

Session II

# Theory of Non-spherical Winds

chair: B. Baschek

# Wind-Compressed Disks

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**Abstract.** We discuss the wind-compressed disk (WCD) model and its ability to produce disks in the outflows from rapidly rotating stars. In particular, we discuss the recently discovered non-radial force components that may inhibit the formation of the disk and present preliminary investigations of the ionization distribution and associated line profiles in the WCD model.

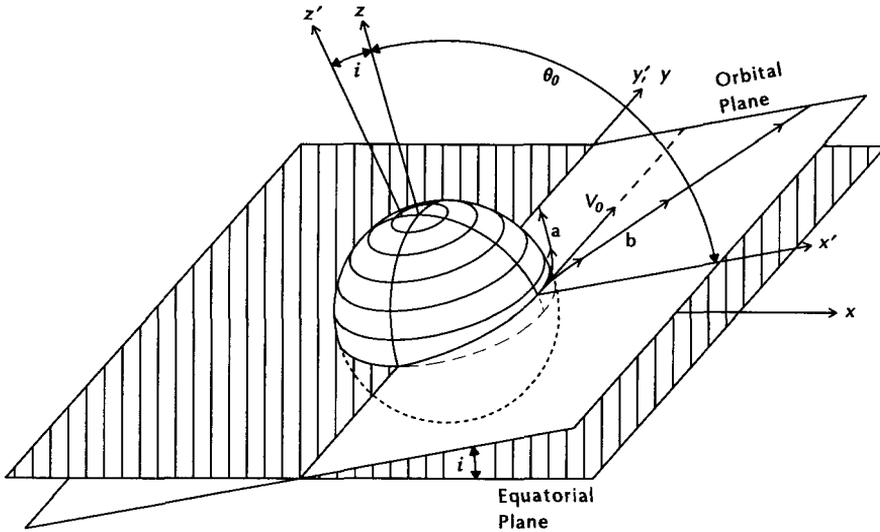
## 1 Introduction

Perhaps the most compelling cases for aspherical mass loss in luminous hot stars are the Be and B[e] stars. Be stars are near-main-sequence stars that have circumstellar disks as evidenced by their double-peaked H $\alpha$  profiles as well as their intrinsic polarization properties (see K. Bjorkman; these proceedings). Another class of stars possessing circumstellar disks are the LMC B[e] supergiants identified by Zickgraf et al. (1985, 1986). Like Be stars, B[e] stars have a two-component outflow (a high speed, high ionization state wind in combination with a dense, low speed, low ionization state disk). In addition to Be and B[e] stars, there have been suggestions that some Wolf-Rayet stars have equatorial density enhancements (to explain their intrinsic polarization), and the bi-polar shape of the nebula surrounding the LBV  $\eta$  Carinae (the so-called homunculus) can also be explained by the presence of a pre-existing disk in the LBV wind (Frank et al. 1995). How such disks may form is not entirely clear, but Be stars are rapid rotators. In their study of rotating stellar winds, Bjorkman and Cassinelli (1993) have shown how rotation can naturally lead to the production of a dense equatorial disk via a mechanism they call the Wind-Compressed Disk (WCD) model.

## 2 Wind Compression

In a rotating two-dimensional axisymmetric model, the wind streamlines do not spiral outward on surfaces (cones) of constant latitude, as is often assumed. Instead they bend toward the equator due to the centrifugal and Coriolis forces.

Consider the forces acting on the fluid. For an axisymmetric geometry, the pressure gradient only has  $r$ - and  $\theta$ -components. Although the  $\theta$ -component is large at the stellar surface (to enforce hydrostatic equilibrium), it drops rapidly beyond the sonic radius. The other forces are gravity and radiation,



**Fig. 1.** Orientation of the orbital plane for a streamline originating at a polar angle  $\theta_0$ . The streamline labeled (a) is a case with a high rotation rate and the streamline labeled (b) denotes a low rotation rate. (Figure from Bjorkman and Cassinelli 1993.)

which are central forces. Thus, beyond the sonic point, there are no external torques, so both the  $\theta$ - and  $\phi$ -components of the velocity are determined by angular momentum conservation and the streamlines are free particle trajectories corresponding to the external forces (radiation and gravity).

### 2.1 Orbital Plane

Much can be learned about the location of the streamline by utilizing the fact that gravity and radiation are central forces. Like a Keplerian orbit, the streamline lies in an orbital plane, containing the center of the star, the initial location, and velocity vector,  $V_0$  (see fig. 1). To find the streamline trajectories, we simply rotate the Friend and Abbott (1986) 1-d solution in the equatorial plane up to the initial latitude of the streamline and adjust the rotation velocity by  $V_{\text{rot}} \rightarrow V_{\text{rot}} \sin \theta_0$ .

Figure 1 shows two trajectories labeled (a) and (b), that correspond to different initial conditions. Trajectory (a) has a slow initial acceleration and occurs when there is a large rotation rate. Trajectory (b) has a fast initial acceleration and occurs when there is a slow rotation rate. Note that as trajectory (a) orbits around the star, it has a decreasing altitude,  $z$ , and eventually crosses the equator.

## 2.2 Disk Formation

Unlike a non-rotating stellar wind (where the streamlines are radial) the fluid elements in a rotating wind tend to orbit the star. Thus material from high latitudes orbits toward the equator where it collides with material from the opposite hemisphere. Since the flow velocities are supersonic, this collision results in a pair of shocks above and below the equatorial plane. Between the shocks, the shock compression produces a thin, dense, wind-compressed disk. Thus the primary effect of rotation is not to change the total mass loss rate from the star; instead it *redistributes* the mass loss such that most of the mass lost from the stellar surface is funneled into the small solid angle of the equatorial disk (typically the opening angle, HWHM, of the disk is only about  $3^\circ$ ), which results in the equatorial disk being about 100 times more dense than the polar outflow.

To form the shock-compressed disk, the star must rotate fast enough that the streamlines attempt to cross the equator. Thus there is a minimum stellar rotation speed, above which a disk forms, and the rotation threshold depends on how rapidly the wind is accelerated. Below this rotation threshold there is still some compression toward the equator, but it only produces a mild density enhancement (typically less than a factor of 10). To distinguish these two classes of equatorial density enhancements, we call the rapidly rotating shock compression disks “WCDs”, while the cases with rotation speeds below the disk formation threshold, we call “wind compressed zone” (WCZ) models.

## 2.3 Bistability Mechanism

Although rotation typically does not appreciably change the mass flux from the stellar surface, there is an important exception that occurs for B[e] stars. This exception is the bistability mechanism discovered by Pauldrach & Puls (1990) in their study of the mass loss from P Cyg (see Lamers; these proceedings). They were studying the effect of lowering the surface gravity (i.e., increasing the stellar radius) on the mass loss rate from the star. They found for an effective temperature of 19000 K, that when the surface gravity dropped below  $\log g_{\text{eff}} = 1.6$ , there was a sudden increase in the mass flux (about a factor of 3) and a corresponding decrease in the wind terminal speed. This was attributed to the wind becoming optically thick in the photoionizing Lyman continuum, which produces a shift in the ionization balance that changes the number of optically thick driving lines in the wind.

This discontinuity in the terminal speed has now been observed in B supergiants by Lamers, Snow, & Lindholm (1995). They find that earlier than  $\log T_{\text{eff}} = 4.3$  the data are consistent with  $v_\infty/v_{\text{esc}} = 2.7$ , and later than that  $v_\infty/v_{\text{esc}}$  drops to 1.3. They also find that there may be a second bistability jump at  $\log T_{\text{eff}} = 4.0$  where  $v_\infty/v_{\text{esc}}$  becomes as small as 0.7.

Since B[e] stars have effective temperatures near the bistability jump, Lamers & Pauldrach (1991) investigated how it may effect the outflow from

B[e] stars. They supposed that the star is rapidly rotating and gravity darkened and estimated the hydrogen Lyman column density as a function of latitude. They found that the star could be optically thin in the polar regions and optically thick in the equatorial latitudes. Thus the bistability jump could occur at some mid latitude, producing denser slower outflow in the equatorial zones, which they argue could be responsible for the B[e] phenomenon. Since the bistability jump is not that large, the density contrast from pole to equator in their models is probably at most about a factor of 10.

Lamers and Pauldrach did not include the 2-D wind compression effects mentioned above. Although we will not attempt to quantitatively combine the bistability mechanism with the wind compressed disk model, it is nonetheless interesting to explore how the two models may interact with each other. If bistability occurs at low latitudes, then the outflow originating at low latitudes has a smaller-than-expected terminal speed and larger mass flux. This implies that the low latitude flow will be more slowly accelerating, which will increase the wind compression effects and lower the threshold stellar rotation speed for forming a disk. On the other hand, wind compression effects increase the low latitude densities, which increases the Lyman column density for two reasons. First, the larger density increases the recombination rates, which lowers the hydrogen ionization fraction and increases the Lyman bound-free opacity. Second, the increased optical depths in the equator prevent the photoionizing flux from reaching the equator and redirect it poleward, since the light will tend to diffuse out the path of least resistance. Decreasing the photoionizing flux also increases the Lyman bound-free opacity, so both these effects imply that the bistability mechanism will occur more easily in the presence of wind-compression effects. Since wind-compression enhances the likelihood of bistability and bistability enhances wind compression, there is now the possibility of feedback that will enhance the disk density even more than one would naively think. Whether or not such a runaway could occur requires detailed computations that we will not attempt. Here we will merely use bistability as a mechanism for motivating the possibility that the low latitude outflow may have terminal speeds as small as  $v_\infty/v_{\text{esc}} = 1.3$ .

## 2.4 Disk Formation Threshold

Since the disk is formed from the low latitude outflow, we use the low latitude terminal speed,  $v_\infty/v_{\text{esc}} = 1.3$ , to determine the disk formation threshold,

$$\left(\frac{V_{\text{rot}}}{V_{\text{crit}}}\right)_{\text{th}} = \begin{cases} 0.3 & (\beta = 3), \\ 0.8 & (\beta = 0.8), \end{cases} \quad (1)$$

where  $\beta$  is the velocity law exponent (i.e.,  $v = v_0 + [v_\infty - v_0][1 - R/r]^\beta$ ). Since we do not know how rapidly a bistable wind accelerates, we have chosen both a slow ( $\beta = 3$ ) and a fast ( $\beta = 0.8$ ) acceleration to illustrate the possible

range. Using a typical supergiant escape speed ( $240 \text{ km s}^{-1}$ ) to determine the critical speed  $V_{\text{crit}} = V_{\text{esc}}/\sqrt{3}$ , the required rotation speed to form a WCD is

$$(V_{\text{rot}})_{\text{th}} = \begin{cases} 40 \text{ km s}^{-1} & (\beta = 3), \\ 110 \text{ km s}^{-1} & (\beta = 0.8). \end{cases} \quad (2)$$

These values imply that it is relatively easy to form a disk, especially if the wind is slowly accelerating ( $\beta = 3$ ).

## 2.5 Non-radial Forces

Recently, Owocki, Cranmer, & Gayley (1996, OCG) have questioned whether or not a WCD can form in a line driven wind as originally proposed by Bjorkman & Cassinelli. The radiation force that drives the wind has non-radial components arising from three effects: 1) The penetration probability  $\beta = [1 - \exp(-\tau)]/\tau$ , where the Sobolev optical depth  $\tau \propto 1/|dv/dl|$ . Since the velocity gradients are asymmetric with respect to the radial direction, the absorption of stellar photons produces a retarding torque that reduces the rotation of the wind. 2) Similarly, the absorption asymmetry also produces a  $\theta$ -component of the force directed away from the equator. 3) A rotating star is oblate and gravity darkened. Although the pole is brighter than the equator, the increased solid angle subtended by the equator causes the net radiative flux vector to have a  $\theta$ -component away from the equator. All three of these effects produce a non-radial component of the force that reduces the amount of wind compression. Although these non-radial forces are an order of magnitude less than the radial component of the radiation force, they are comparable to the *net* radial force, because the radiation force only marginally exceeds the force of gravity. Consequently, OCG find that the non-radial forces produced by these three effects inhibit the formation of a WCD. However, we should also keep in mind that non-radial forces are not necessarily bad; it depends on their direction. For example, if the terminal speed were to increase toward the equator instead of decreasing (as usual), then the direction of the non-radial forces would be toward the equator, enhancing the disk (see Puls; elsewhere these proceedings).

OCG use the Castor, Abbot, and Klein (1975, CAK) parameterization of the line driving force as modified by Abbot (1982), which uses a (photospheric) core and (optically thin) halo approximation for the photoionization balance of the wind. In OCG's calculation of the WCD inhibition, they assume that the wind is smoothly accelerating (so that the Sobolev approximation determines the radiation force) and that the CAK parameters ( $k, \alpha, \delta$ ) are constant throughout the wind. As we discussed earlier, an optically thick disk redirects photoionizing radiation away from the equatorial regions producing lower ionization states. There is also observational evidence for latitudinal ionization gradients in the winds of rapidly rotating stars (Bjorkman et al. 1994), and in the next section we will see that the

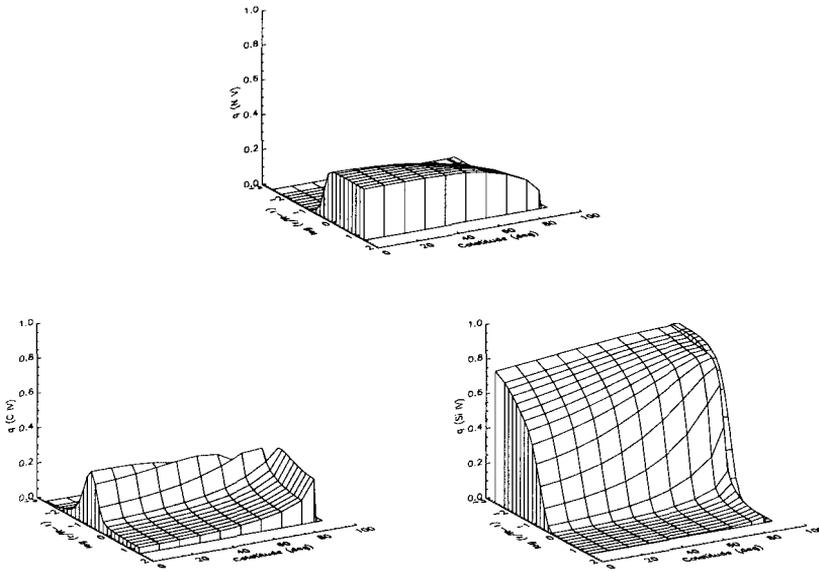
wind can dramatically increase its ionization state as the wind moves away from the star. This position dependence of the ionization balance implies that the CAK force parameters are not constant throughout the wind as assumed by OCG. Although changing the force parameters ( $k, \alpha, \delta$ ) will not alter the direction of the non-radial force, it could seriously change its magnitude. For example, if the ionization balance shifts to drastically decrease the radiation force shortly after the wind is initiated, then gravity will dominate thereafter, and a WCD will form. Similarly if the wind driving lines become optically thin, the escape probability is isotropic, so there would no longer be any non-radial force components (unless the star is oblate). Finally if the wind is clumpy and not smoothly accelerating, the radiation force is no longer determined by the Sobolev approximation; it is determined instead by the physical geometry of the clump. The acceleration of the clump depends inversely on its mass; thus, if the clump is massive enough and forms close enough to the star (so that its outward velocity is less than the escape speed), then gravity would dominate, causing the clump to orbit/fall into the disk according to the WCD mechanism. The above issues indicate that, to determine when and where WCDs form, it is essential that we perform detailed calculations of the wind dynamics and ionization to determine how the CAK parameters ( $k, \alpha, \delta$ ) change with location in the wind.

### 3 WCD Ionization

To begin a preliminary investigation of the ionization distribution within the WCD model, we employed MacFarlane's NLTE wind ionization code, which is a spherically symmetric model using the Sobolev approximation for the line transfer (MacFarlane et al. 1993). For this initial attempt to calculate the 2-D ionization distribution, we used a piecewise spherical approximation; i.e., a series of spherically symmetric models, one for each latitude.

#### 3.1 Ionization Distribution

Figure 2 shows the ionization fraction of N v, C iv, and Si iv throughout the wind of a B2.5 IV star rotating at 80% critical with an X-ray emission measure of  $10^{53} \text{ cm}^{-3}$ , which corresponds roughly to  $L_X \sim 10^{30} \text{ ergs}^{-1}$ , the maximum B star X-ray luminosity observed by ROSAT (Cohen et al. 1997). Note that as the wind moves away from the star, Si iv is rapidly ionized (between 1 and 2 stellar radii). At large radii, Si vi and Si vii are the dominant states. Similarly, there is a latitudinal ionization gradient. Since Si iv is below the dominant state, it is enhanced in the denser equatorial regions. Conversely N v, which is above the dominant state, is enhanced at the poles and destroyed near the equator. Finally although C iv is mostly produced close to the star (from 1.1 to 2 stellar radii), it does persist to large radii (albeit at somewhat low levels). It is also enhanced near the equator, but destroyed in the disk.

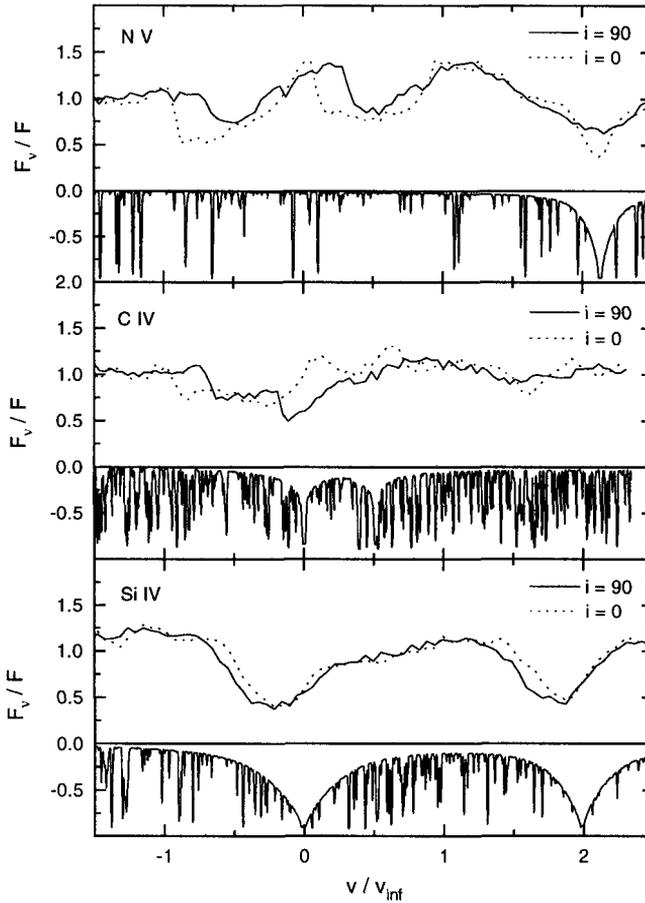


**Fig. 2.** WCD Ionization Fractions. Shown are the ionization fractions,  $q$ , of N v (top), C iv (bottom left), and Si iv (bottom right) as a function of position,  $\log(r/R - 1)$ , and co-latitude,  $\theta$ .

### 3.2 Line Profiles

Figure 3 shows the UV resonance line profiles associated with the WCD ionization fractions shown in Figure 2. These profiles were calculated using a Monte Carlo simulation that properly accounts for non-monotonic velocity fields and any associated non-local coupling of the diffuse radiation between common point surfaces in the WCD model. It also includes a realistic line blanketed stellar photosphere for the lower boundary. Note that the Monte Carlo simulation includes stellar rotation and that the small spectral features in the calculated profiles are rotationally broadened photospheric features and are not “statistical noise” in the simulation.

The solid profiles are for edge-on ( $i = 90^\circ$ ) observers, while the dashed profiles are for pole-on ( $i = 0^\circ$ ) observers. Note that the blue edge of C iv and N v is quite sharp (because the ions persist to large radii) and that the polar edge velocity is larger than the equatorial. This is because the terminal speed of the wind decreases toward low latitudes. On the other hand, Si iv has a shallow return to the continuum without a sharp blue edge and no emission. Surprisingly, the polar edge velocity of Si iv is *smaller* than the equatorial edge velocity (opposite to the trend with C iv and N v). This peculiar behavior is a result of two factors. First, Si iv is destroyed close to



**Fig. 3.** WCD Line profiles. Shown are the resonance line doublet profiles of N v (top), C IV (middle), and Si IV (bottom) for a pole-on observer (dotted line) and an edge-on observer (solid line). Shown below each profile is the input stellar photospheric spectrum.

the star, which explains the shallow return to the continuum without a sharp blue edge. It also explains the lack of emission as a result of occultation by the star. Second, Si IV is below the dominant ionization stage and is enhanced near the equator. Thus Si IV persists to larger radii in the equator than in the pole. As a result it can absorb to higher velocities in the equator than in the pole despite the fact that the wind terminal speed is lower in the equator.

## 4 Conclusions

The ionization distribution clearly shows a large sensitivity to location in the wind. In particular the rapid ionization destroying Si IV close to the star while creating Si VII far away implies that the CAK line parameters are very likely to change quite dramatically. In general we expect that the large increase in ionization state implies that most strong lines will exist in the EUV where there is very little stellar flux. The remaining lines in the Balmer continuum are likely to be weak optically thin lines. Thus we speculate that the line driving will shift from strong lines to weak lines causing a decrease in the CAK parameter  $\alpha$  quite close to the stellar surface. Whether this could remove the disk inhibition by non-radial forces remains to be seen.

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## Discussion

**A. Feldmeier:** You mentioned that the azimuthal velocity law in the disk is non-Keplerian. Can you actually specify  $v_\phi(r)$ ? Shouldn't it drop faster than  $1/r$ ?

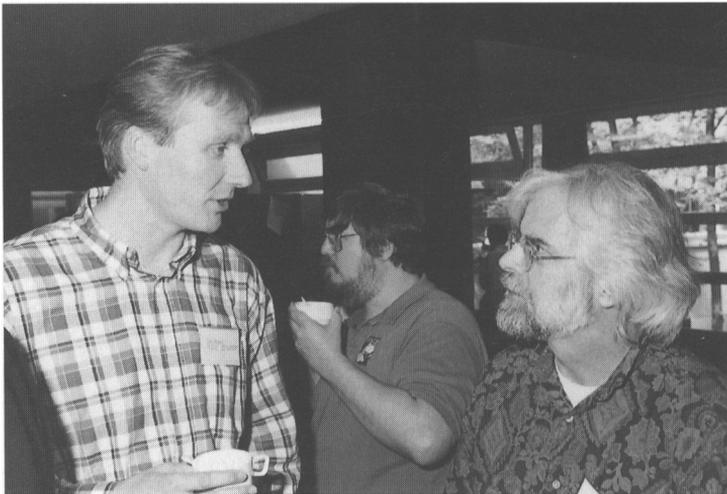
**J. Bjorkman:** It depends on how the low specific angular momentum material mixes as it is added to the disk. Numerical simulations by Owocki show that  $v_\phi$  in the equator does in fact fall as  $1/r$ , so this mixing must not be large.

**A. Maeder:** The simple application of the wind momentum – luminosity relation,  $\dot{M} v_\infty = L/c$ , suggests an enhanced equatorial mass loss rather than the opposite. This is because the local luminosity on the right side goes like the effective gravity, while the terminal velocity goes like the square root of  $g_{\text{eff}}$ .

**J. Bjorkman:** As you suggest, using the von Zeipel theorem to obtain the radiation flux vs. latitude can cause the polar mass loss rate to be larger than the equatorial one. Stan Owocki has studied this effect and the results depend on the value of the CAK parameter, which determines the scaling of the mass flux vs. radiative flux. He finds that indeed the polar mass loss rate can exceed the equatorial one.

**H. Henrichs:** Since Be-star disks are time-variable, your new considerations that make the disk exist or not strongly improve the chances that the WCD model is applicable to such systems. May I encourage you to consider the implications of these “other effects”, in particular what the relevant time scales are to build up or destroy a disk.

**J. Bjorkman:** Since the model is a time-independent model, any time dependence must be put in by hand. For example, one can vary the input parameters to get time variability. The time scale for variability is the radial flow time scale in the disk which is of the order of a few days. Any longer time scale would have to arise from the input parameters.



Alexander Fullerton and Derck Massa (Jon Bjorkman in the background)