CORRESPONDENCE.

ON A METHOD OF USING THE "TABLE OF QUARTER SQUARES."

To the Editor of the Assurance Magazine.

SIR,—Allow me to call your attention, and that of your readers, to a method of using the "Table of Quarter Squares" computed by me and published in 1855,* which, although not pointed out in the introduction to that work, is one which will extend the usefulness of the table, and, in certain cases, materially diminish labour.

A constant factor is of frequent occurrence in some of the calculations of an actuary; for my present purpose it will be sufficient to instance the expression for the value of a policy when the premium is just due and not paid, viz.—

$$1-\frac{1+a_{m+n}}{1+a_m}=1-(1+a_{m+n})\left(\frac{1}{1+a_m}\right),$$

in which the term $\frac{1}{1+a_m}$ is constant, whatever may be the value of the other factor. Supposing a computer to be about to form a table of the values of a policy for a series of years—say from 1 to n years—it will manifestly be advantageous if he can save one reference to the table in each calculation.

* *Table of Quarter Squares of all Integer Numbers up to* 100,000, *by which the Product of Two Factors may be found by the aid of Addition and Subtraction alone.* C. & E. Layton, Fleet Street.

This saving can be effected, where one of the factors is a constant; as may be seen by reference to the equation [2], at p. xiv. of that work, where it is shown that

$$ab=2\left(\frac{a^2}{4}+\frac{b^2}{4}-\frac{(a-b)^2}{4}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [2]$$

But this expression requires the tabular results in every case to be multiplied by 2; and the object of the present communication is to show that where a constant factor occurs, as in the instance above quoted, the trouble of doubling the tabular results may be saved. For, transforming the above expression [2], by doubling the factor a, we obtain

$$(2a+b)^2=4a^2+4ab+b^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [A]$$

$$(2a-b)=4a^2-4ab+b^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [B]$$

By addition of [A] and [B], we obtain

$$4a^2+4ab+b^2+(2a-b)^2=8a^2+2b^2;$$

$$\text{hence } 4ab=4a^2+b^2-(2a-b)^2,$$

$$\text{and } ab=a^2+\frac{b^2}{4}-\frac{(2a-b)^2}{4};$$

and since $a^2=\frac{(2a)^2}{4}$, the expression may be further transformed into

$$ab=\frac{(2a)^2}{4}+\frac{b^2}{4}-\frac{(2a-b)^2}{4}.$$

It will, therefore, be obvious, that if, before entering the table of quarter squares, we double the factor a, we shall obtain precisely the same result as by the use of the expression [2].

I am aware that, in an isolated case, it is immaterial which of these methods be adopted; in general, in such a case, it will be preferable to use the other formula stated in the work, viz.—

$$ab=\frac{(a+b)^2}{4}-\frac{(a-b)^2}{4}. \quad . \quad . \quad . \quad . \quad . \quad . \quad [1]$$

It may be said that, by the process here recommended, three tabular entries are necessary, while by the use of the formula [1] two tabular entries only are needed; but it must be borne in mind, that in the operation by the former of these modes we have only to take the difference of the factor, while in the latter both the sum and difference of the factors must be found. In isolated cases, the amount of work by either formula is about equal; but with a series of quantities to be multiplied into a constant factor, the computer will, I think, find it more convenient to adopt the method now suggested. It may further be remarked, that the process here pointed out is quite as simple, if not more so, than if done by logarithms. The amount of work is not greater, and we obtain a result directly in natural numbers.

In illustration of the above remarks, I append an example of the values of a policy taken out at age 25, at the end of 1, 2, 3, &c., years, the premium being just due and not paid (Carl. 3 per cent.). For the sake of comparison, I place in juxtaposition the logarithmic process. The constant is supposed to be written on a card, and moved onward for each operation, and for this reason the constant figures do not appear except at the head of each column.

Here the constant factor is $\frac{1}{1+a_{25}}=\frac{1}{21\cdot666}=\cdot0461575$, and this quantity being doubled, for the reason before pointed out, $=\cdot092315$.

$2.\left(\frac{1}{1+a_{25}}\right)$	(1) Constant factor, ·092315.	Quarter square of (1), 213052.	Colog. $(1+a_{25})$, 8·66424.
$1+a_{26}$	21·442	11494	1·33127
Difference	70873	Ar. Co. $\bar{1}874425$	9·99551=·98971
		·98971	
$1+a_{27}=$	21·212	11249	1·32658
	71103	$\bar{1}873609$	9·99082=·97909
		·97910	
$1+a_{28}=$	20·981	11005	1·32183
	71334	$\bar{1}872787$	9·98607=·96844
		·96844	
$1+a_{29}=$	20·761	10776	1·31725
	71554	$\bar{1}872001$	9·98149=·95828
		·95829	
$1+a_{30}=$	20·556	10564	1·31294
	71759	$\bar{1}871266$	9·97718=·94882
		·94882	
$1+a_{31}=$	20·348	10351	1·30852
	71967	$\bar{1}870519$	9·97276=·93921
		·93922	
$1+a_{32}=$	20·134	10134	1·30393
	72181	$\bar{1}869748$	9·96817=·92933
		·92934	
$1+a_{33}=$	19·910	9910	1·29907
	72405	$\bar{1}868938$	9·96331=·91900
		·91900	
$1+a_{34}=$	19·675	9678	1·29392
	72640	$\bar{1}868086$	9·95816=·90816
		·90816	
$1+a_{35}=$	19·433	9441	1·28854
	72882	$\bar{1}867205$	9·95278=·89698
		·89698	

The foregoing results, subtracted from unity, give the value of a policy at the end of 1, 2, 3, &c., years.

I have the honour to be, Sir,

Your obedient Servant,

SAMUEL L. LAUNDY.

Eagle Life Office,
26th March, 1860.

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