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A REMARK ON AN INTEGRAL INEQUALITY

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A counter example is constructed to show that an integral inequality established by Sree Hari Rao (Theorem 3.1, *J. Math. Anal. Appl.* **72** (1979), 545-550) is erroneous.

With the notations of [2], the main result (namely Theorem 3.1) in [2] is

THEOREM 1. *Let* $y(t)$ *and* $u(t)$ *be non-negative functions of bounded variation with* $u(t)$ *increasing. Let* $k(t)$ *be a non-negative function integrable with respect to* u *on* $[0, T]$ *for which the inequality*

$$(1) \qquad y(t) \leq c + \int_0^t k(s)y(s)du(s) \ , \quad 0 \leq t \leq T \ ,$$

holds, where $c > 0$ *is a constant. Then*

$$(2) \qquad y(t) \leq c\left[1 + \int_0^t k(s)\exp\left(\int_s^t k(l)du(l)\right)du(s)\right]$$

for t *in* $[0, T]$.

The following example shows that Theorem 1 is not true.

EXAMPLE. Choose $c = 1$, $k(t) \equiv 1$ and $1 < T < \infty$. Also let

$$u(t) = \begin{cases} t & \text{for } 0 \leq t < 1 \ , \\ a + t & \text{for } 1 \leq t \leq T \ , \end{cases}$$

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where a is an arbitrary number in $(0, 1)$. The initial value problem

(3) $$Dy = yDu \ , \quad y(0) = 1 \ ,$$

is equivalent to (for a proof refer to Das and Sharma [1]) the integral equation

(4) $$y(t) = 1 + \int_0^t y(s)du(s) \ .$$

It can be checked that

(5) $$y(t) = \begin{cases} \exp(t) & \text{if} \quad 0 \le t < 1 \ , \\ \big(\exp(t)\big)/(1-a) & \text{if} \quad 1 \le t \le T \ . \end{cases}$$

From (5) it is clear that $y(t)$ satisfies the conditions stated in Theorem 1. Also a simple computation shows that

$$\int_s^t k(l)du(l) \le \exp(T) - 1 + a \ .$$

Let M denote the right side of (2). It is easy to show that

$$M \le c\big[1+\exp(T)\exp\big(\exp(T)\big)\big]$$

and clearly M is bounded and is independent of a. On the other hand from (5) we see that $y(t)$ can be made greater than M by a suitable choice of a (say for an example $a = 1 - 1/M'$, where $M < M' < \infty$ and $M' > 1$). To conclude we have:

(1) $y(t) > M \ , \quad T \ge t \ge 1$;

(2) $y(t)$ satisfies the inequality (1); and

(3) all the rest of the hypotheses of Theorem 1 are satisfied.

Thus Theorem 1 seems to be false.

References

[1] P.C. Das and R.R. Sharma, "Existence and stability of measure differential equations", *Czechoslovak Math. J.* 22 (97) (1972), 145-158.

[2] V. Sree Hari Rao, "Integral inequalities of Gronwall type for distributions", *J. Math. Anal. Appl.* 72 (1979), 545-550.

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