ORBITAL EVOLUTION OF TROJAN ASTEROIDS

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ABSTRACT

The motion of the orbits of Trojan asteroids are investigated. Four asteroids around L_5 are shown to have librating perihelion. A criterion for the libration of the perihelion is derived and expressed by the initial conditions. This criterion is solved for the initial conditions in two special cases. Thus regions in the configuration space of the initial conditions yielding libration of the orbits of Trojan asteroids are established.

1. INTRODUCTION

According to the Lagrangian solutions of the three-body problem a small body of negligible mass resting at the relative equilibrium points L_4 or L_5 of the Sun-Jupiter system moves around the Sun on an orbit which is similar to Jupiter's orbit but the perihelions of the two orbits are at 60° from each other. If the body suffers a perturbation it leaves L_4 or L_5 and its orbit around the Sun also changes. The stability investigations of the Lagrangian points usually deal with the motion of the small body around L_4 or L_5. This paper studies the behaviour of the orbit of the small body around the Sun.

The well-known examples for the Lagrangian solutions of the three-body problem are the Trojan asteroids. The main perturbations of the eccentricity e and the longitude $\tilde{\omega}$ of the perihelion of the orbits of these asteroids can be described by the equations (Érdi, 1979)

$$\begin{aligned} e \sin\tilde{\omega} &= a - c \sin\chi , \\ e \cos\tilde{\omega} &= b - c \cos\chi \end{aligned} \tag{1}$$

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where a, b and $c \geq 0$ are constants and γ is a slowly changing function of the time. It can be shown that if

$$c \leq \sqrt{a^2+b^2}$$

then the perihelion of the asteroids librates around a direction which is about at 60° from Jupiter's perihelion. If

$$c > \sqrt{a^2+b^2}$$

then the perihelion of the asteroids circulates.

For an asteroid which is exactly at L_4, $c=0$ and $e=e_J$ where e_J is the orbital eccentricity of Jupiter. Moreover, $\tilde{\omega}-\tilde{\omega}_J=60^\circ$ where $\tilde{\omega}_J$ is the longitude of the perihelion of Jupiter. If the asteroid undergo a small perturbation it leaves L_4 and its orbit begins libration around the equilibrium orbit. The parameter c can be considered as the measure of the perturbation. As c increases so does the amplitude of the libration of the perihelion. When the difference between the two extremum values of $\tilde{\omega}$ exceeds the value 180° the orbit of the asteroid begins to circulate.

The purpose of this paper is to find initial conditions which result in the libration of the perihelion. Knowing the initial position and velocity of an asteroid near L_4 the question is whether the orbit of the asteroid will librate or circulate. For the solution of this problem the results of the author's previous papers (Érdi, 1978, 1979, 1981) are applied.

2. THE MOTION OF THE PERIHELION

Equations (1) were derived from an asymptotic solution for the motion of Trojan asteroids. First a short summary of that solution is given here. Under the assumptions that the motion of a Trojan asteroid around the Sun is perturbed only by Jupiter and Jupiter's orbit around the Sun is an ellipse the equations of motion of the asteroid are (Érdi, 1978)

$$\begin{aligned}
&\frac{d^2r}{dv^2}-r\left(\frac{d\alpha}{dv}\right)^2-2r\frac{d\alpha}{dv}=\frac{1}{1+e_J\cos v}\left[r-\frac{1-\mu}{R_1^3}r+\mu\left(\frac{\cos\alpha-r}{R_2^3}-\cos\alpha\right)\right],\\
&\frac{d}{dv}\left(r^2\frac{d\alpha}{dv}+r^2\right)=\frac{\mu r\sin\alpha}{1+e_J\cos v}\left[1-\frac{1}{R_2^3}\right],\\
&\frac{d^2z}{dv^2}+z=\frac{z}{1+e_J\cos v}\left[1-\frac{1-\mu}{R_1^3}-\frac{\mu}{R_2^3}\right]
\end{aligned} \tag{2}$$

where r, α, z are the cylindrical coordinates of the asteroid (see Figure 1), v is the true anomaly of Jupiter, μ is the mass of Jupiter divided by the total mass of the Sun-

Jupiter system and

$$R_1 = \sqrt{r^2 + z^2}\,, \qquad R_2 = \sqrt{1 + r^2 - 2r\cos\alpha + z^2}\,.$$

The coordinates r and z are dimensionless, the Sun-Jupiter distance is supposed to be unity.

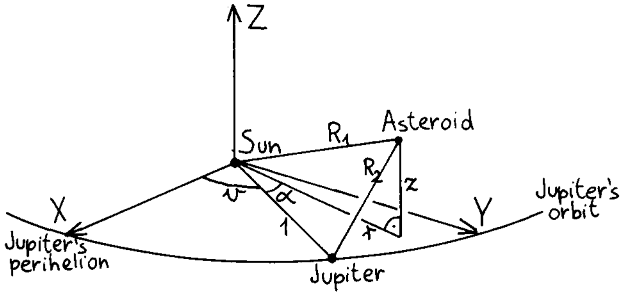

Figure 1. The coordinates r, α, z in the Cartesian coordinate system SXYZ.

The solution of Equations (2) were derived in the form of a three-variable asymptotic expansion (Erdi, 1978, 1981)

$$\begin{aligned} r &= 1 + \sum_{n=1}^{N} \varepsilon^n r_n(v, u, \tau) + O(\varepsilon^{N+1}), \\ \alpha &= \alpha_0(u, \tau) + \sum_{n=1}^{N} \varepsilon^n \alpha_n(v, u, \tau) + O(\varepsilon^{N+1}), \\ z &= \varepsilon^{1/2}\left[\sum_{n=0}^{N} \varepsilon^n z_n(v, u, \tau) + O(\varepsilon^{N+1})\right] \end{aligned} \tag{3}$$

where $\varepsilon = \sqrt{\mu}$, $u = \varepsilon(v - v_0)$, $\tau = \varepsilon^2(v - v_0)$

and v_0 is the initial value of v at the epoch. This solution is a generalization of Kevorkian's two-variable solution for the planar problem (Kevorkian, 1970).

The functions r_n, α_n, z_n depend on the variables v, u, τ representing three different time-scales of the motion of the Trojan asteroids. The variable v corresponds to the orbital revolution of the asteroids around the Sun. The time-scale of the long-periodic librational motion around L_4 or L_5 is represented by the variable u. The variable τ is connected with the motion of the perihelion of the asteroids. The approximate periods are 12, 150, 3600 years.

The determination of the functions r_n, α_n, z_n is deduced to a system of partial differential equations. One of the equations to be solved is

$$\frac{\partial^2 \alpha_0}{\partial u^2} + 3\left[1 - 2^{-3/2}(1 - \cos\alpha_0)^{-3/2}\right]\sin\alpha_0 = 0. \tag{4}$$

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The function $\alpha_0(u)$ describing the main part of the librational motion around L_4 or L_5 can be determined from Equation (4). An integral of Equation (4) is

$$\frac{1}{2}\left(\frac{\partial\alpha_0}{\partial u}\right)^2 - 3\left[\cos\alpha_0 - 2^{-1/2}(1-\cos\alpha_0)^{-1/2}\right] = h \tag{5}$$

where h might depend on τ, but it can be shown that h is a constant.

For moderate librational amplitudes which occur among the known Trojan asteroids the solution of Equation (4) is (Érdi, 1978, 1981)

$$\begin{aligned}\alpha_0 = {} & \frac{\pi}{3} + \frac{3\sqrt{3}}{2^3}\ell^2 + \frac{13\sqrt{3}}{2^8}\ell^4 + \\ & + \ell\cos\phi - \left(\frac{\sqrt{3}}{2^3}\ell^2 + \frac{\sqrt{3}}{2^8 3^2}\ell^4\right)\cos 2\phi + \\ & + \left(\frac{5}{2^6}\ell^3 - \frac{65}{2^{12}}\ell^5\right)\cos 3\phi - \frac{25\sqrt{3}}{2^7 3^2}\ell^4\cos 4\phi + \\ & + \frac{1283}{2^{12}\cdot 3\cdot 5}\ell^5\cos 5\phi + O(\ell^6)\end{aligned} \tag{6}$$

where
$$\phi = \sqrt{\frac{27}{2^2}\left(1 - \frac{3}{2^3}\ell^2 - \frac{37}{2^9}\ell^4\right)}\, u + \delta$$

and ℓ is a constant and δ depends on τ. The parameter ℓ means the approximate librational amplitude around L_4. For most of the known Trojan asteroids $\ell < 0.5$.

Substituting the solution (6) into Equation (5) it follows

$$h = \frac{3}{2} + \frac{27}{2^3}\ell^2 - \frac{81}{2^8}\ell^4 + O(\ell^6). \tag{7}$$

Equations (1) were derived from the solution (3) using the formulas of the two-body problem. In Equations (1) the parameters a, b, c and χ mean

$$a = -e_J\frac{A_2}{A_0}, \quad b = -e_J\frac{A_1}{A_0}, \tag{8a}$$

$$c = \varepsilon\varrho_{11}, \tag{8b}$$

$$\chi = A_0\tau + \psi_{11} \tag{8c}$$

where
$$A_0 = \frac{27}{2^3} + \frac{129}{2^6}\ell^2 - \frac{87}{2^7}\ell^4 + O(\ell^6), \tag{8d}$$

$$\frac{A_1}{A_0} = -\frac{1}{2} - \frac{17}{2^4 3}\ell^2 - \frac{329}{2^8 3^3}\ell^4 + O(\ell^6), \tag{8e}$$

$$\frac{A_2}{A_0} = -\frac{\sqrt{3}}{2} + \frac{73\sqrt{3}}{2^4 3^2}\ell^2 - \frac{6233\sqrt{3}}{2^8 3^4}\ell^4 + O(\ell^6) \tag{8f}$$

and ϱ_{11} and ψ_{11} are constants. Note, that Equations (1) are valid for asteroids around L_4. In case of asteroids around L_5 the parameter α must be changed by $-\alpha$.

In the paper (Érdi, 1979) Equations (1) were used to study the motion of the perihelion of 30 known Trojan asteroids. The parameter ℓ and the constants ϱ_{11}, ψ_{11} can be determined from the osculating orbital elements of the asteroids. The parameter ℓ can also be calculated from the endpoints of the libration around L_4 or L_5. Thus it can be determined which Trojan asteroids have librating perihelion. From the 30 investigated asteroids 10 asteroids, all around L_4, proved to show the libration of the perihelion. A numerical integration of the planar elliptic restricted three-body problem (Érdi and Presler, 1980) confirmed the perihelion-libration of the same ten asteroids. In the case of the asteroid PL 4572 the libration of the perihelion was also shown by Bien (1980).

Equations (1) are applied here for 5 Trojan asteroids, all around L_5, which were not included in the earlier investigation. Table 1 shows the limits of the variations of e and $\tilde{\omega}$ of these asteroids and also the approximate periods of the variations of e and $\tilde{\omega}$, obtained from Equations (1) using the osculating orbital elements at the epoch December 27.0, 1980 ET.

Table 1. Variations of e and $\tilde{\omega}$ of 5 Trojan asteroids around L_5.

	e	$\tilde{\omega}$	$T_{e,\tilde{\omega}}$
1871 Astyanax	0.029-0.059	291°3-331°3	3230 years
1872 Helenos	0.029-0.060	287.7-329.6	3320
1870 Glaukos	0.030-0.066	279.4-323.4	3600
1867 Deiphobus	0.012-0.077	262.9-356.1	3290
1873 Agenor	0.077-0.172	0 -360	3590

Thus in the case of the asteroid 1873 Agenor the perihelion circulates and in the other four cases the perihelion librates. That means that from 35 known Trojan asteroids 14 asteroids have librating perihelion. (It must be mentioned that in Equations (1) and also in Table 1 $\tilde{\omega}$ is counted from the perihelion of Jupiter.)

Figure 2 shows the $e,\tilde{\omega}$ trajectories for 10 known Trojan asteroids around L_5. For those asteroids which are not included in Table 1 the trajectories were taken from the paper (Érdi, 1979).

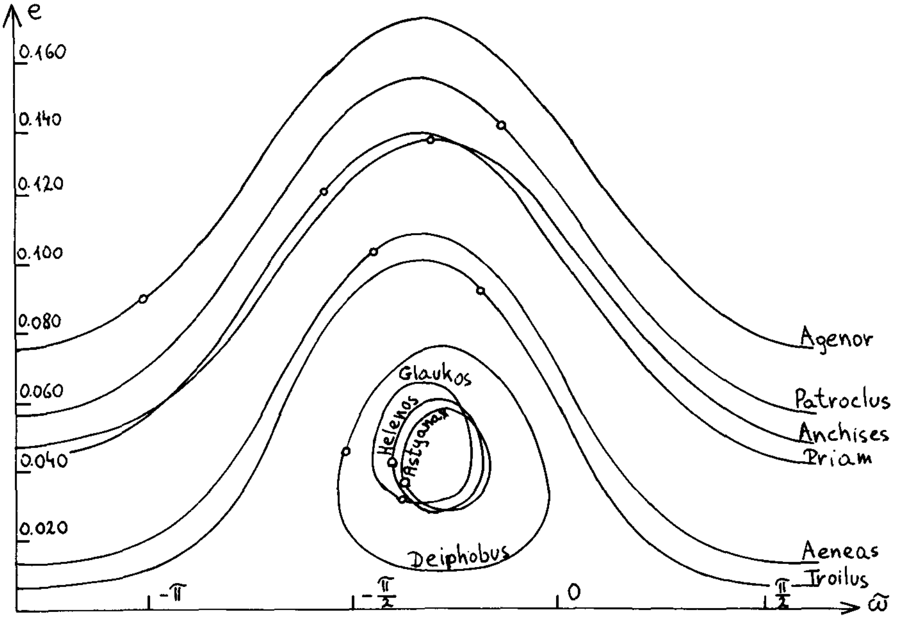

Figure 2. Variations of e and $\tilde{\omega}$ of 10 Trojan asteroids around L_5. The circles indicate the values at the epoch Dec. 27, 1980.

3. A CRITERION FOR THE LIBRATION OF THE PERIHELION

Considering Equations (1), (8a) and (8b) the condition for the libration of the perihelion is

$$0 \leqq \varrho_{11} \leqq e_1 \frac{\sqrt{A_1^2 + A_2^2}}{A_0} \tag{9}$$

where e_1 is defined by the equation

$$e_J = \varepsilon e_1 . \tag{10}$$

In order to determine such initial conditions which result in the libration of the perihelion the condition (9) should be expressed by the initial conditions of the motion. For the sake of simplicity only the planar motion of Trojan asteroids will be considered here.

Let the initial conditions at $v = v_0$ be

$$\begin{aligned} r &= 1 + \varepsilon r_0 , \qquad & \alpha &= \alpha_{00} , \\ \frac{dr}{dv} &= \varepsilon \dot{r}_0 , \qquad & \frac{d\alpha}{dv} &= \varepsilon \dot{\alpha}_{00} . \end{aligned} \tag{11}$$

Then using the approximate solution (Érdi, 1981)

$$\tau = 1 + \varepsilon\left[\rho_1 \cos(v + \psi_1) - \frac{2}{3}\frac{\partial \alpha_0}{\partial u}\right] + O(\varepsilon^2),$$
$$\alpha = \alpha_0 + \varepsilon\left[-2\rho_1 \sin(v + \psi_1) + q_1\right] + O(\varepsilon^2)$$

where q_1 is a known function of u and τ, and

$$\frac{d\tau}{dv} = -\varepsilon \rho_1 \sin(v + \psi_1) + O(\varepsilon^2),$$
$$\frac{d\alpha}{dv} = \varepsilon\left[-2\rho_1 \cos(v + \psi_1) + \frac{\partial \alpha_0}{\partial u}\right] + O(\varepsilon^2),$$

it follows that at $v = v_0$

$$\alpha_0 = \alpha_{00}, \quad \frac{\partial \alpha_0}{\partial u} = -3(2\tau_0 + \dot{\alpha}_{00}), \tag{12a}$$

$$\begin{aligned} \rho_1 \cos\psi_1 &= -(3\tau_0 + 2\dot{\alpha}_{00})\cos v_0 - \dot{\tau}_0 \sin v_0, \\ \rho_1 \sin\psi_1 &= -\dot{\tau}_0 \cos v_0 + (3\tau_0 + 2\dot{\alpha}_{00})\sin v_0, \end{aligned} \tag{12b}$$

$$q_1 = -2\dot{\tau}_0. \tag{12c}$$

Using the equations (Érdi, 1981)

$$\rho_1 \cos\psi_1 = \rho_{10} \cos(\alpha_0 + \psi_{10}) + e_1,$$
$$\rho_1 \sin\psi_1 = \rho_{10} \sin(\alpha_0 + \psi_{10}),$$
$$\rho_{10} \cos\psi_{10} = \rho_{11} \cos(A_0\tau + \psi_{11}) + e_1 \frac{A_1}{A_0},$$
$$\rho_{10} \sin\psi_{10} = -\rho_{11} \sin(A_0\tau + \psi_{11}) - e_1 \frac{A_2}{A_0}$$

and Equations (12b), the constant ρ_{11} can be expressed by the initial conditions. Thus the criterion (9) can be substituted by the following criterion for the libration of the perihelion

$$\begin{aligned} -e_1^2 \frac{A_1^2 + A_2^2}{A_0^2} \leqq\ & e_1^2 + (3\tau_0 + 2\dot{\alpha}_{00})^2 + \dot{\tau}_0^2 + \\ & + 2e_1^2\left(\frac{A_1}{A_0}\cos\alpha_{00} + \frac{A_2}{A_0}\sin\alpha_{00}\right) + \\ & + 2e_1\left[\frac{A_1}{A_0}(3\tau_0 + 2\dot{\alpha}_{00}) - \frac{A_2}{A_0}\dot{\tau}_0\right]\cos(v_0 + \alpha_{00}) + \\ & + 2e_1\left[\frac{A_2}{A_0}(3\tau_0 + 2\dot{\alpha}_{00}) + \frac{A_1}{A_0}\dot{\tau}_0\right]\sin(v_0 + \alpha_{00}) + \\ & + 2e_1(3\tau_0 + 2\dot{\alpha}_{00})\cos v_0 + 2e_1\dot{\tau}_0 \sin v_0 \leqq 0. \end{aligned} \tag{13}$$

Note, that in this inequality A_1/A_0 and A_2/A_0 depend on the initial conditions through Equations (12a), (5), (7), (8e), (8f). Given the initial conditions τ_0, α_{00}, $\dot{\tau}_0$, $\dot{\alpha}_{00}$ at $v = v_0$ it can be determined from (13) whether the perihelion makes libration (the inequality is satisfied) or circulation (the inequality is not satisfied). Two special cases will be considered next.

4. INITIAL CONDITIONS FOR PERIHELION-LIBRATION

4.1 Limit-velocity curves at L_4

The following problem is considered. Putting a small body into the point L_4 and starting it in a given direction with different initial velocities increasing from zero, determine that critical velocity where the libration of the perihelion of the orbit of the body changes to circulation. The critical velocities in different directions form a limit-velocity curve inside which all velocities, given to the body as initial velocity, will result in the libration of the perihelion. At different values of v_0, that is at different initial configurations of the three bodies, the limit-velocity curves are different.

Substituting $r_0=0$, $\alpha_{00}=\pi/3$ into (13) the condition for the libration of the perihelion in this case is

$$-e_1^2\frac{A_1^2+A_2^2}{A_0^2}\leqq K\leqq 0 \tag{14}$$

where
$$K=\dot{r}_0^2+B\dot{r}_0+C \tag{15}$$

and
$$\left.\begin{aligned}
B&=2e_1(F\sin v_0+G\cos v_0),\\
C&=4\dot{\alpha}_{00}^2+e_1^2(2F-1)+4e_1\dot{\alpha}_{00}(F\cos v_0-G\sin v_0),\\
F&=1+\frac{1}{2}\frac{A_1}{A_0}+\frac{\sqrt{3}}{2}\frac{A_2}{A_0},\\
G&=\frac{\sqrt{3}}{2}\frac{A_1}{A_0}-\frac{1}{2}\frac{A_2}{A_0}.
\end{aligned}\right\} \tag{16}$$

According to Equations (12a) and (5) in the case $r_0=0$, $\alpha_{00}=\pi/3$

$$h=\frac{3}{2}+\frac{9}{2}\dot{\alpha}_{00}^2$$

and from Equation (7) approximately

$$\ell^2=\frac{2^4}{3}\left(1-\sqrt{1-\frac{1}{2}\dot{\alpha}_{00}^2}\right). \tag{17}$$

Considering Equations (8e), (8f) and (16) now it can be seen that in (15) B and C depends only on $\dot{\alpha}_{00}$ and v_0. It can be shown that the minimum of K according to v_0 is

$$K_{min}=4\dot{\alpha}_{00}^2+e_1^2(2F-1)-4e_1\dot{\alpha}_{00}\sqrt{F^2+G^2}$$

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and $$-e_1^2 \frac{A_1^2 + A_2^2}{A_0^2} \leq K_{min} \tag{18}$$

for every $\alpha_{00}^{\cdot}$ in the interval (-0.6, 0.6) where Equation (17) gives those values of ℓ for which the solution (6) is sufficiently accurate. The equality holds at $\alpha_{00}^{\cdot}=0$, when $K_{min}=-e_1^2(A_1^2+A_2^2)/A_0^2 = -e_1^2$.

It follows from (15) and (18) that for every values of v_0 and at a given value of $\alpha_{00}^{\cdot}$ from the interval (-0.6, 0.6)

$$K \leqq 0$$

if $$r_{02}^{\cdot} \leqq r_0^{\cdot} \leqq r_{01}^{\cdot}$$

where $$r_{01}^{\cdot} = -\frac{B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 - C}\,, \quad r_{02}^{\cdot} = -\frac{B}{2} - \sqrt{\left(\frac{B}{2}\right)^2 - C}\,.$$

At $\alpha_{00}^{\cdot}=0$: $r_{01}^{\cdot} = e_1$, $r_{02}^{\cdot} = -e_1$ for every v_0.

Figure 3 shows the limit-velocity curves for $v_0=0$, $\pi/2$, π, $3\pi/2$. The curves for any v_0 and $v_0+\pi$ are mirror-images of each other for the centre of the coordinate system. The curves are open in the investigated region of $\alpha_{00}^{\cdot}$. For larger values of $|\alpha_{00}^{\cdot}|$ a numerical integration is necessary to determine the limit-velocity curves.

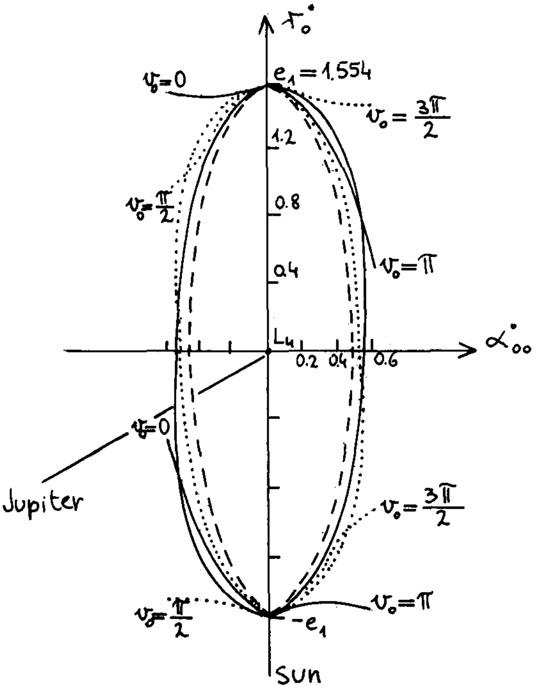

Figure 3. Limit-velocity curves at L_4.

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Considering the extremum values of $\dot{\tau}_{01}$ and $\dot{\tau}_{02}$ according to ϑ_0, an estimation can be derived for the allowed region of $\dot{\tau}_0$ at a given value of $\dot{\alpha}_{00}$ which is valid for every ϑ_0. According to this estimation

$$|\dot{\tau}_0| \leqq -e_1\sqrt{F^2+G^2} + \sqrt{-4\dot{\alpha}_{00}^2 + e_1^2(1-2F) - 4e_1\dot{\alpha}_{00}\sqrt{F^2+G^2}} \quad \text{if } \dot{\alpha}_{00} \geqq 0$$

$$|\dot{\tau}_0| \leqq -e_1\sqrt{F^2+G^2} + \sqrt{-4\dot{\alpha}_{00}^2 + e_1^2(1-2F) + 4e_1\dot{\alpha}_{00}\sqrt{F^2+G^2}} \quad \text{if } \dot{\alpha}_{00} < 0$$

for every ϑ_0. The corresponding limit-velocity curve is also shown on Figure 3 (dashed line). Inside that curve every velocity as initial velocity at L_4 will result in the libration of the perihelion of the small body for every initial configurations of the three bodies.

4.2 Libration with zero initial velocity

Let us determine that region around L_4 in which a small body starting from any point with zero initial velocity (in the rotating coordinate system in which Equations (2) are valid) will have an orbit with librating perihelion. A similar problem was studied by McKenzie and Szebehely (1981) but they determined initial positions around L_4 and L_5 with zero initial velocities for librational motions around L_4 and L_5 in the Earth-Moon system.

Now $\dot{\tau}_0 = 0$, $\dot{\alpha}_{00} = 0$, and (13) takes the form

$$-e_1^2\frac{A_1^2 + A_2^2}{A_0^2} \leqq K \leqq 0 \tag{19}$$

where $$K = 9\tau_0^2 + B\tau_0 + C, \tag{20}$$

and
$$\left.\begin{aligned} B &= 6e_1(F\cos\vartheta_0 - G\sin\vartheta_0), \\ C &= e_1^2(2F-1), \\ F &= 1 + \frac{A_1}{A_0}\cos\alpha_{00} + \frac{A_2}{A_0}\sin\alpha_{00}, \\ G &= \frac{A_1}{A_0}\sin\alpha_{00} - \frac{A_2}{A_0}\cos\alpha_{00}. \end{aligned}\right\} \tag{21}$$

Those values of τ_0, α_{00} are to be determined for which the inequality (19) is satisfied.

It follows from Equations (12a), (5) and (7) that

$$\ell^2 = \frac{2^4}{3}\left(1 - \sqrt{\frac{7}{6} - 2\tau_0^2 + \frac{1}{3}\left[\cos\alpha_{00} - 2^{-1/2}(1-\cos\alpha_{00})^{-1/2}\right]}\right) \tag{22}$$

and according to Equations (8e), (8f) and (21) the parameters B and C in (20) depend on both τ_0 and α_{00}. Thus the solution of the inequality (19) can be obtained only numerically.

Figure 4. shows a solution which has been obtained by the following method. For a given value of α_{00} the values of K are calculated for different values of r_0 increasing and decreasing from zero. Then it is decided whether (19) is satisfied or not. The procedure is repeated for different values of α_{00} around $\alpha_{00} = \pi/3$.

According to Equation (22) as $|r_0|$ increases ℓ also increases and after a value of ℓ about 0.6 the solution (6) will not be accurate enough. Figure 4. has been obtained by fixing the upper value of ℓ as 0.5 which is valid for most of the known Trojan asteroids. Thus the curve on Figure 4 corresponds to those values of r_0 and α_{00} for which $\ell = 0.5$. Along and inside this curve the inequality (19) is satisfied for every value of v_0.

The region in which the small body at any point with zero initial velocity will have an orbit with librating perihelion is certanly more extended than shown on Figure 4. However, its accurate size can be determined only by numerical integration.

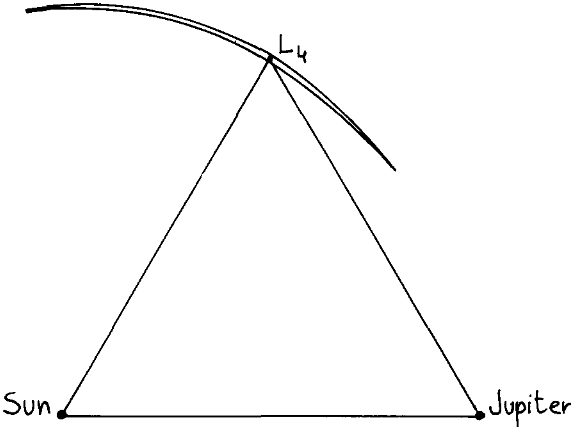

Figure 4. A region around L_4 for the libration of the perihelion with zero initial velocity.

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