

index and extensive bibliography. This is a well-written book containing full details of proofs of the main theorems. It should prove particularly valuable to the research worker in the field.

H. R. DOWSON

OGILVY, C. STANLEY, *Excursions in Geometry* (Oxford University Press, 1970), vi + 178 pp., £2.60.

This pleasantly written little book is an account of some especially attractive topics in elementary geometry aimed at "people who liked geometry when they studied it . . . but who sensed a lack of intellectual stimulus in the traditional course and . . . felt that the play was ending just when the plot was beginning to become interesting". Given this objective the choice of subject matter follows the expected lines. Topics discussed include circle geometry, with emphasis on inversion; conics from the standpoint of their focal distance and mirror properties and as sections of a cone; projective geometry introduced via conical projection and the invariance of cross-ratio; the golden section; some unsolved and unsolvable problems.

One naturally makes a comparison with an established text such as H. S. M. Coxeter's *Introduction to Geometry*. The work under review is, of course, much smaller, less ambitious and less technically detailed (for instance coordinate techniques are little used, which is rather a pity now that they are being given greater prominence in elementary school work). The common aim of selecting entertaining material means inevitably that some topics are dealt with by both authors; however there are differences of approach and depth. The present book, besides being very welcome in its own right, would serve as a good *apéritif* for the larger one. Its purpose is certainly to be commended. Worthy of particular mention is the discussion of Soddy's hexlet and its modifications. The book is produced to the press's usual high standard. A nice prize for a sixth-former!

D. MONK

NAIMPALLY, S. A. and WARRACK, B. D., *Proximity Spaces* (Cambridge Tracts in Mathematics and Mathematical Physics No. 59, Cambridge University Press, 1970), x + 128 pp., £3.

The subject of proximity spaces is a sufficiently compact subset of topology that a tract of this size can be introductory in character and yet succeed in its aim of enabling the reader to understand current literature. I conjecture that a necessary and sufficient condition to comfortably read the first half of the book would be knowledge of the Stone-Céch compactification, though it receives mention but twice. However at least a nodding acquaintance with uniform spaces is required for the second half.

I welcome the inclusion of an excellent historical introduction, which reveals, as do all histories of recent events, that the nearer one comes to the present day the harder it becomes to unravel a sense of direction in the subject. The following lines are developed in the text:

The axioms for a proximity on a set are motivated by five properties of "nearness" between pairs of subsets of a pseudo-metric space. A proximity induces a completely regular topology. A subspace proximity and proximity mapping are defined in a natural way.

The main result in the first half of the book is the construction of the Smirnov compactification of a proximity space by means of clusters, revealing the order-isomorphism between the proximities and compactifications of a completely regular space.

Proximity lies between uniformity and topology in the sense that a uniformity induces a proximity and a proximity induces a topology. The equivalence class of

uniformities corresponding to a given proximity is studied. It always contains a minimum element, which is totally bounded, though in general not a maximum. However the existence of the latter has been satisfied by introducing a generalized uniform structure with a weaker intersection axiom. Two uniformities of the same height and in the same proximity equivalence class induce equivalent uniformities in the hyperspace.

The final chapter considers proximal convergence and makes brief mention of symtopogeneous structures, sequential proximity, and four generalizations of proximity structures.

A very comprehensive bibliography containing 138 papers up to 1969 and the notes and references at the end of each chapter contribute to the book's value as a survey for the intending research worker. However it will also be enjoyed by those merely wishing to add proximity spaces to their store of knowledge.

J. R. MCCARTNEY

ROOM, T. G. and KIRKPATRICK, P. B., *Miniquaternion Geometry* (Cambridge Tracts in Mathematics and Mathematical Physics, No. 60, Cambridge University Press, 1971), viii + 176 pp., £4.

The system of miniquaternions is the 9-element near-field in which multiplication is right-distributive but not left-distributive over addition, and miniquaternion geometry is the study of three non-desarguesian projective planes of order 9 (i.e. with 10 points on each line) that can be constructed and coordinatized by means of this near-field. Lest such a bald definition of the meaning of the book's title should give the false impression that it is devoted exclusively to very specialized topics, it must at once be added that the sub-title, "An Introduction to the Study of Projective Planes", summarizes much more revealingly the book's aims and contents.

The 41-page second chapter is the most fundamental. It starts with the definition of a projective plane and surveys important generalities such as the well-definedness of the order of a finite plane, properties of central and axial collineations, and the Desargues configuration. It then turns attention to planes over fields and discusses matrix representation of projectivities, correlations, polarities, and conics when the field has characteristic different from 2. This chapter stands apart from the rest of the book in that other chapters aim to acquaint the reader with projective planes through the medium of detailed analysis of particular cases, attention being focussed almost exclusively on planes of order 9: thus Chapter 3 is devoted mainly to the plane over  $GF(9)$ , and Chapters 4 and 5 to miniquaternion geometry proper. The so far unmentioned first chapter is a short account of the miniquaternion system itself and, like the rest of the book, demands no sophisticated algebraic knowledge on the part of the reader.

Apart from a few curious lapses, the authors have presented their material in a well-organized form, and they are good at providing sign-posts for readers who are strangers to this area of mathematics. Chapter 2 deserves repeated mention as a welcome and very readable addition to the few available introductory accounts of projective planes. On the other hand, some readers who are largely familiar with the contents of Chapter 2 may regret that the emphasis throughout the rest of the book is so strongly on the particular as opposed to the general. In defence of the authors' approach, one can point out that their discussions of planes of order 9 provide an attractive and uncluttered setting for the communication of many ideas of more general importance, that in any case finite geometry is a subject where much of the interest and the charm resides in the particular, and that it would be impossible to