tangent planes are concerned, is limited by two pinch points so that the whole model is in a finite space. As it happens also to be one of the models of the real projective plane it duly appears later; *105–114* show various plane sections.

This brings us to the cyclides which, although algebraic, indeed quartic, surfaces figure in the section of the commentary concerned with differential geometry presumably because the circles that are their lines of curvature are so prominent. The standard 1822 reference to Dupin is given but, as this review is edited from headquarters named after James Clerk Maxwell, one may be allowed to regret that there is no reference to him: he gave stereoscopic diagrams of horned, parabolic, ring and spindle cyclides; see *71–76*. H. F. Baker, in his fourth volume (1925) on *Principles of Geometry*, gave the cyclides full treatment, using projection from four dimensions to derive many of their properties as envelopes of spheres, inverses of tori, and so on, and duly referred to Maxwell who arrived at his conclusions by optical considerations. The reference was adequate, but in his 1940 reprint he was moved to supply a postscript detailing some of Maxwell's calculations. What Baker, perhaps writing "off duty", describes as "the surface of two bananas placed with the ends of either coincident with the ends of the other" is *74*, while that "of the shape of a distorted anchor ring" is *71* which it is easy to visualize as the envelope of spheres touching it internally along the circles shown. Clear diagrams of ring, spindle and horned tori, with the two orthogonal families of circles marked on each, are on p. 28.

Photos *79–88* are of surfaces of constant Gaussian curvature; *90–96* are of minimal surfaces; *98–102* of convex bodies of constant width. To derive these, two plane curves of constant width are used: (a) an involute of the 3-cusped hypocycloid, (b) the Reuleaux triangle of three circular arcs: each joins two vertices of an equilateral triangle and is centred at the third. These provide surfaces of revolution of constant width (*98, 99*). A surface not of revolution that has this property (*100–102*) is not so simple, but its construction is fully described (pp. 54–55). The next photos are of regular star polyhedra (*103–106*) and of models of the real projective plane (*107–120*).

From now onwards the photos illustrate topics in analysis, the first two being almost too familiar to university teachers. The surface $z(x^2+y^2)=xy(x^2-y^2)$ of *121* would seem visually non-singular at $x=y=0$ with a unique tangent plane meeting it in four equally spaced lines; but although the mixed second partial derivatives of z with respect to x and y both exist at $x=y=0$ they are not equal. But on $z=(2x^2-y)(y-x^2)$ pictured in *122* $x=y=0$ is genuinely non-singular, the tangent plane $z=0$ meeting it in two parabolas, both clear on the model, with a common tangent. This phenomenon of a Peano saddle point, signalized by him in 1884, occurs when the section of a surface by its tangent plane at a non-singular point P has a tacnode at P, the two touching branches being real (not complex conjugates). The distance of a point on the surface from the tangent plane at P has then neither a maximum nor a minimum at P.

Later photos depict real and imaginary parts of Riemann surfaces (*123–125*) and of the Weierstrass elliptic function and its derivative (*129–131*).

The photos are a pleasure to contemplate, the commentary a pleasure to read, the books a pleasure to handle. A table at the close of the commentary gives, so far as they are known, the authors of the models, the dates of their construction and their present location.

W. L. EDGE

RUSTON, A. F., *Fredholm theory in Banach spaces* (Cambridge Tracts in Mathematics 86, Cambridge University Press, 1986), x + 293 pp. £30.

Books by pioneers are always valuable and the author is one of the pioneers of the important theory of determinantal expansions of the resolvent of certain operators in Banach spaces associated with Fredholm type solutions of integral equations. In the L^2-case if the underlying operator is compact the technique consists of finding determinantal expansions (using tensor product notation) for resolvents of finite rank operators and then approximating. A stylish exposition of this theory was given by Smithies in his Cambridge Tract *Integral Equations* in

1958. Here the author generalises this theory to encompass larger classes of operators such as the Riesz operators (or as he calls them asymptotically quasi-compact operators—those with quasinilpotent images in the Calkin algebra) in Banach spaces.

Considerable attention is directed to the problem of putting the appropriate norms on the tensor products of Banach spaces and then using this notation the author gives exhaustive discussions of the various expansion formulae associated with the names of Sikorski, Lezanski, Plemelj and Grothendieck besides surveying his own work commenced as a graduate student under Frank Smithies in Cambridge.

The reader should be warned, however, that the author has been somewhat ingenuous in his choice of title. To a modernist, a work on Fredholm theory in Banach spaces without a reference to the Fredholm index would be unthinkable. This book does not attempt to survey the spectral theory which has given Fredholm and semi-Fredholm operators in Banach spaces such a central role in modern analysis. Rather is it a specialist text on Fredholm's theory of determinantal expansions of the resolvent by an expert in the field.

T. T. WEST

KLINE, M., *Mathematics and the search for knowledge* (Oxford University Press, New York, 1985), pp. 257, £21.

Professor Kline has made use of his encyclopaedic knowledge of the history of mathematics to give a readable and coherent résumé of his view of the changing role of Mathematics from ancient times to the twentieth century. Briefly his thesis is that mathematics was first honoured in the ancient world for semi-mystical reasons by philosophers; that after the renaissance the revolution in the human conception of the universe wrought by the mathematically based work of Copernicus, Galileo, Kepler and Newton led to a quasi-religious respect for mathematics as revealing the ways of God in creating and ordering the world; that in the later eighteenth and early nineteenth centuries this religious faith declined but the reliance on mathematics as a source of ultimate truth increased; but that recently causes of doubt arising in the heart of Mathematics and Physics—the existence of non-euclidean geometries and the uncertainty principles of quantum mechanics for example—have led modern thinkers to abandon absolute truth as a standard against which Mathematics is measured.

The whole book requires a minimum of technical mathematical knowledge and gives an illuminating oversight of the historical developments sketched above with many interesting observations along the way.

I was left wondering at the end why the book seemed vaguely unsatisfactory to me. There appear on reflection to be two reasons. The first is that a rather large number of imprecisely worded sentences have been allowed to remain in the final version of the text. Examples from Chapter VII are "...the science of electricity, the Greek word for amber" and "...the peculiar property of magnets is their power of attracting unmagnetized iron or steel, a stronger magnet being able to pull a heavier piece of iron to itself" and "Volta realized that the two unlike metals were producing a force, now called electromotive force...". There is a correct idea in each case which can be understood by one who knows it, but the uninstructed reader may well be slightly misled.

The second reason is more fundamental and is implicit in the theme Professor Kline has chosen. When the book has been read one is left with the feeling that the golden days of certainty in mathematical truth are done, and that the Götterdämmerung is on us. Although much about our present era is dark I cannot accept that no adequate successors to early nineteenth-century attitudes to Mathematics have been proposed or explored. There is indeed no single unified Philosophy of Mathematics, but it is surely partly for this reason that the subject is alive and interesting.

M. PETERSON