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GLOBAL ASYMPTOTIC STABILITY FOR GENERAL SYMMETRIC RATIONAL DIFFERENCE EQUATIONS

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Abstract

We investigate the global asymptotic stability for positive solutions to a class of general symmetric rational difference equations and prove that the unique positive equilibrium 1 of the general symmetric rational difference equations is globally asymptotically stable. As a special case of our result, we solve the conjecture raised by Berenhaut, Foley and Stević ['The global attractivity of the rational difference equation $y_n = (y_{n-k} + y_{n-m})/(1 + y_{n-k}y_{n-m})$', *Appl. Math. Lett.* **20** (2007), 54–58].

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1. Introduction

Recently there has been great interest in studying the qualitative properties of rational differences equations. Some prototypes for development of the basic theory for the global behaviour of nonlinear difference equations of order greater than 1 come from the results for rational equations (see, for example, [1, 4–6] and the references therein).

In 2007, Berenhaut *et al.* [2] studied the global asymptotic stability for positive solutions to the difference equations

$$y_n = \frac{y_{n-k} + y_{n-m}}{1 + y_{n-k}y_{n-m}}, \quad n \in N_0 = \{0, 1, 2, \ldots\}$$

with $y_{-m}, y_{-m+1}, \ldots, y_{-1} \in (0, \infty)$ and $1 \le k < m$. At end of the paper, they raised two conjectures as follows.

CONJECTURE 1.1. Suppose that $\{y_i\}$ satisfies

$$y_n = \frac{y_{n-k}y_{n-l}y_{n-m} + y_{n-k} + y_{n-l} + y_{n-m}}{y_{n-k}y_{n-l} + y_{n-k}y_{n-m} + y_{n-l}y_{n-m} + 1}, \quad n \in N_0,$$

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with $y_{-m}, y_{-m+1}, \ldots, y_{-1} \in (0, \infty)$ and $1 \le k < l < m$. Then the sequence $\{y_i\}$ converges to the unique equilibrium 1.

CONJECTURE 1.2. Suppose that v is odd and $1 \le k_1 < k_2 < \cdots < k_v$, and define $S = \{1, 2, \ldots, v\}$. If $\{y_i\}$ satisfies

$$y_n = \frac{f_1(y_{n-k_1}, y_{n-k_2}, \ldots, y_{n-k_v})}{f_2(y_{n-k_1}, y_{n-k_2}, \ldots, y_{n-k_v})}, \quad n \in N_0,$$

where

$$f_1(x_1, x_2, \ldots, x_v) = \sum_{\substack{r=1 \\ r \text{ odd}}}^{v} \sum_{\substack{\{t_1,t_2,\ldots,t_r\} \subset S \\ t_1 < t_2 < \cdots < t_r}} x_{t_1} x_{t_2} \cdots x_{t_r}$$

and

$$f_2(x_1, x_2, \ldots, x_v) = 1 + \sum_{\substack{r=2 \\ r \text{ even}}}^{v-1} \sum_{\substack{\{t_1,t_2,\ldots,t_r\} \subset S \\ t_1 < t_2 < \cdots < t_r}} x_{t_1} x_{t_2} \cdots x_{t_r}$$

with $y_{-k_v}, y_{-k_v+1}, \ldots, y_{-1} \in (0, \infty)$, then the sequence $\{y_i\}$ converges to the unique equilibrium 1.

Conjecture 1.1 was solved by Berenhaut and Stević [3]. However, to the best of our knowledge, Conjecture 1.2 has not hitherto been solved.

Let $v \ge 2$ be an arbitrary integer. Denote by v_o (v_e) the largest odd (even) number not greater than v. We consider the general symmetric rational difference equation

$$y_n = \frac{f_1(y_{n-k_1}, y_{n-k_2}, \ldots, y_{n-k_v})}{f_2(y_{n-k_1}, y_{n-k_2}, \ldots, y_{n-k_v})}, \quad n = 0, 1, 2, \ldots, \tag{1.1}$$

where $1 \le k_1 < k_2 < \cdots < k_v$ and $y_{-k_v}, y_{-k_v+1}, \ldots, y_{-1} \in (0, \infty)$, and for $S = \{1, 2, \ldots, v\}$,

$$f_1(x_1, x_2, \ldots, x_v) = \sum_{\substack{r=1 \\ r \text{ odd}}}^{v_o} \sum_{\substack{\{t_1,t_2,\ldots,t_r\} \subset S \\ t_1 < t_2 < \cdots < t_r}} x_{t_1} x_{t_2} \cdots x_{t_r} \tag{1.2}$$

and

$$f_2(x_1, x_2, \ldots, x_v) = 1 + \sum_{\substack{r=2 \\ r \text{ even}}}^{v_e} \sum_{\substack{\{t_1,t_2,\ldots,t_r\} \subset S \\ t_1 < t_2 < \cdots < t_r}} x_{t_1} x_{t_2} \cdots x_{t_r}. \tag{1.3}$$

In this paper, we investigate the global asymptotic stability for positive solutions to the general symmetric rational difference equation (1.1) and prove that the unique positive equilibrium 1 of the general symmetric rational difference equation (1.1) is globally asymptotically stable. As a special case of our result, we solve Conjecture 1.2 raised by Berenhaut *et al.* [2].

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2. Preliminaries

For the sake of simplicity, for $\bar{S} \subset S$ let $o(\bar{S})$ denote the largest odd number, and $e(\bar{S})$ the largest even number, not greater than the number of the elements in $\bar{S}$,

$$f_1(\bar{S}) = \sum_{\substack{r=1 \\ r \text{ odd}}}^{o(\bar{S})} \sum_{\substack{\{t_1,t_2,\ldots,t_r\}\subset\bar{S} \\ t_1<t_2<\cdots<t_r}} x_{t_1}x_{t_2}\cdots x_{t_r} \tag{2.1}$$

and

$$f_2(\bar{S}) = 1 + \sum_{\substack{r=2 \\ r \text{ even}}}^{e(\bar{S})} \sum_{\substack{\{t_1,t_2,\ldots,t_r\}\subset\bar{S} \\ t_1<t_2<\cdots<t_r}} x_{t_1}x_{t_2}\cdots x_{t_r}. \tag{2.2}$$

LEMMA 2.1. *Let*

$$f(x_1, x_2, \ldots, x_v) = \frac{f_1(x_1, x_2, \ldots, x_v)}{f_2(x_1, x_2, \ldots, x_v)}, \tag{2.3}$$

where f_1 and f_2 are defined by (1.2) and (1.3), respectively. Then

$$\frac{\partial f}{\partial x_i} = (f_2)^{-2} \prod_{j\neq i}(1 - x_j^2), \quad i = 1, 2, \ldots, v.$$

PROOF. For any $i \in \{1, 2, \ldots, v\}$, let $S_i = S\backslash\{i\}$. From (2.1) and (2.2), it is easy to obtain that

$$f_1(S) = x_i f_2(S_i) + f_1(S_i) \tag{2.4}$$

and

$$f_2(S) = x_i f_1(S_i) + f_2(S_i). \tag{2.5}$$

For any $j \neq i$, set $S_{ij} = S_i\backslash\{j\}$. Then it follows from (2.4) and (2.5) that

$$f_1(S_i) = x_j f_2(S_{ij}) + f_1(S_{ij}) \tag{2.6}$$

and

$$f_2(S_i) = x_j f_1(S_{ij}) + f_2(S_{ij}). \tag{2.7}$$

Now from (2.4)–(2.7),

$$\begin{aligned}
\frac{\partial f_1(S)}{\partial x_i} f_2(S) - \frac{\partial f_2(S)}{\partial x_i} f_1(S) &= f_2(S_i)(x_i f_1(S_i) + f_2(S_i)) \\
&\qquad - f_1(S_i)(x_i f_2(S_i) + f_1(S_i)) \\
&= f_2^2(S_i) - f_1^2(S_i) \\
&= (x_j f_1(S_{ij}) + f_2(S_{ij}))^2 - (x_j f_2(S_{ij}) + f_1(S_{ij}))^2 \\
&= (1 - x_j^2)(f_2^2(S_{ij}) - f_1^2(S_{ij})).
\end{aligned}$$

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Noticing that $j \neq i$ was arbitrary and that $f_2^2(S_{ij}) - f_1^2(S_{ij})$ does not depend on i or j, it is easy to see that

$$\frac{\partial f_1(S)}{\partial x_i} f_2(S) - \frac{\partial f_2(S)}{\partial x_i} f_1(S) = \prod_{j \neq i} (1 - x_j^2)$$

and so

$$\frac{\partial f(S)}{\partial x_i} = (f_2(S))^{-2} \prod_{j \neq i} (1 - x_j^2).$$

This completes the proof. □

By (1.1), (2.4) and (2.5), it is easy to obtain that $\bar{x} = 1$ is the unique positive equilibrium of (1.1). The following concepts can be found in [7]. A subsequence $\{y_n\}$ of a solution of (1.1) is called *trivial* if it is eventually identical to the equilibrium 1. Otherwise it is *nontrivial*. The *sign* of a subsequence $\{y_n\}$ of a solution of (1.1) is defined as the sequence which is composed of the signs of the terms of $\{y_n - 1\}$. If $y_n - 1 = 0$, then the sign of the nth term of $\{y_n - 1\}$ is denoted by 0.

Let $\{y_n\}$ be a positive solution of (1.1), $m = k_v$ and $A_i = \{y_{nm+i}\}_{n=-1}^{\infty}$ for $i = 0, 1, \ldots, m-1$. Then $\{y_n\}$ is divided into m subsequences $A_0, A_1, \ldots, A_{m-1}$. Set $A_i = B^i \cup B_i$ with $B^i = \{y \in A_i \mid y \geq 1\}$ and $B_i = \{y \in A_i \mid y < 1\}$. For the sake of simplicity, for $n \geq 0$, $j = 1, 2$ and $i = 0, 1, \ldots, m-1$, let

$$f_j(n, i) = f_j(y_{nm+i-k_1}, y_{nm+i-k_2}, \ldots, y_{nm+i-k_v}) \tag{2.8}$$

and

$$f_j(n, i, v) = f_j(y_{nm+i-k_1}, y_{nm+i-k_2}, \ldots, y_{nm+i-k_{v-1}}), \tag{2.9}$$

where f_1 and f_2 on the right-hand side of (2.8) (or (2.9)) are given by (2.1) and (2.2) with $\bar{S} = S$ (or $\bar{S} = S \backslash \{v\}$), respectively. It follows from (2.4), (2.5), (2.8) and (2.9) that, for any $n \geq 0$,

$$f_1(n, i) = y_{(n-1)m+i} f_2(n, i, v) + f_1(n, i, v) \tag{2.10}$$

and

$$f_2(n, i) = y_{(n-1)m+i} f_1(n, i, v) + f_2(n, i, v). \tag{2.11}$$

LEMMA 2.2. *For each $i \in \{0, 1, \ldots, m-1\}$, there exists some $L_i \geq 1$ such that B^i converges to L_i if B^i is infinite, and B_i converges to $1/L_i$ if B_i is infinite.*

PROOF. If A_i is trivial, the proof is trivial.

If A_i is nontrivial, we can assert that $y_{nm+i} \neq 1$ for all $n \geq -1$. Otherwise, $y_{\bar{n}m+i} = 1$ for some $\bar{n} \geq -1$, and by (2.8)–(2.11),

$$\begin{aligned} y_{(\bar{n}+1)m+i} &= \frac{f_1(\bar{n}+1, i)}{f_2(\bar{n}+1, i)} \\ &= \frac{y_{\bar{n}m+i} f_2(\bar{n}+1, i, v) + f_1(\bar{n}+1, i, v)}{y_{\bar{n}m+i} f_1(\bar{n}+1, i, v) + f_2(\bar{n}+1, i, v)} \\ &= 1, \end{aligned}$$

so that A_i is trivial, a contradiction. It follows that $y_{nm+i} = 1$ for $n \geq \bar{n}$ by induction, which contradicts the fact that A_i is nontrivial. Thus B^i is in fact $\{y \in A_i | y > 1\}$, and it follows from (2.8)–(2.11) that, for any $j = 0, 1, \ldots, m-1$ and $n \geq 0$,

$$\begin{aligned} y_{nm+j} - y_{(n-1)m+j} &= \frac{f_1(n, j) - y_{(n-1)m+j} f_2(n, j)}{f_2(n, j)} \\ &= \frac{y_{(n-1)m+j} f_2(n, j, v) + f_1(n, j, v) - y_{(n-1)m+j}(y_{(n-1)m+j} f_1(n, j, v) + f_2(n, j, v))}{f_2(n, j)} \\ &= \frac{f_1(n, j, v)}{f_2(n, j)}(1 - y_{(n-1)m+j})(1 + y_{(n-1)m+j}) \neq 0. \end{aligned}$$

This implies that

$$(y_n - y_{n-m})(y_{n-m} - 1) < 0, \quad n = 0, 1, 2, \ldots. \tag{2.12}$$

We suppose that $y_{-m+i} > 1$. The proof of the case $y_{-m+i} < 1$ is analogous, so we omit it. Assume that the sign of A_i is $q_1^+, q_2^-, q_3^+, q_4^-, \ldots$, where q_1^+ means q_1 successive positive signs and q_2^- means q_2 successive negative signs. We consider two cases as follows.

Case 1. The sign sequence is finite; that is, there exists a positive N such that $q_N = \infty$. Without loss of generality we may assume $N = 1$, that is, $q_1 = \infty$ and B_i is empty. By (2.12),

$$y_{-m+i} > y_i > y_{m+i} > \cdots$$

and so $\{y_{nm+i}\}$ is decreasing with lower bound 1. This implies that $\lim_{n\to\infty} y_{nm+i} = L_i$ for some $L_i \geq 1$.

Case 2. The sign sequence is infinite. Then each q_j is a positive integer. Letting

$$s(0) = -1,\ s(n) = s(n-1) + q_n, \quad n = 1, 2, \ldots,$$

then

$$B^i = \{y_{(s(2n)+j)m+i} \mid n \geq 0,\ j = 0, 1, \ldots, q_{2n+1} - 1\} \tag{2.13}$$

and

$$B_i = \{y_{(s(2n+1)+j)m+i} \mid n \geq 0,\ j = 0, 1, \ldots, q_{2n+2} - 1\}. \tag{2.14}$$

It is easy to obtain from (2.12) that

$$y_{s(2n)m+i} > y_{(s(2n)+1)m+i} > \cdots > y_{(s(2n+1)-1)m+i} > 1 \tag{2.15}$$

and

$$y_{s(2n+1)m+i} < y_{(s(2n+1)+1)m+i} < \cdots < y_{(s(2n+2)-1)m+i} < 1 \tag{2.16}$$

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for $n \geq 0$. By (2.8)–(2.11) and (2.15),

$$\begin{aligned} y_{s(2n+1)m+i} &= \frac{f_1(s(2n+1), i)}{f_2(s(2n+1), i)} \\ &= \frac{y_{(s(2n+1)-1)m+i} f_2(s(2n+1), i, v) + f_1(s(2n+1), i, v)}{y_{(s(2n+1)-1)m+i} f_1(s(2n+1), i, v) + f_2(s(2n+1), i, v)} \\ &> \frac{f_2(s(2n+1), i, v) + f_1(s(2n+1), i, v)}{y_{(s(2n+1)-1)m+i}(f_1(s(2n+1), i, v) + f_2(s(2n+1), i, v))} \\ &= \frac{1}{y_{(s(2n+1)-1)m+i}} \end{aligned} \tag{2.17}$$

for $n \geq 0$. Similarly, by (2.8)–(2.11) and (2.16), we can get

$$y_{s(2n+2)m+i} < \frac{1}{y_{(s(2n+2)-1)m+i}} \tag{2.18}$$

for $n \geq 0$. Define a sequence $\{x_n\}$ by

$$x_n = \begin{cases} y_n & \text{if } y_n \in B^i, \\ \dfrac{1}{y_n} & \text{if } y_n \in B_i. \end{cases}$$

Then (2.13)–(2.18) imply that $\{x_n\}$ is decreasing with lower bound 1 and so $\{x_n\}$ has a limit $L_i \geq 1$ as desired. This completes the proof. □

For our main result, we also need the following lemma.

LEMMA 2.3 [5, Corollary 1.3.2]. *Assume that $F = F(u_0, \ldots, u_k)$ is a C^1 function and let $\bar{x}$ be an equilibrium of the rational difference equations*

$$x_{n+1} = F(x_n, x_{n-1}, \ldots, x_{n-k}), \quad n = 0, 1, 2, \ldots, \tag{2.19}$$

where $k \geq 0$ is an integer. If all the roots of the polynomial equation

$$\lambda^{k+1} - \sum_{i=0}^{k} \frac{\partial F}{\partial u_i}(\bar{x}, \ldots, \bar{x})\lambda^{k-i} = 0$$

lie in the open unit disk $|\lambda| < 1$, then the equilibrium $\bar{x}$ of (2.19) is asymptotically stable.

3. Main Results

THEOREM 3.1. *The unique positive equilibrium 1 of (1.1) is globally asymptotically stable.*

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PROOF. The linearized equation of (1.1) with respect to the positive equilibrium $\bar{x} = 1$ is

$$y_n = \sum_{i=1}^{v} \frac{\partial f(1, 1, \ldots, 1)}{\partial x_i} y_{n-k_i}, \quad n = 1, 2, \ldots,$$

where $f = f(x_1, \ldots, x_v)$ is given by (2.3). By Lemma 2.1, we know that

$$\frac{\partial f(1, 1, \ldots, 1)}{\partial x_i} = 0, \quad i = 1, 2, \ldots, v.$$

It follows from Lemma 2.3 that $\bar{x} = 1$ is locally asymptotically stable.

Now it is sufficient to prove that 1 is a global attractor for the positive solutions of (1.1). Moreover, by Lemma 2.2, it is sufficient to prove that $L_i = 1$ for $i = 0, 1, \ldots, m-1$, where L_i is the same as in Lemma 2.2.

Let $i \in \{0, 1, \ldots, m-1\}$. Without loss of generality, we may assume that B^i is infinite. Thus, Lemma 2.2 implies that, for any $\varepsilon > 0$ sufficiently small, there exist some $\bar{n}$ and

$$P_l \in \{L_0, \ldots, L_{m-1}, L_0^{-1}, \ldots, L_{m-1}^{-1}\}, \quad l = 0, 1, \ldots, m-1,$$

such that $y_{\bar{n}m+i} \in B^i$,

$$|y_{\bar{n}m+i} - L_i| < \varepsilon, \quad |y_{\bar{n}m+i-k_l} - P_l| < \varepsilon \tag{3.1}$$

with

$$|y_{(\bar{n}-1)m+i} - L_i| < \varepsilon \quad \text{if } y_{(\bar{n}-1)m+i} \in B^i \tag{3.2}$$

and

$$|y_{(\bar{n}-1)m+i} - 1/L_i| < \varepsilon \quad \text{if } y_{(\bar{n}-1)m+i} \in B_i. \tag{3.3}$$

For the sake of simplicity, let

$$f_j(P, \pm\varepsilon) = f_j(P_1 \pm \varepsilon, P_2 \pm \varepsilon, \ldots, P_{v-1} \pm \varepsilon), \quad j = 1, 2, \tag{3.4}$$

and

$$f_j(P) = f_j(P_1, P_2, \ldots, P_{v-1}), \quad j = 1, 2, \tag{3.5}$$

where f_1 and f_2 on the right-hand side of the above two equations are the same as in (2.9). Notice that f_1 and f_2 are increasing. If $y_{(\bar{n}-1)m+i} \in B^i$, then (2.8)–(2.11), (3.1), (3.2) and (3.4) imply that

$$\begin{aligned} L_i - \varepsilon &< y_{\bar{n}m+i} \\ &= \frac{f_1(\bar{n}, i)}{f_2(\bar{n}, i)} \\ &= \frac{y_{(\bar{n}-1)m+i} f_2(\bar{n}, i, v) + f_1(\bar{n}, i, v)}{y_{(\bar{n}-1)m+i} f_1(\bar{n}, i, v) + f_2(\bar{n}, i, v)} \\ &\le \frac{(L_i + \varepsilon) f_2(P, +\varepsilon) + f_1(P, +\varepsilon)}{(L_i - \varepsilon) f_1(P, -\varepsilon) + f_2(P, -\varepsilon)} \end{aligned} \tag{3.6}$$

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and

$$\begin{aligned} L_i + \varepsilon > y_{\bar{n}m+i} \\ &= \frac{y_{(\bar{n}-1)m+i} f_2(\bar{n}, i, v) + f_1(\bar{n}, i, v)}{y_{(\bar{n}-1)m+i} f_1(\bar{n}, i, v) + f_2(\bar{n}, i, v)} \\ &\geq \frac{(L_i - \varepsilon) f_2(P, -\varepsilon) + f_1(P, -\varepsilon)}{(L_i + \varepsilon) f_1(P, +\varepsilon) + f_2(P, +\varepsilon)}. \end{aligned} \tag{3.7}$$

Since ε is arbitrary and f_1, f_2 are continuous, it follows from (3.6), (3.7) and (3.5) that

$$L_i = \frac{L_i f_2(P) + f_1(P)}{L_i f_1(P) + f_2(P)},$$

which yields that $L_i = 1$. Similarly, if $y_{(\bar{n}-1)m+i} \in B_i$, by (2.8)–(2.11), (3.1), (3.3)–(3.5) we obtain

$$L_i = \frac{(1/L_i) f_2(P) + f_1(P)}{(1/L_i) f_1(P) + f_2(P)}.$$

This also leads to $L_i = 1$. This completes the proof. □

Letting $v = 3$ in Theorem 3.1, we have the following corollary.

COROLLARY 3.2. *Suppose that* $\{y_i\}$ *satisfies*

$$y_n = \frac{y_{n-k} y_{n-l} y_{n-m} + y_{n-k} + y_{n-l} + y_{n-m}}{y_{n-k} y_{n-l} + y_{n-k} y_{n-m} + y_{n-l} y_{n-m} + 1}, \quad n \in N_0,$$

with $y_m, y_{-m+1}, \ldots, y_1 \in (0, \infty)$ *and* $1 \leq k < l < m$. *Then the sequence* $\{y_i\}$ *converges to the unique equilibrium* 1.

For $v \geq 3$ and v odd in Theorem 3.1, we have the following corollary.

COROLLARY 3.3. *Suppose that* v *is odd,* $1 \leq k_1 < k_2 < \cdots < k_v$, *and* $S = \{1, 2, \ldots, v\}$. *If* $\{y_i\}$ *satisfies*

$$y_n = \frac{f_1(y_{n-k_1}, y_{n-k_2}, \ldots, y_{n-k_v})}{f_2(y_{n-k_1}, y_{n-k_2}, \ldots, y_{n-k_v})}, \quad n \in N_0,$$

where

$$f_1(x_1, x_2, \ldots, x_v) = \sum_{\substack{r=1 \\ r \text{ odd}}}^{v} \sum_{\substack{\{t_1, t_2, \ldots, t_r\} \subset S \\ t_1 < t_2 < \cdots < t_r}} x_{t_1} x_{t_2} \cdots x_{t_r}$$

and

$$f_2(x_1, x_2, \ldots, x_v) = 1 + \sum_{\substack{r=2 \\ r \text{ even}}}^{v-1} \sum_{\substack{\{t_1, t_2, \ldots, t_r\} \subset S \\ t_1 < t_2 < \cdots < t_r}} x_{t_1} x_{t_2} \cdots x_{t_r}$$

with $y_{-k_v}, y_{-k_v+1}, \ldots, y_{-1} \in (0, \infty)$, *then the sequence* $\{y_i\}$ *converges to the unique equilibrium* 1.

REMARK 3.4. Corollary 3.3 solves Conjecture 1.2 raised by Berenhaut *et al.* [2].

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