alpha, \quad (6)$$

where $\mathbf{\Sigma}_\alpha$ is the summation of all forces acting on an elementary volume of the plasma species α, including the friction force caused by elastic collisions, i.e. the term $(\partial f_\alpha/\partial t)_{\text{el}}$, as well as the term caused by inelastic collisions $(\partial f_\alpha/\partial t)_{\text{inel}}$.

Here, $\mathbf{v}_\alpha$ is now the macroscopic velocity of the species α. The Poisson bracket in (6) is subject to a transformation of the form $\nabla \cdot \{\mathbf{a}, \mathbf{b}\} \equiv \mathbf{b}(\nabla \cdot \mathbf{a}) + (\mathbf{a} \cdot \nabla)\mathbf{b}$, so that (6) becomes

$$n_\alpha \left[\frac{\partial \mathbf{v}_\alpha}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\mathbf{v}_\alpha\right] + \mathbf{v}_\alpha \left[\frac{\partial n_\alpha}{\partial t} + \nabla(n_\alpha \mathbf{v}_\alpha)\right] = \mathbf{\Sigma}_\alpha. \quad (7)$$

Consequently, using (4), the equation of motion reduces to

$$n_\alpha \left[\frac{\partial \mathbf{v}_\alpha}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\mathbf{v}_\alpha\right] + \mathbf{v}_\alpha S_\alpha = \cdots + \mathbf{v}_\alpha S_\alpha + n_\alpha \left(\frac{\partial \mathbf{v}_\alpha}{\partial t}\right)_{\text{inel}}, \quad (8)$$

since Σ_α in (6) contains the term

$$\int \mathbf{v}d^3\mathbf{v}\left(\frac{\partial f_\alpha}{\partial t}\right)_{\text{inel}} = \frac{\partial}{\partial t}\left(\int f_\alpha \mathbf{v} d^3\mathbf{v}\right)_{\text{inel}}$$
$$= \left[\frac{\partial(n_\alpha \mathbf{v}_\alpha)}{\partial t}\right]_{\text{inel}} = \left(\frac{\partial n_\alpha}{\partial t}\right)_{\text{inel}} \mathbf{v}_\alpha + n_\alpha \left(\frac{\partial \mathbf{v}_\alpha}{\partial t}\right)_{\text{inel}}. \quad (9)$$

It is seen that the term $\mathbf{v}_\alpha S_\alpha$ in the left-hand side of (8) cancels out with the identical term on the right-hand side (given in (9)). Hence, the only term that should affect the dynamics is entirely due to the inelastic collisions which remain under the term Σ_α in the equation of motion

$$n_\alpha \left(\frac{\partial \mathbf{v}_\alpha}{\partial t}\right)_{\text{inel}} \equiv c_{\text{r}} n_\alpha \mathbf{v}_\alpha. \quad (10)$$

Note that exactly the same procedure is presented in a classical book [2].

As for the specific concrete expressions for the source–sink contribution (10) in the momentum equation, we point out [3] and [4] where the equations are derived from

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the first principles. In [3] (using the same notation, which is, however, equivalent to our notation used above) the ion momentum equation is written in the form

$$\frac{\partial}{\partial t}(MN\mathbf{V}) + \nabla \cdot (NT\mathbf{I} + MN\mathbf{vv} + \pi_{\rm i}) = \mathbf{F} + eN(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \nu_x Mn(\mathbf{V} - \mathbf{U}) - \nu_{\rm r} MN\mathbf{V} + \nu_z Mn\mathbf{U}. \tag{11}$$

Here $\mathbf{I}$ is the unit diad, $\nu_{\rm r}$ denotes the recombination rate, ν_z the electron impact ionization rate, ν_x the charge exchange, and $n, \mathbf{U}$ the number density and velocity of neutrals. The corresponding continuity equation is given by

$$\frac{\partial N}{\partial t} + \nabla \cdot (N\mathbf{V}) = -\nu_{\rm r} N + \nu_z n. \tag{12}$$

Combining these two equations with the cancellation as in the previous text, we obtain the following two terms on the right-hand side of the momentum equation

$$\cdots = -\nu_x Mn(\mathbf{V} - \mathbf{U}) - \nu_z Mn(\mathbf{V} - \mathbf{U}). \tag{13}$$

The equations should be supplemented by an appropriate set of equations for neutrals, especially if neutrals are created/lost whenever ions are lost/created; more details on this issue can be found in [3] and [4]. This inclusion is in fact simple because the source/sink terms enter with opposite signs in the equations for ions and neutrals. One can argue that (11) and (12) are also based on some model and limited by assumptions used in the derivations. Yet, the point is that the source–sink term used in the momentum equation of the commented paper is not correct. Our Comment suggests that using the source–sink term approach in studying physical phenomena, such as the ionization instability, is suspect and more care must be taken to provide the necessary physical insight.

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