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Attention is called to the fact that the JOURNAL invites submission of (1) original technical papers in the field of symbolic logic, (2) expository papers in this field, (3) philosophical papers which bear upon this field or make use of its methods, and (4) studies in the history of logic in which modern technical developments are taken into account.

Authors and publishers are requested to send, for review, copies of books and reprints of articles in journals to *Alonzo Church, Fine Hall, Princeton University, Princeton, New Jersey.*

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# ERRATA

## VOLUME 2

Page 47, line 9. For "intuitionistischer", read "intuitionistischer".

## VOLUME 5

Page 166, line 15 from the bottom. For "metafizyka", read „metafizyką”.

## VOLUME 13

Page 159, line 20. For "Hungtington's", read "Huntington's".

## VOLUME 16

Page 83, line 14. For "wenn  $M$  eine", read "wenn  $M'$  eine".

Page 84, line 17. For " $-b$ ", read " $< b$ ".

Page 87, line 2. For "Reinhenfolge", read "Reihenfolge" and replace the comma at the end of the line by a period.

Page 98, line 28. For "Typ  $c$ ", read "Typ  $r$ ".

Page 156, line 14 from the bottom. For  $K_3$ , read  $K_3$ .

Page 317, second column, line 7 from the bottom. For "infinto", read "infinito".

Page 327, column 2, line 9 from the bottom. For the period at the end of the line, read a comma.

## VOLUME 17

Page 69, line 12. Delete the first right parenthesis.

Page 79, line 19 from the bottom. For "vol. 28", read "vol. 29".

Page 146, line 14. For "aglebra", read „algebra”.

Page 154, line 5 from the bottom. For "p. 72", read "pp. 71–72".

Page 223, line 12 from the bottom. For "relations", read "relations".

Page 238. In the upper left corner. For "Number 9", read "Number 4".

Page 271, line 5. For "ZÉGALKIN", read "ŽÉGALKIN".

Page 259, bottom line. Delete the initial "H".

Page 280, line 24 from the bottom. After "Large-Scale", insert "Digital".

## VOLUME 18

Page 2, line 2 from bottom. For

$$(x)a\{p \cdot [x \frown w]p \cdot [x \frown L]p \cdot [x \frown R]p\},$$

read

$$(x)a\left\{p \cdot \left[ \overset{x}{x \frown w} \right] p \cdot \left[ \overset{x}{x \frown L} \right] p \cdot \left[ \overset{x}{x \frown R} \right] p \right\}$$

Page 5, line 20 from bottom. For  $\begin{bmatrix} a \\ x \end{bmatrix}$ , read  $\begin{bmatrix} x \\ a \end{bmatrix}$ .

Page 38, line 10. For  $\langle M \rangle$ , read  $[M]$ .

Page 54, line 7. Change "recursion" to "induction".

Page 56, line 17. Change "recursion" to "induction".

Page 61, lines 2 and 3. For  $A$  and  $B$ , read  $A$  and  $B$ .

Page 62, line 7. Same correction as on page 61.

Page 65, lines 15, 16 and 18. For  $X$ , read  $X$  (five times), and for  $Y$ , read  $Y$  (once).

Page 66, last line. For "of", read "if".

Page 68, line 5. For "mimeographed", read "photo-offset".

Page 75, line 14. The subscript after the last  $A$  in the line should be  $2^n$ .

Page 75, line 22. The subscript after the second  $i$  should be  $2^n$ .

## ERRATA

- Page 78, line 4 from the bottom. For "1942", read "1952".
- Page 84, line 17 from the bottom. For "seen", read "seem".
- Page 91, line 30. Delete the parenthesis enclosing 4916.1.
- Page 93, line 6. For the comma, read a period.
- Page 95, line 24 from the bottom. For the comma, read a period.
- Page 96, line 2. For the hyphen at the end of the line, read a dash.
- Page 149, lines 4, 5 and 8 from the bottom (of text), for *V* read *V*.
- Page 185, line 4 from the bottom. For "the oremin", read "theorem in".
- Page 190, line 12 from the bottom. For "Stumblingblock", read "stumblingblock".
- Page 257, at the end of line 27 from the bottom. For *e* read *é*.
- Page 259, line 17. For the second occurrence of "'English'", read "'Englisch'".
- Page 262, line 21. After "subsystems", insert "of".