3. Internal Structure (J. Christensen-Dalsgaard)

In this section I concentrate on the spherically symmetric aspects of solar structure, corresponding to "classical" stellar evolution models. Such models are characterized by a number of simplifying assumptions, as well as by the physical properties of matter in the star, conveniently labeled "micro-physics". The latter include descriptions of the equation of state, the opacity and the nuclear reaction rates; in addition, molecular diffusion, included in several recent calculations, should be considered as part of the micro-physics. The assumptions in the standard calculations, simplifying what might be called the macro-physics, include the neglect of effects of rotation and magnetic fields (implicit in the assumption of spherical symmetry), as well as the assumption that material mixing occurs only in convectively unstable regions, or possibly as a result of molecular diffusion and settling; also, convective energy transport is treated crudely through some form of mixing-length approximation and the contribution to hydrostatic balance from the turbulent motion in the convection zone, usually called turbulent pressure, is ignored.

While we have no direct evidence that the approximations are inadequate in the solar interior, it is obvious that our treatment of layers near the solar surface, and the solar atmosphere, is grossly over-simplified. This must be kept in mind in analyses of solar oscillation frequencies. On the other hand, the bulk of the convection zone, apart from the region near the surface where a substantial superadiabatic gradient is required to drive the convective flux, has a fairly simple structure. Since energy transport is almost entirely through convection, the opacity has no effect; also, the stratification is very nearly adiabatic and is therefore essentially determined by the equation of state. It follows that the structure of the convection zone is characterized by the equation of state, the composition and the (constant) value of the specific entropy. This makes the convection zone particularly well suited for testing the equation of state.

Detailed analyses of the observed frequencies must be based on numerically computed frequencies, obtained by solving the equations of stellar oscillation; however, a great deal of insight, as well as quantitative results of considerable precision, have resulted from asymptotic relations for acoustic modes. In its simplest form, the relation for the angular frequency ω can be written as (*e.g.* Gough, 1984)

$$\frac{(n+\alpha)\pi}{\omega} = \int_{r_t}^{R} \left(1 - \frac{L^2}{\omega^2}\frac{c^2}{r^2}\right)^{1/2} \frac{\mathrm{d}r}{c} , \qquad (1)$$

where n is the order of the mode and $L = l + 1/2$, l being the degree; c is the adiabatic sound speed, which is a function of the distance r to the center, the lower turning point r_t is located where the the integrand vanishes, and R is the surface radius. Finally, $\alpha(\omega)$ depends on the properties of the region near the solar surface. Equation (1) can be refined, for example by including the effect of the perturbation in the gravitational field and the dependence of the near-surface reflection on degree (*e.g.* Brodsky & Vorontsov, 1991; Gough & Vorontsov, 1995); however, here the present form is sufficient.

For low-degree modes equation (1) can be expanded, to yield the cyclic frequency $\nu = \omega/2\pi$ as

$$\nu_{nl} \simeq \Delta\nu \left(n + \frac{l}{2} + \alpha + \frac{1}{4}\right) + \epsilon_{nl} \; , \tag{2}$$

where the large separation $\Delta\nu = \int_0^R \mathrm{d}r/c$ and ϵ_{nl} is a small correction term (*e.g.* Vandakurov, 1967; Tassoul, 1980). If this term is neglected, equation (2) predicts a spectrum of uniformly spaced peaks alternating between modes of even and odd degree; including ϵ_{nl} introduces a small separation $\delta\nu_{nl} = \nu_{nl} - \nu_{n-1,l+2}$ which may be shown to depend predominantly on conditions in the stellar core.

It is convenient to analyze the frequencies in terms of corrections relative to a reference model. In equation (1) these can be characterized by the change $\delta c(r)$ in the sound speed, evaluated at fixed r, as well as $\delta\alpha(\omega)$ determined by changes near the surface. From equation (1) one finds for the resulting change $\delta\omega$ in the frequency that

$$S \frac{\delta\omega_{nl}}{\omega_{nl}} = \mathcal{H}_1\left(\frac{\omega_{nl}}{L}\right) + \mathcal{H}_2(\omega_{nl}) \; , \tag{3}$$

where

$$S_{nl} = \int_{r_t}^{R} \left(1 - \frac{L^2 c^2}{r^2 \omega_{nl}^2}\right)^{-1/2} \frac{\mathrm{d}r}{c} - \pi \frac{\mathrm{d}\alpha}{\mathrm{d}\omega} \; ,$$

$$\mathcal{H}_1(w) = \int_{r_t}^{R} \left(1 - \frac{c^2}{r^2 w^2}\right)^{-1/2} \frac{\delta c}{c} \frac{\mathrm{d}r}{c} \; , \tag{4}$$

$$\mathcal{H}_2(\omega) = \frac{\pi}{\omega} \delta\alpha(\omega)$$

(Christensen-Dalsgaard, Gough & Pérez Hernández, 1988). Thus the frequency change contains a contribution, $\mathcal{H}_1(\omega/L)$, from the sound-speed difference in the bulk of the model and another, $\mathcal{H}_2(\omega)$, from the near-surface changes; because of their different dependence on ω and l these two terms can be separated in an analysis of the data.

The effects of the inadequate treatment of the physics in the model near the surface are contained in the term $\mathcal{H}_2$. The separation in equation (3) in effect removes from the frequency differences the uncertainties introduced by our ignorance about the physics in the superficial layers of the Sun, leaving in $\mathcal{H}_1$ a clean signal which can subsequently be analyzed to determine the correction δc to the sound speed.

The goal of structure inversion is to infer corrections to the solar models from the differences between the observed and computed frequencies. In principle, therefore, one seeks for example the correction $\delta c(r)$ to the sound speed as a function of r throughout the Sun. In practice, it is obvious that analysis of a finite set of data cannot provide complete information about such a function. Further restrictions follow from the requirement that the random errors in the data have an acceptable effect on the inferred solution. Thus the inversion involves constraints which typically impose desirable properties, such as smoothness, on the solution or limit the propagation of errors. The weight given to these constraints is controlled by trade-off parameters which must be chosen to make optimal use of the given data.

The asymptotic expression (1) is at the origin of simple, yet powerful inversion techniques. In equation (1) the left-hand side can be determined from the observed frequencies; the right-hand side can then be inverted to determine $c(r)$, without the use of a reference model (*e.g.* Gough, 1984; Christensen-Dalsgaard *et al.*, 1985). Similarly, the function $\mathcal{H}_1(w)$ can be determined by fitting an expression of the form given in equation (3) to differences between observed frequencies and frequencies of a reference model. The correction $\delta c/c$ to the model sound speed can subsequently be determined by inverting the expression given in equation (4) (Christensen-Dalsgaard, Gough & Thompson, 1989). Finally, linearization of the full expressions for the frequencies lead to inversion procedures that do not depend on the asymptotic approximation; the resulting linear relations between the frequency differences between the Sun and a model, and the corresponding differences in structure, can be analyzed by means of a variety of techniques (*e.g.* Gough & Thompson, 1991; Goode, section 4; Basu, section 6).

Although some form of inverse analysis is required to obtain detailed information about the solar interior, simple comparisons of frequencies, or combinations of frequencies, may still yield insight into particular aspects of the Sun. An especially important example is the small frequency separations $\delta\nu_{nl}$ between almost equal frequencies of low-degree modes. As discussed above, these are predominantly sensitive to the structure of the solar core. For normal solar models the separations are in good agreement with observations, particularly when helium settling is taken into account (see also Elsworth, section 10). In contrast to that, models where the neutrino fluxes have been reduced through partial mixing of the core or the inclusion of energy transport by Weakly Interacting Massive Particles are

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inconsistent with the observed separations.

As indicated at the begin of this section, analysis of modes trapped in the convection zone provides a test of the equation of state, together with the potential for determining the helium abundance in the outer parts of the Sun (see, for instance, Vorontsov, Baturin & Pamyatnykh, 1991; Christensen-Dalsgaard & Däppen, 1992; Kosovichev *et al.*, 1992). The dominant effect comes from the second ionization zone of helium which is sensitive to the equation of state and the helium abundance, yet sufficiently deep to be largely unaffected by the near-surface uncertainties. By analyzing $\mathcal{H}_2$ Pérez Hernández & Christensen-Dalsgaard (1994) showed that the so-called MHD equation of state provided a substantially better fit in the second helium ionization zone to the observed frequencies than did a somewhat simpler description based on the Eggleton *et al.* (1973) formulation but including Coulomb effects in the Debye-Hückel approximation. On the other hand, Dziembowski, Pamyatnykh & Sienkiewicz (1992) inferred that there are significant departures from the MHD treatment in somewhat deeper regions in the convection zone. These results are striking illustrations of the power of helioseismology to investigate details of the thermodynamic properties of the solar plasma.

4. Seismic solar model (P. Goode)

The method of frequency inversion reveals that within the quoted observational errors, it is possible to achieve a precision of $\sim 10^{-3}$ in the sound speed determination through most of the Sun's interior. Only for $r < 0.05 R_\odot$ is the precision $\sim 10^{-2}$. The accuracy of the density and pressure determinations is only slightly worse. Such restrictions impose significant constraints on the microscopic physical data, i.e. opacities, nuclear reaction cross-sections, and diffusion coefficients as well as on the solar age. The helioseismic age is consistent with that from meteorites.

Recently released low-l solar oscillation data from the BISON network combined with BBSO data yield the most up-to-date *solar seismic model* of the Sun's interior. For the core, the *solar seismic model* from the new data are consistent with the best, current standard solar models. An astrophysical solution to the *solar neutrino problem* fades away.

In constructing this model from seismic data, we make two assumptions: 1) we assume that the only forces acting in the Sun's interior are gravity and pressure, P and 2) we know the adiabatic exponent, Γ_1, as a function of P and the density, ρ, and the chemical composition; and further, that Γ_1 departs significantly from the model value only in the homogeneous outer layers covering the convective envelope and atmosphere. We note that the first assumption implies spherical symmetry and parenthetically that this