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ARTICLE

Broad Divisia money, supply pressures, and U.S. inflation following the COVID-19 recession

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Abstract

The rise of U.S. inflation in 2021 and 2022 and its partial subsiding have sparked debates about the relative role of supply and demand factors. The initial surge surprised many macroeconomists despite the unprecedented jump in money growth in 2020–21. We find that the relationship between consumption and the theoretically based Divisia M3 measure of money (velocity) can be well modeled both in the short- and long-runs. We use the estimated long-run relationship to calculate the deviation of actual velocity from its long-run equilibrium and incorporate it into a P-Star framework. Our model of velocity significantly improves the performance of the P-Star model relative to using a one-sided HP filter to calculate trend velocity as used by other researchers. We also include a global supply pressures index in the model and find that recent movements in U.S. inflation largely owed to aggregate demand driven macroeconomic factors that are tracked by Divisia money with a smaller role played by supply factors.

Keywords: money; Divisia; inflation; supply chain disruptions

JEL classifications: E51; E41; E52; E58

1. Introduction

The rise and partial ebbing of U.S. inflation since the COVID Recession (Figure 1) has spawned studies assessing the roles played by supply chain disruptions and excess demand growth. Some analysts argue that supply chain disruptions can largely account for the recent swings in inflation. However, most analysts argue that the combination of supply constraints and a strengthening of aggregate demand from stimulatory monetary and fiscal policy actions had created excessive upward pressures on inflation (see Blanchard et al. 2022; Bordo and Levy, 2022; Bolhuis et al. 2022; DeSoyres et al. 2022; Farie-e-Castro, 2025). Other studies decompose these influences using VARs, such as Gordon and Clark (2023), Hall et al. (2023), and Liu and Nguyen (2023) or use model-based estimates (Bernanke and Blanchard 2025; Koch and Noureldin, 2023) or model-based calibrations (Di Giovanni et al. 2022). Beyond food and energy cost shocks, the latter studies consider shocks emanating from either supply chain disruptions or COVID-related shifts in the composition of demand that spawned imbalances between supply and demand.

The view that supply shocks alone cannot explain U.S. inflation is consistent with nominal GDP exceeding its pre-COVID trend and core PCE inflation exceeding a 2 percent path since late 2021 (Bordo and Duca, 2025). And while the inflation surge in 2020-21 coincided with the rise in

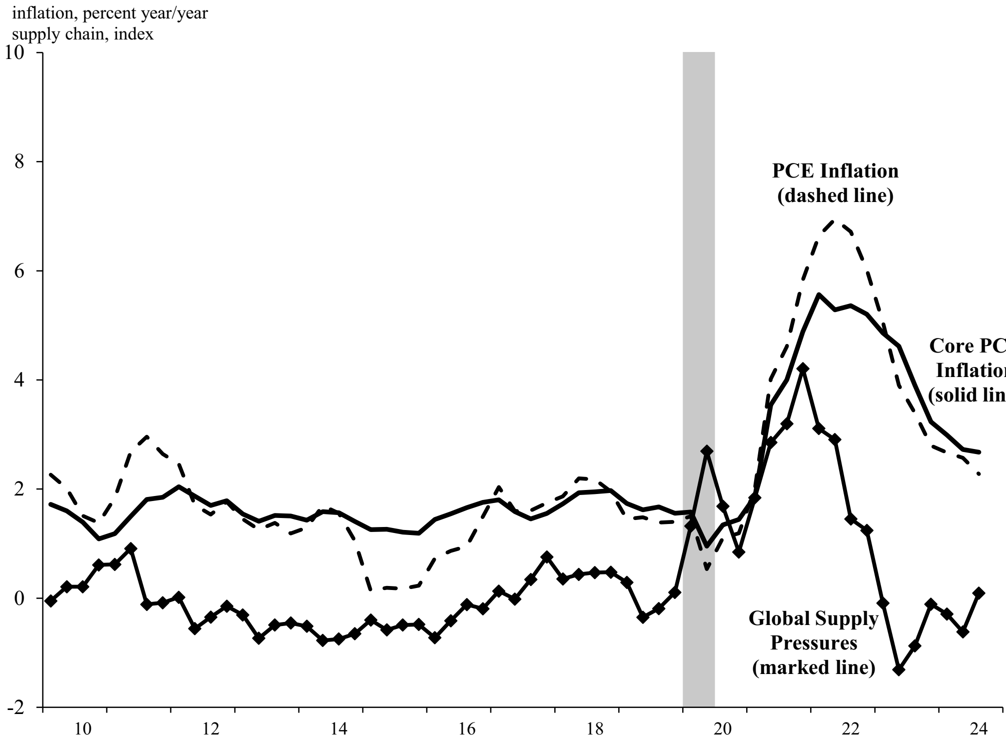

Figure 1. U.S. inflation and supply chain pressures post-Global Financial Crisis (GFC).
Sources: BEA and FRBNY.

the Federal Reserve Bank of New York's (FRBNY) global index of supply pressures (Figure 1), the price level did not fully return to its pre-pandemic path consistent with 2 percent inflation after supply pressures fell back to their pre-pandemic range. Actual nominal GDP growth and inflation imply that aggregate demand contributed to inflation as the pandemic unwound.

Although broad Divisia money growth surged in 2020 (Figure 2) many macro-economists ignored this signal because of perceived instability in the demand for conventionally defined monetary aggregates.[1] By weakening the link between money (M) and GDP, such instability undermined the link between money and inflation. Hallman et al.'s (1991) original P-Star model of inflation rearranges the quantity theory of money equation, $MV = PY$, where Y is real income and P is the price level, to infer the long-run price level, $P^* = MV^*/Y^*$, where Y^* is long-run or trend real income, M is a monetary aggregate (originally simple-sum M2), and V^* is the long-run equilibrium velocity of money toward which velocity gradually converges (error-corrects). In this framework, inflation depends not only on its own lags, but also on the *price gap*, the extent to which the price level deviates from its long-run value. The price gap can be decomposed into the gap between long-run equilibrium velocity and its current level and the cyclical component of real income. That is, $\ln(P^*/P) = \ln(V^*/V) + \ln(Y/Y^*)$, where the first term is the velocity gap and the latter is the cyclical component of real income.

If long-run velocity is unstable or difficult to model, the velocity gap will be less informative, thereby undermining the usefulness of the P-Star approach. Belongia and Ireland (2015, 2017) estimate P-Star models for inflation and nominal income using a one-sided (i.e., real-time) HP filter to model trend velocity.[2] An innovative aspect of their work is that they use Divisia monetary aggregates rather than conventional money measures to define velocity. Building on Bordo and Duca's (2025) analysis of GDP measures of velocity, we employ a small-scale error-correction

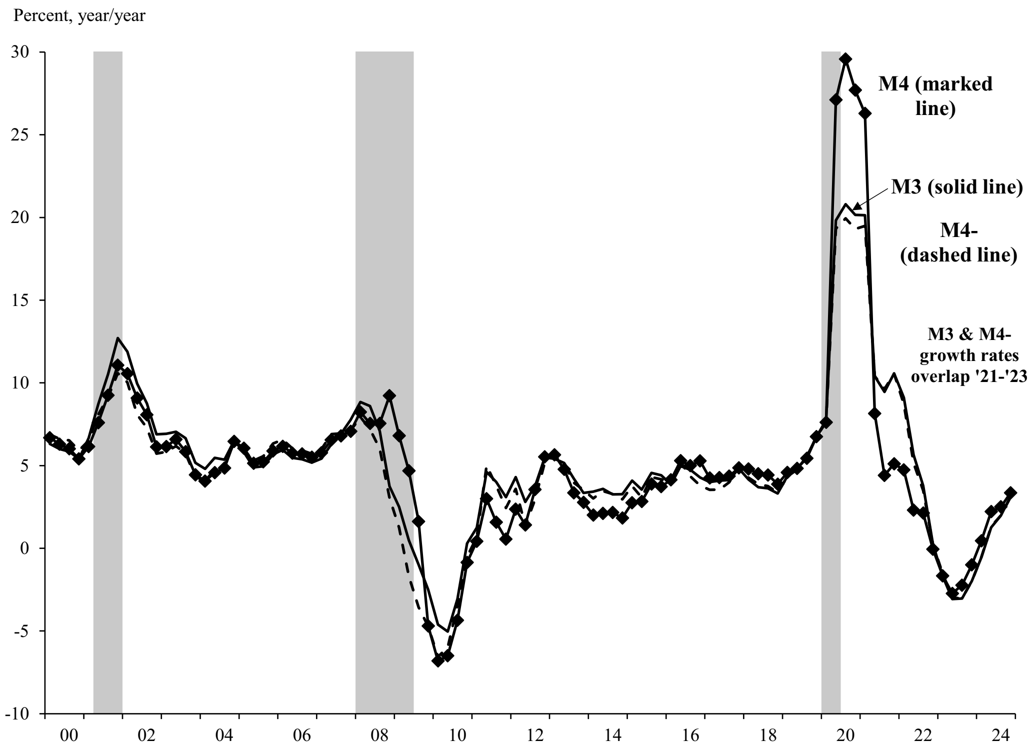

Figure 2. U.S. broad Divisia money growth surges in the COVID-19 recession in sharp contrast to the great recession.
Sources: Center for Financial Stability and authors' calculations.

model to estimate trend consumption velocity and incorporate the resulting velocity gap into a P-Star model of PCE inflation.[3] Our approach focusses on the percentage fees (loads) for buying or selling mutual funds, declines in which in turn led to a permanent decline in the demand for money in the late 1980s and 1990s–especially small time deposits (Duca, 2000; Bordo and Duca, 2025). This caused an upward level shift in velocity that led the P-Star model to underpredict inflation, discrediting simple-sum M2 versions of this framework (see Becsi and Duca, 1994; Ireland, 2022, 2023). Simple sum monetary aggregates assume perfect substitutability between their components implying that they may be distorted by optimal portfolio reallocation in response to interest rate changes and, moreover, M2 may be too narrow to capture some of the substitutions involved (see Barnett, 1982; Barnett et al. 2013).

We find that the consumption velocity of Divisia M3 has a stable long-run relationship with the mutual fund cost variable we employ when we control for the effects of the Commodity Futures Modernization Act (CFMA). We accomplish the latter by estimating a broken constant version of the standard cointegration framework from Johansen et al. (2000). We then use the estimated long-run relationship to calculate the velocity gap out-of-sample, which we incorporate into a P-Star framework from 2013 to 2024. Following Ireland (2024), we center our analysis on PCE-based measures of inflation. We add a role for supply pressures using an index from the FRBNY, and we add dummies for extreme declines in energy prices and for 2020 to account for the unprecedented scale of the pandemic shock.

As in Bordo and Duca (2025), the acute initial phase of the pandemic is associated with a very steep drop in Divisia M3's velocity reflecting both the scale of the real economic shock and the massive money growth shown in Figure 2. Divisia M3's level abruptly shifted upwards relative to

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its pre-pandemic trend and its annual growth rate was over 18% between 2020Q2 and 2021Q1 and was over 9% for the following three quarters. Divisia's annual growth fell rapidly during 2022 and turned negative in 2023 as the Fed tightened monetary policy. Divisia M3's velocity correspondingly declined sharply in 2020Q2, rebounded sharply in 2020Q3, and then gradually rose, surpassing its 2020Q1 level in mid-2023. It took a while for inflation to reach its peak and largely ebb as shown in Figure 1. As COVID-19 waned, much pent-up aggregate demand was released that bolstered inflation due to both supply- and demand-side factors in 2021 to early 2023, reflected in the unwinding of earlier pandemic-related declines in velocity. The disinflationary impact of monetary policy tightening took hold starting in early 2023.

We find that the P-Star model is able to explain much of the increase and subsequent ebbing of inflation in terms of the deviation of velocity from its long-run equilibrium level even after controlling for supply pressures. In fact, we find that while the estimated contribution of the index of global supply pressure to the temporary run-up of core inflation is notable, more of it stems from aggregate demand pressures reflected by the price and velocity gap variables from the P-star model.[4] Finally, we show that our cointegration model of velocity significantly enhances the performance of the P-Star model relative to using a one-sided HP filter to calculate trend velocity.

The paper is organized as follows. Section 2 discusses Divisia monetary aggregates and presents our empirical specification of consumption velocity. Section 3 presents results from estimating velocity models. Section 4 lays out our P-Star model, which models core and overall PCE inflation from 2013 to 2024. Those models track aggregate demand pressures by incorporating detrended real consumption and the gaps between velocity and its estimated long-run equilibrium value, where the latter uses velocity coefficients estimated through 2012 to gauge equilibrium velocity since 2013. While we find that global supply pressures explain some of the large swings in core inflation since 2020, our results imply that swings in aggregate demand pressures driven by money growth played a larger role. Section 5 concludes by discussing our findings in a broader context.

2. The demand for Divisia monetary services

This section begins with a brief comparison of Divisia versus simple-sum measures of money before discussing the determinants of the demand for Divisia monetary services. Based on this discussion, Section 3 specifies estimable models of the velocity of Divisia money.

2.1. Broad Divisia measures of money versus narrow Divisia and simple-sum measures of money

Conventional (simple-sum) monetary aggregates implicitly assume that different monetary components are perfect substitutes by adding them up when aggregating. This is inconsistent with empirical estimates of elasticities of substitution from demand models for the underlying components (e.g., Jones et al. 2008a, b; Fleissig and Jones, 2015; Jadidzadeh and Serletis, 2019; Xu and Serletis, 2022). Divisia monetary measures do not make implicit assumptions about substitution elasticities between the monetary assets that they include. However, the oft stated ability of Divisia aggregates to internalize pure substitution effects between its components does require weak separability without which demand for the aggregate can become unstable (see Barnett, 1982). Jones et al. (2005) found that separability tests favored broad over narrow aggregates when the monetary assets were adjusted for retail sweep accounts but not for commercial sweep accounts. Jadidzadeh and Serletis (2019) tested various separability structures within the Divisia M4 monetary aggregate published by the Center for Financial Stability (CFS), but rejected all of them, leading them to favor the broadest available monetary aggregate. Hjertstrand et al. (2016) found that M1 and a broad aggregate similar to M4 were weakly separable from consumption and leisure, but the evidence for M2 and M3 was weaker. We focus on Divisia M3, noting that movements in it are very similar to CFS's Divisia M4- and Divisia M4.[5]

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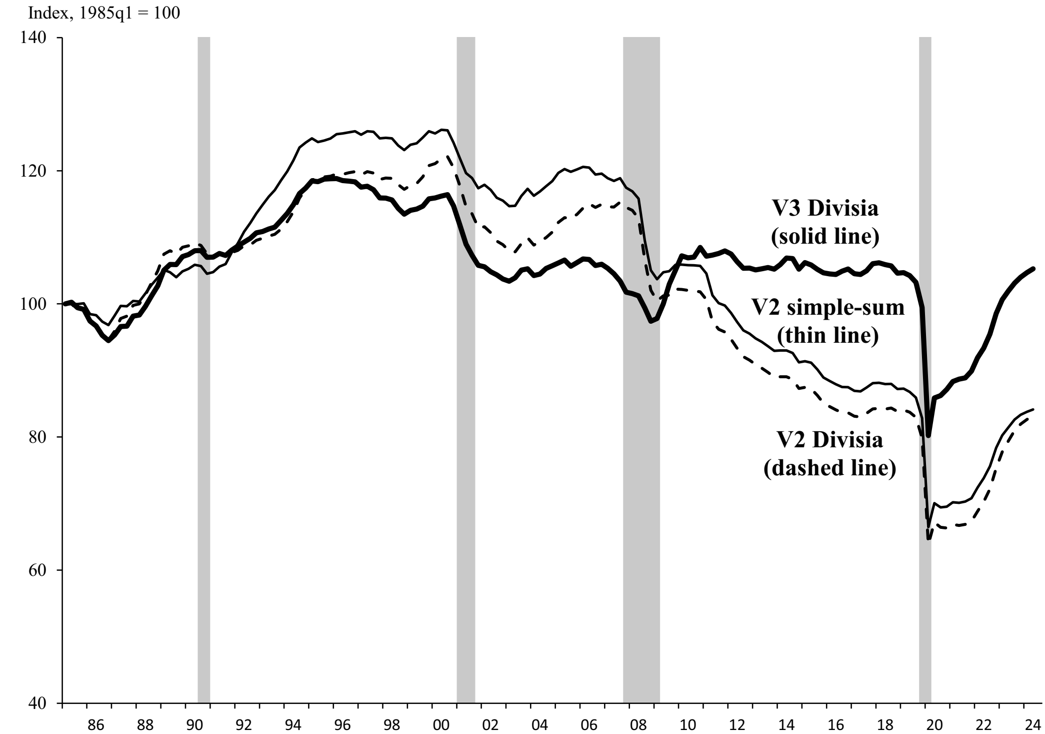

Figure 3. Since the mid-1980s, the consumption velocity of broader Divisia money (M3) is more stable than that of simple-sum and Divisia M2 in the U.S.
Sources: CFS, Federal Reserve, and authors' calculations. Shaded areas are NBER recessions.

The demand for Divisia money can be affected by changes in the liquidity of assets outside of it. This limitation suggests that broader Divisia indexes (e.g., M3) may be preferable to Divisia M2, because they are better able to internalize shifts between liquidity provided by insured deposits in M2 *vis-à-vis* uninsured liabilities issued by banks (e.g., large time deposits), institutional money funds, and repurchase agreements (RPs) outside of M2.[6] While the velocity of broad Divisia money may be affected by shifts in the liquidity of assets not spanned by them, the factors driving those changes are less numerous than for Divisia M2 and are feasible to track for Divisia M3 (Bordo and Duca, 2025). Indeed, consumption velocity (nominal PCE divided by money) is more mean reverting for Divisia M3 than for simple sum or Divisia M2 over the past four decades (Figure 3). As shown in Figure 3, Divisia M3 velocity does not exhibit any clear discernable trend.

2.2. *Pre-pandemic long-run demand for Divisia monetary services*

The long-run demand for money depends on overall transactions, as well as factors altering the relative liquidity of money versus that of other assets. We now provide an overview of the determinants of money demand that we include in our error-correction model of velocity. Our focus on stock loads and CFMA is very similar to Bordo and Duca (2025), although we use consumption as the scale variable rather than GDP and consider several opportunity cost variables.

2.2.1. Tracking the transactions demand for Divisia M3

Since Friedman (1956) redefined the quantity theory of money by replacing overall transactions with nominal GDP, most money demand models have tracked the transactions demand for money with nominal GDP. However, for five reasons and in line with Ireland (2024), we use a version of

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the P-Star model that relates PCE inflation to the gap between current and long-run consumption velocity rather than GDP velocity. First, we focus on modeling PCE measures of inflation, which the Federal Reserve monitors and targets. Second, consistent with Friedman (1956, 1957) and later empirical models of money demand, the demand for money is more linked to permanent income and consumption than to GDP, and the Life-Cycle/Permanent Income Hypotheses suggest that PCE can proxy for permanent income (Cochrane, 1994). Third, the bulk of monetary assets within the Divisia aggregates are held by households, whose main spending is tracked by PCE. These reasons suggest that consumption velocity (PCE/M) should be less variable than GDP velocity (GDP/M), which is consistent with a lower standard deviation of consumption versus GDP velocity. Fourth, large swings in net exports are associated with nominal spending and output diverging. If spending is more pertinent to money demand than output (GDP), then consumption may better track the transactions demand for money. Fifth, and likely reflecting the impact of the second through fourth of these considerations when we use PCE instead of GDP, we do not need any non-COVID short-run shock variables in our cointegration models to avoid serial correlation in the residuals.

2.2.2. Factors affecting the liquidity provided by assets not covered by Divisia measures

The literature on Divisia indexes tends to overlook how and why the demand for broad indexes can shift in the long run and vary in the short run owing to factors other than income (consumption) and user costs or interest rates. We show that by addressing these issues Divisia M3 can be useful in tracking inflation. In doing so, our analysis helps address the skepticism about tracking money growth in light of past instability in the demand for simple-sum monetary aggregates such as M2.

Our specifications of the demand for monetary services from liquid assets emphasize two major factors that shift the demand between conventional monetary assets, and assets that traditionally have been viewed as less liquid. The first is the cost for the marginal agent holding money of shifting between monetary assets and stocks. The bulk of M2 and M3 assets are held by middle- and upper-middle income households, for whom stock mutual funds have been the main vehicle to hold a diversified portfolio of stocks (Duca, 2000). Lower proportional transfer costs reduce money demand in contrast to the implications of the overly stylized Baumol-Tobin framework (see Brunner and Meltzer, 1967; Duca, 2000). Following Anderson et al. (2017), who extend Duca's (2000) analysis of simple-sum M2 velocity, we assume the transactions costs of switching between monetary assets and stock mutual funds is proportional to the average load fee for purchasing or selling a stock mutual fund in a one-year horizon ($SLoad$), based on extending Anderson et al.'s (2017) series. This variable is discussed at length in Anderson et al. (2017) and in Bordo and Duca (2025). The data we use are identical to those calculated by the latter. As these costs fall, the liquidity of stock funds rises which lowers the demand for Divisia indexes that omit such assets.

The second major factor that affected the demand for Divisia indexes is a financial regulation that induced changes in financial intermediation by nonbank financial intermediaries ("shadow banks") that issue higher-liquidity short-term debt to fund lower-liquidity (often long maturity) assets. The passage of the Commodity Futures Modernization Act (CFMA) in late 2000 made it feasible for credit default swaps (CDS) to be widely used to reduce the tail risk of assets (e.g., non-government issued MBS), after which CDS issuance became notable (Duca and Ling, 2020).[7] Such enhancements allowed many nonbanks (e.g., conduits, investment banks, and special investment vehicles) to use derivatives to protect their portfolios from tail risk and helped them obtain investment grade ratings on the short-term debt that they used to fund investments. In turn, the investment grade short-term debt was purchased by banks and institutional money market mutual funds, the latter of which tripled their liabilities between the passage of CFMA and mid-2009. In periods of low financial market stress, CFMA also induced increased repo activity, as formerly ineligible paper became acceptable as collateral, thereby expanding the pool of assets used in repurchase agreements and the monetary services they provide. In seven years, institutional money funds and security broker RPs soared from 13 percent of M3 (OECD) to 25 percent until Lehman failed. By inducing large increases in highly liquid institutional money funds and RPs, CFMA was associated with an upward level shift in the broader Divisia aggregates and a corresponding downward shift in their velocities starting in 2001 when the law took effect. This is evident in the sharp jump in CDS issuance in early 2001 and likely reflects that the law's passage was highly anticipated in 2000Q4, when the bill passed the House by a large majority and by unanimous consent by the U.S. Senate. We account for CFMA by allowing for a break in the constant in a model of long-run velocity coinciding with the passage of CFMA as detailed below.

2.2.3. Opportunity cost of money

Monetary aggregation theory builds upon the user cost of monetary assets; see Barnett (1978) and Donovan (1978). From the user cost formula, the opportunity cost of the monetary services derived from holding a particular asset corresponds to the foregone interest from holding that asset relative to a non-monetary "benchmark" asset. The growth rate of a Divisia index is the average expenditure share weighted sum of the growth rates of its component monetary assets, where the expenditure on each monetary asset depends on its user cost. Thus, monetary asset user costs are a key feature of how Divisia aggregates are computed. In addition, money demand models of Divisia aggregates often feature an opportunity cost variable. Some authors, such as Belongia and Ireland (2016, 2018, 2019), emphasize the use of aggregation-theoretic dual user cost measures, such as those produced by CFS, to model Divisia money demand.[8]

Others, however, use a standard interest rate as an opportunity cost variable. Barnett et al. (2022), for example, estimate cointegration relationships between money, output, interest rates, and real effective exchange rates for several countries including the U.S. They note that short-term interest rates are typically used to model narrow money measures, but long-term interest rates are typically used for broad money, such as Divisia M3 explaining that "[t]he conventional rationale for this choice is that many components of broad money, such as time deposits, are associated with long-term interest rates." Following this reasoning, we assess both the Divisia M3 dual user cost index (*Usercost*) and the ten-year Treasury yield (*Bond*) as additional variables in our models.[9] As shown in Figure 4, the *Usercost* series is more variable and mean reverting than the slower moving stock load series, which experiences substantial declines in the 1980s and 1990s.

The zero lower bound on short-term interest rates binds in much of the latter part of our sample (2008–2015 and 2020–2021), which by compressing user costs to a low positive range, could complicate the interpretation of dual user costs over those periods. Mattson and Valcarcel (2016) show that the fall of the federal funds rate "likely passed through to the user costs of many of the assets" in Divisia M4 resulting in "unprecedented compression in user costs" and to convergence between Divisia and simple-sum M4 starting in 2009. Nevertheless, the dual user cost does not hit the zero lower bound for any of the CFS Divisia measures (Chen and Valcarcel, 2024).[10]

2.3. Modelling the consumption velocity of Divisia M3

Our analysis focuses on the consumption velocity of CFS's Divisia M3 aggregate, which measures the monetary services from the components of the Federal Reserve's simple-sum M2 aggregate as well as from large time deposits, institutional money market mutual funds, and repurchase agreements (RPs). The Federal Reserve's simple sum M3 aggregate, which was published through early 2006, added institutional money funds, total large time deposits, total repurchase agreements and total Eurodollar deposits to M2; making its components roughly comparable, but not identical, to CFS's Divisia M3 measure. As Bordo and Duca (2025) stress, Divisia M3 helps internalize substitution between the liquidity provided by M2 and non-M2 liabilities, where the latter help track shifts in the role of liquidity provided by instruments funding the shadow banking sector. Of the three broad Divisia measures (M3, M4-, and M4) constructed by CFS, Bordo and Duca (2025) were slightly better able to track the GDP velocity of Divisia M3.

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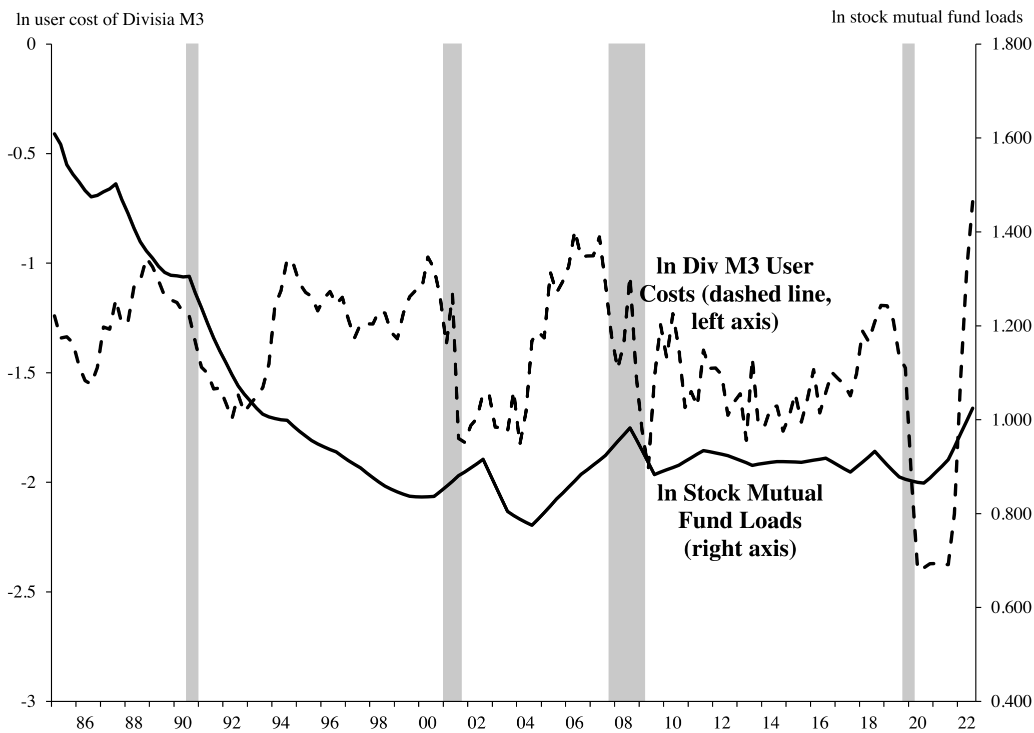

Figure 4. Stock fund loads and the CFS measure of Divisia M3 user cost.
Sources: CFS and Bordo and Duca (2025).

The preceding discussion implies that the long-run demand for the log of real Divisia M3 (*LDM*), takes the form:

$$LDM^* = \gamma_0 + \gamma_1 CFMA + \gamma_2 LSLoad + \gamma_3 LPCE + \gamma_4 LOC \quad (1)$$

where *CFMA* is a level shift dummy equal to one following passage of *CFMA* and zero for earlier periods. The remaining variables are the logs of the stock load variable (*LSLoad*), real consumption (*LPCE*), and the opportunity cost (*LOC*), which is either the long-term bond yield or the dual user cost index. For consistency, Divisia M3 should be converted to real terms using the PCE deflator. As discussed above, CFMA was associated with increased money holdings implying that $\gamma_1 > 0$. Higher transfer costs should increase money demand implying that $\gamma_2 > 0$ by reducing the liquidity of assets outside of Divisia M3 and higher opportunity costs are expected to reduce money demand implying $\gamma_4 < 0$. Finally, the elasticity of money demand with respect to consumption, γ_3, should be positive. For later reference, omitting the opportunity cost variable results in the simplified expression:

$$LDM^* = \gamma_0 + \gamma_1 CFMA + \gamma_2 LSLoad + \gamma_3 LPCE \quad (1')$$

If the consumption elasticity of long-run Divisia money demand is unity, $\gamma_3 = 1$, (as Bordo and Duca (2025) impose for GDP velocity), then eqs. (1) and (1') can be rearranged into corresponding models of long-run consumption velocity (in logs):

$$LV^* = \theta_0 + \theta_1 CFMA + \theta_2 LSLoad + \theta_3 LOC \quad (2)$$

and

$$LV^* = \theta_0 + \theta_1 CFMA + \theta_2 LSLoad \quad (2')$$

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respectively, since $LV = LPCE - LDM$. Assuming that *CFMA* bolstered Divisia M3, as argued earlier, then it would have correspondingly lowered its velocity implying that $\theta_1 < 0$. Previous arguments likewise imply that $\theta_2 < 0$ (higher transfer costs raise money demand and hence lower velocity) and $\theta_3 > 0$ (higher opportunity costs lower money demand and hence increase velocity).

We posit that velocity (real money) adjusts gradually to the deviation of it from its long-run equilibrium, $LV_t - LV_t^*$ ($LDM_t - LDM_t^*$), which gives rise to an error-correction model. However, including the CFMA level shift dummy in the long-run relationship must be properly incorporated into the estimation, and there can be transitional short-run dynamics immediately following passage of the Act. To handle these issues, we estimate broken constant models from Johansen et al. (2000). These models generalize the standard cointegration models (see, e.g., Johansen, 1996) to allow for structural breaks in the deterministic components where the breakpoints are known. Since we are concerned with breaks associated with the adoption of the CFMA in our analysis, this is an attractive framework for our empirical design.

2.4. Technical details

The standard VECM framework consists of a model in the form:

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \Pi_1 \mathrm{t} + \mu + \varepsilon_t \tag{3}$$

where k is the number of lags of the underlying VAR model (in levels) and the vector X includes LV, $LSload$, and LOC. Let p be the number of variables in the VAR. Table 1 reports unit root tests for each of the variables over our sample period (1985–2022). These tests indicate that LV and $LSLoad$ are $I(1)$, but that both $LBond$ and $LUsercost$ may be $I(0)$. While the VECM model can include stationary variables, doing so would increase the rank of the system. The table also includes tests for real Divisia M3 and real PCE (both in logs), which are used in Appendix 1.

Under cointegration, Π is of reduced rank and is usually written as $\Pi = \alpha\beta'$, with α, β both being p by r. In that case, $\beta' X_{t-1}$ represents the (r) cointegration relations and α represents the corresponding impact coefficients of those relations on the first differences of each of the system variables in the VAR. Π_1 is typically imposed to be of the form $\Pi_1 = \alpha\gamma'$, so that the model does not have a quadratic trend in levels (see Johansen et al. 2000). Consequently, with the reduced rank condition imposed, the cointegration relations would be trend stationary, since $\Pi X_{t-1} + \Pi_1 t = \alpha(\beta' X_{t-1} + \gamma' t)$. In applications, $\gamma = 0$ can be further imposed, which eliminates the trend from the cointegration space, but not from the levels. Assuming $\gamma = 0$, if it is further imposed that $\mu = \alpha\rho'$ then the constant is restricted to the cointegrating relationship, and the model will not have a linear trend in any direction (Johansen et al. 2000, p. 218).

Johansen et al. (2000) generalize this framework by introducing indicator variables for different sample periods:

$$E_{j,t} = \begin{cases} 1 & \text{for } T_{j-1} + 1 + k \leq t \leq T_j \\ 0 & \text{o.w.} \end{cases} \tag{4}$$

where $j = 1, \ldots, q$. Setting $T_0 = 0$ for convenience, the effective sample is $t = 1 + k, \ldots, T_q = T$ with $X_1, \ldots, X_k$ being used as initial observations and there are q subsamples with effective sample periods running from $1 + k$ to T_1, $T_1 + 1 + k$ to T_2, and so on. Following Johansen et al. (2000, p. 219), we define the vector of sub-sample indicator variables as $E_t = (E_{1,t}, E_{2,t}, \ldots, E_{q,t})'$.

Using this notation, Johansen et al. (2000) proposes several models that allow for breaks in the deterministic components. We impose $\gamma = 0$, which eliminates the possibility of trends entering the long-run relationship and we focus on the *broken constant* specification from

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Table 1. Unit root tests

Variable	Augmented Dickey Fuller test (SIC lag)	Reject unit root?	Phillips–Perron (bandwidth)	Reject unit root?
1984–2022 for velocity and money demand models				
LDM (DivM3)	−1.871 (1)	No	−1.937 (5.43)	No
Δ*LDM* (DivM3)	−7.687** (0)	Yes	−7.573** (1.48)	Yes
LPCE	−1.872 (0)	No	−1.750 (1.97)	No
Δ*LPCE*	−14.503** (0)	Yes	−14.503** (0.506)	Yes
LV (DivM3)	−2.138 (0)	No	−2.155 (1.38)	No
Δ*LV* (DivM3)	−12.062** (0)	Yes	−12.062** (0.354)	Yes
LSload	−1.191 (1)	No	−1.239 (13.1)	No
Δ*LSload*	−4.561** (0)	Yes	−4.729** (1.82)	Yes
LUserCost (DivM3)	−3.654* (1)	Yes	−3.609* (2.83)	Yes
Δ*LUserCost* (DivM3)	−10.844** (0)	Yes	−10.844** (0.493)	Yes
LBond	−4.452** (1)	Yes	−3.692* (4.67)	Yes
Δ*LBond*	−8.684** (1)	Yes	−8.823** (1.73)	Yes
1984–2024 Q3 for P-Star model variables				
Δ*LPPCE*	−6.986** (0)	Yes	−6.986** (1.19)	Yes
Δ^2*LPPCE*	−13.829** (1)	Yes	17.420** (1.67)	Yes
Δ*LPcorePCE*	−3.277+ (1)	Borderline	−4.256** (2.3)	Yes
Δ^2*LPcorePCE*	−13.330** (1)	Yes	−21.094** (1.78)	Yes
SupPress	−3.997* (0)	Yes	−4.110** (1.75)	Yes
Δ*SupPress*	12.762** (0)	Yes	−12.762** (0.44)	Yes

Notes: +, * and ** denote 90%, 95% and 99% significance. Lag length or bandwidth statistics are in parentheses. Tests for *SupPress* begin in 1998 when the series starts. Lag lengths for ADF tests selected using the SIC. Phillips–Perron tests use the quadratic spectral kernel with an Andrews Bandwidth. All tests include an intercept and a time trend.

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Johansen et al. (2000, p. 225). That specification is as follows:

$$\Delta X_t = (\Pi, \mu)\begin{pmatrix} X_{t-1} \\ E_t \end{pmatrix} + \sum\nolimits_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \sum\nolimits_{i=1}^{k} \sum\nolimits_{j=2}^{q} \kappa_{j,i} D_{j,t-i} + \varepsilon_t \tag{5}$$

where $D_{j,t-i} = 1$ if $t = T_{j-1} + i$ (and zero otherwise) and $\mu = (\mu_1, \ldots, \mu_q)$ is a p by q matrix. The term, μE_t, allows for the constant in the VAR to differ across subsamples. As Johansen et al. (2000, p. 220) explain, the model dummies out the k initial observations following each breakpoint, $X_{T_1+1}, \ldots, X_{T_1+k}, X_{T_2+1}, \ldots, X_{T_2+k}, \ldots$ rendering "the corresponding residuals zero thereby essentially eliminating the corresponding factors from the likelihood function, and hence producing the conditional likelihood function given the initial values in each period." The relevant reduced rank cointegration hypothesis (Johansen et al. (2000, p. 225) is

$$H_c(r): \ rank \ (\Pi, \mu) \le r \tag{6}$$

so that $(\Pi, \mu) = \alpha(\beta', \rho')$, where α, β are both p by r matrices and ρ is q by r. With the reduced rank condition imposed, $\Pi X_{t-1} + \mu E_t = \alpha(\beta' X_{t-1} + \rho' E_t)$, corresponding to a set of r cointegration relations with different constants in each of the q subsamples. Within the same specification, Johansen et al. (2000, p. 219) also consider an alternative reduced rank hypothesis:

$$H_{\ell c}(r): \ rank \, \Pi \le r \tag{7}$$

Johansen et al. (2000, pp. 218–220) consider the following specification as well:

$$\Delta X_t = (\Pi, \Pi_1, \ldots, \Pi_q)\begin{pmatrix} X_{t-1} \\ tE_t \end{pmatrix} + \sum\nolimits_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \sum\nolimits_{i=1}^{k} \sum\nolimits_{j=2}^{q} \kappa_{j,i} D_{j,t-i} + \mu E_t + \varepsilon_t \tag{8}$$

For this specification, the reduced rank cointegration hypothesis is

$$H_\ell(r): \ rank \ \left(\Pi, \Pi_1, \ldots, \Pi_q\right) \le r \tag{9}$$

so that

$$\left(\Pi, \Pi_1, \ldots, \Pi_q\right) = \alpha \begin{pmatrix} \beta \\ \gamma_1 \\ \vdots \\ \gamma_q \end{pmatrix}' \tag{10}$$

where γ_j is 1 by r for all j. Imposing the reduced rank condition, the cointegration relations have *broken trends* denoted by $\gamma_j' t$ in each subsample j. We do not consider this specification further in this paper.

Johansen et al. (2000) derive likelihood ratio tests for $H_c(r)$ and $H_{\ell c}(r)$ against $H_{\ell c}(p) = H_c(p)$ and for $H_\ell(r)$ against $H_\ell(p)$, which are in the usual trace test form: i.e. $LR\{H_c(r)|H_c(p) = -T \sum_{i=r+1}^{p} \ln(1 - \hat{\lambda}_i)$. Corresponding tables of critical values for $H_c(r)$ and $H_\ell(r)$ can be found in Giles and Godwin (2012). The critical values depend on the relative breakpoints, $v_j = T_j/T$, where T_j is the last observation of the jth subsample and T is the full sample size. Johansen et al. (2000, p. 219) find the $H_{\ell c}(r)$ hypothesis to be "less attractive" than the other two hypotheses, however, in part due to their finding that "the asymptotic analysis is heavily burdened with nuisance parameters" in that case. Consequently, we focus on testing the $H_c(r)$ hypothesis.

In our application, the only structural break we consider is associated with the adoption of CFMA. To align with our notation, we set the number of sub-samples, q, equal to 2 and set T_1 to be 2000Q3 corresponding to the end of the pre-CFMA period, since CFMA passed in 2000Q4. As in eq. (4), $E_{2,t}$ is an indicator variable that is one from $T_1 + 1 + k$ through the end of the sample, where k is the number of lags in the VAR, and zero for earlier periods. The inclusion of the $D_{j,t-i}$

dummies in eq. (5) implies that we do not need to model the transitional impacts of CFMA and that $E_{2,t}$ is, therefore, effectively an indicator for the post-CFMA period. We follow Johansen et al. (2000) as laid out above with one exception. We set $E_{1,t}=1$ over the entire sample rather than over just the sub-sample covering the pre-CFMA period. This allows us to present the results in terms of a constant for the full sample and a shift in the constant for the post-CFMA period as in eq. (2). This minor change does not affect the estimation of any other parameters nor does it affect the test procedures employed. We do this to simplify exposition regarding the effect of CFMA.

Estimating eq. (5) and setting $r=1$ yields coefficients on the X vector components for a single long-run relationship, $\beta' X_{t-1}$, with constant term, $\rho' E_t$. The corresponding impact coefficients, α, determine how the endogenous variables each adjust to the long-run relationship. Since we focus on velocity, the impact of the long-run relationship on the stock load, the long-term Treasury rate, or the user cost are not of particular interest and are not reported. Similarly, we only report first-order short-run dynamics for the change in velocity, as captured by the corresponding row of Γ_1 (full results are available upon request). By normalizing the β vector on the element corresponding to log velocity, $\beta' X_{t-1}$ can be represented in the following form:

$$LV_{t-1} + \beta_2 LSLoad_{t-1} + \beta_3 LOC_{t-1} \tag{11}$$

Ignoring the constants in the cointegration, we can define long-run equilibrium velocity as

$$LV^*_{t-1} \equiv -\beta_2 LSLoad_{t-1} - \beta_3 LOC_{t-1} = \theta_2 LSLoad_{t-1} + \theta_3 LOC_{t-1}$$

so that the corresponding error-correction term is as follows:

$$EC_{t-1} \equiv LV_{t-1} - LV^*_{t-1} = LV_{t-1} + \beta_2 LSLoad_{t-1} + \beta_3 LOC_{t-1} \tag{12}$$

which are the deviations from long-run equilibrium. Incorporating the constants in the cointegration relations results in

$$LV^* = \theta_0 + \theta_1 CFMA + \theta_2 LSLoad + \theta_3 LOC \tag{13}$$

as in eq. (2).[11] As discussed previously, theoretical priors imply $\theta_1,\ \theta_2 < 0$ and $\theta_3 > 0$.

Our model of consumption velocity builds on Bordo and Duca (2025) who model the GDP velocity of broad Divisia but differs in three important respects: First, we use PCE to track the aggregate transactions instead of GDP as discussed above. Second, we adopt the technically more correct approach of estimating the broken constant specification rather than treating *CFMA* as a regular variable within the VECM as in Bordo and Duca (2025). The trace test for the broken constant model can be compared to tables of critical values and are interpreted in the usual way as described above. Third, Bordo and Duca (2025) model the impact of COVID-19 with a continuous variable that they construct from an index of government mobility restrictions and vaccination rates. For the sample that includes post-2019 observations, we instead just include simple impact dummies for each of the quarters of 2020, which are sufficient to produce stable long-run relationships over our full sample. Modelling velocity in terms of PCE is more attractive than GDP, because the Fed's price stability goal is in terms of PCE inflation. We later incorporate estimates from our model of consumption velocity into a P-Star model of PCE inflation.

3. Estimation

3.1. Sample periods

We estimate our consumption velocity model over four sample periods that all start in 1985Q3 but have different end-of-sample periods. Earlier observations are used as initial starting values in the models. We generally find clean residuals for our estimated models with lag lengths of 7 quarters or less as described below. For 7 lags, the 1985Q3 start-of-sample implies that eq. (5) is estimated using only data (in differences) going back to 1984Q1. The starting point is chosen to

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avoid using earlier data that could be affected by deposit deregulation and financial innovations that led to the introduction of new types of monetary assets.

During the 1970s and early 1980s, there were shifts away from savings and time deposits with binding deposit interest rate ceilings (whose measured user costs were high, because Regulation Q capped deposit interest rates below market levels), to money market mutual funds (MMMFs), and later to money market deposit accounts (MMDAs). Super NOW accounts and MMDAs were both introduced beginning in late 1982. Super NOWs, MMDAs, and MMMFs all paid higher interest rates than regulated deposits and hence had user costs that were relatively low and MMDAs and MMMFs also had checking features unlike traditional savings and small-time deposits. In light of this, the shifts out of regulated deposits would be weighted by a Divisia index using expenditure shares based on relatively high user costs. In contrast, the movement of funds into the assets paying higher rates would receive expenditure share weights based on relatively low user costs. As a result, these shifts would be associated with a distorted decline in measured Divisia (but not simple-sum) money (see Bordo and Duca, 2025, for a discussion). Indeed, over this period, there was a corresponding upward level shift in Divisia velocity in the late-1970s and early 1980s. Chen and Valcarcel (2024) find structural breaks in Divisia M2 and Divisia M3 around 1980 and subsequent breaks in 2012 and 1988 respectively. They also find structural breaks in the corresponding user cost measures in 2008, and that "[o]verall, these statistical tests show a preponderance of evidence for a structural break in 1980."[12]

The earliest end of sample period we consider is 2005Q4 (Model 1), which is just before the bursting of the subprime mortgage bubble and the GFC. Model 2 ends in 2012Q4 to allow ample time for the adjustment of money to the staggered implementation of new financial regulations under the Dodd-Frank Act, through which the U.S. implemented the Basel III reforms. Model 3 ends in 2019Q4, to span the pre-COVID period, while Model 4 uses a full sample ending in 2022Q4. The full sample is based on the availability of the underlying data used to construct the stock load variable. The full sample also differs in that we include dummies for each quarter in 2020 to control for the unprecedented impact of the pandemic during its acute initial phase.

3.2. Estimation results

Our specification of long-run equilibrium consumption velocity is spelled out in eq. (13) with the corresponding error-correction term in eq. (12) which enters into the short-run dynamics of the model. We estimate three versions of the model that differ in terms of opportunity costs. We first assess models that include stock fund loads and the long-term bond yield. We then assess models including stock fund loads and the Divisia M3 user cost measure from the CFS. Finally, we present models that just include stock mutual fund loads. In our estimations, we select lag lengths using two criteria: First, we estimate a simple unrestricted VAR in levels and employ LM tests for serial correlation (up to four quarters) for lag lengths from 2 to 8. From models having clean residuals, we select the lag length that minimizes the Schwartz Information Criterion (SIC).

3.2.1. Estimation results using the long-term bond rate

Table 2 reports results for models that use the long-term bond rate as the opportunity cost variable using the lag length prescribed by our procedure (described above) over the four different sample periods. For these models, there are three variables ($p = 3$), so we can use the trace test to evaluate $H_c(r)$ for $r = 0, 1, 2$. If $r = 0$, then the system is difference stationary. If $0 < r < p$, then the model has r cointegrating vectors and if $r = p$, then the system is stationary. Throughout this section, we report results in terms of the coefficients of the long-run equilibrium velocity relationship, eq. (13), since these are easily interpretable. For the shortest subsample, 1985Q3 to 2005Q4 (Model 1), the trace test rejects $r = 0$ at the 99% level.[13] The rank (r) is ambiguous though, since the trace test for $r = 1$ is borderline at the 90% significance level and the test for $r = 2$ is rejected

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Table 2. Quarterly models of U.S. consumption velocity including long treasury yields.

Long-run relationship: $LV^* = \theta_0 + \theta_1 CFMA + \theta_2 LSLoad + \theta_3 LBond$				
	Model 1 1985Q3-2005Q4	Model 2 1985Q3-2012Q4	Model 3 1985Q3-2019Q4	Model 4 1985Q3-2022Q4
Constant	1.972 (31.19)	2.627 (64.39)	2.660 (108.83)	2.697 (40.14)
$CFMA_t$	0.033 (1.87)	−0.114 (7.22)	−0.120 (10.04)	−0.126 (4.11)
$LSload_t$	−0.643 (16.17)	−0.316 (10.55)	−0.277 (15.58)	−0.300 (6.07)
$LBond_t$	0.539 (10.64)	0.011 (0.38)	−0.026 (2.35)	−0.043 (1.62)
Trace ($r = 0$)	66.20**	55.02**	55.52**	48.83*
Trace ($r = 1$)	23.27†	25.79+	18.14	21.52
Trace ($r = 2$)	11.12+	10.51	6.54	5.33
v_1 or $1 - v_1$	0.244	0.422	0.469	0.426
Rank (90%):	Ambiguous	$r = 2$	$r = 1$	$r = 1$
Lag length (k)	4	6	7	5
Short-Run: $\Delta LV_t = \alpha_1 EC_{t-1} + \Gamma_1(1,1)\Delta LV_{t-1} + \Gamma_1(1,2)\Delta LSload_{t-1} + \Gamma_1(1,3)\Delta LBond_{t-1} + \ldots$				
EC_{t-1}	0.006 (0.30)	−0.088 (2.97)	−0.159 (5.41)	−0.042 (3.24)
ΔLV_{t-1}	0.179 (1.62)	0.406 (4.11)	0.349 (4.17)	0.330 (3.82)
$\Delta LSload_{t-1}$	−0.036 (0.48)	−0.110 (1.44)	−0.091 (1.29)	−0.090 (1.29)
$\Delta LBond_{t-1}$	0.048 (4.09)	0.019 (1.91)	0.016 (2.43)	0.018 (2.93)
$D2020Q1_t$				−0.036 (5.33)
$D2020Q2_t$				−0.198 (26.17)
$D2020Q3_t$				0.146 (8.11)
$D2020Q4_t$				0.000 (0.004)
Log-Likelihood:	1078.6	1403.4	1746.2	1868.2

Notes: Significance is determined using critical values for the trace test from Giles and Godwin (2012) for various values of v_1 as described in footnote 13. +, ∗, and ∗∗ denote 90%, 95%, and 99% significance respectively. † indicates significance at the 90% level for the lower bracketing value only. Long-term coefficient estimates are for $r = 1$. Absolute t-statistics are in parentheses. For the short-run model, all coefficients are for the velocity equation. $\Gamma_1(i, j)$ is the i, j element of Γ_1 in eq. (5). Short run coefficients corresponding to higher order lags ($\Gamma_2, \ldots \Gamma_{k-1}$) are omitted to save space.

at the 90% level suggesting the possibility that the system is stationary. Our main objective is to derive long-run values for consumption velocity for use in the P-star model and we will focus on a single cointegrating relationship throughout this section. Setting $r = 1$, which corresponds to a single cointegrating relation, results in reasonably signed coefficient estimates for the load variable and for the long-term rate. The coefficient on *LSLoad* is −0.643 with a *t*-stat of 16.17

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implying that higher loads raise money demand and thereby reduce velocity. The coefficient on *LBond* is 0.539 with a t-stat of 10.64 implying that higher long-term rates lower money demand and correspondingly raise velocity. The coefficient on *CFMA* is positive, contrary to expectations. Although it is small, its t-stat is 1.87. The model performs poorly out-of-sample in that the estimated error-correction term begins to trend significantly during the GFC and remains elevated through the onset of the pandemic.[14]

Next, we consider the subsample from 1985Q3 to 2012Q4 (Model 2). For this sample, we find that there is at least one cointegrating vector ($r = 0$ is rejected at the 99% confidence level), but we cannot rule out two vectors at the 90% confidence level, so once again, there is some ambiguity about the rank. For this sample, imposing one cointegrating vector (setting $r = 1$) yields different results than the pre-GFC sample (Model 1). The coefficient on *LSload* is −0.316, which is again consistent with theory. While the coefficient on the long-term rate is still positive (0.011), it is close to zero and has a t-stat of just 0.38. The coefficient on *CFMA* is −0.114, consistent with the view that CFMA promulgated the widespread use of credit default swaps. By enhancing the liquidity of assets held by shadow banks, CFMA thereby increased the demand for shadow bank liabilities in Divisia M3 and correspondingly lowered its velocity.

For the pre-pandemic subsample (Model 3), we find a single cointegrating relationship ($r = 1$) with sensible coefficients on *LSload* (−0.277) and *CFMA* (−0.12) with high t-stats. The coefficient on the long-term Treasury rate, however, is −0.026 with a t-stat of 2.35. While the coefficient is small, its sign is inconsistent with theory.[15] Finally, we estimate the model over the full sample through 2022Q4 adding dummy variables for each of quarter in 2020 to control for the onset of the pandemic. For the full sample, $r = 0$ is rejected at the 95% significance level and $r = 1$ cannot be rejected at the 90% level again indicating a single cointegrating relation. In this sample, the coefficient on *LSload* is −0.3 and the coefficient on *CFMA* is −0.126. The coefficient on the long-term rate is −0.043, however, with a t-stat of 1.62. The COVID dummies effectively delete the observations from 2020 from the likelihood function and their coefficients pick up the steep drop in velocity in 2020Q2 and its partial recovery in 2020Q3.

We conclude that the long-term bond yield only has the anticipated effect on velocity in the pre-GFC sample, but the model did not perform well out-of-sample in that case. In the other three samples, the coefficient on the long-term bond yield is small and often has a counterintuitive sign. Thus, the results do not favor including the long-term Treasury yield as a long-run determinant of consumption velocity.

3.2.2. Estimation results using the dual user cost measure

As Barnett et al. (2022) explain, the Divisia dual user cost index is the opportunity cost measure that is most internally consistent with a Divisia monetary aggregate, since it is constructed from the same underlying data on user costs and asset stocks and is derived within the same aggregation-theoretic framework. Moreover, if a long-term rate was the appropriate alternative rate of return, then the user costs underlying Divisia and its dual should employ such a rate as the benchmark.[16] While the dual user cost is very attractive on theoretical grounds, we were unable to obtain reasonable results using that variable. We now summarize our findings in this regard.

Our lag selection procedure did not produce clean residuals for any lag length between 2 and 8 quarters for the 1985Q3 to 2012Q4 sample (Model 2) when the user cost is used as the opportunity cost variable. Consequently, we estimated the model using several different lags and compared results. At 4 lags, we found that $r = 1$ and the coefficient on *LSLoad* is −0.271 and the coefficient on *CFMA* is −0.16, which differ somewhat from the results using the bond rate, but seem reasonable. The coefficient on *LUsercost*, however, was −0.118 with a t-stat of 4.25. According to the theory, we would expect a higher user cost to negatively affect money demand and, consequently, to positively affect velocity. At 6 lags, there was some ambiguity about the number of cointegrating vectors ($r = 1$ would be rejected at the 90% level). We found negative coefficients for both *LSLoad* (−0.244) and *CFMA* (−0.221) when setting $r = 1$ but again found a counterintuitive

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negative coefficient for *LUsercost* (−0.222) with a t-stat of 5.65. For the pre-pandemic sample, 1985Q3-2019Q4 (Model 3), we estimated the model using the same lag lengths for comparison. At 4 lags, $r = 1$ is rejected at the 90% level, but if we nevertheless impose $r = 1$, then the estimated cointegration vector is very similar to the estimates for the shorter sample period. At 6 lags, $r = 1$ was rejected at the 95% level, but setting $r = 1$ yielded coefficients of −0.252 for *LSLoad*, −0.192 for *CFMA*, and −0.169 for *LUsercost*, which had a *t*-stat of 5.79.[17]

Thus, while the dual user cost is theoretically more attractive, we find that it has a counter-intuitive negative effect on velocity in both sample periods for a range of lag lengths. The exact reason why this happens is beyond the scope of the current paper, but it potentially reflects the impact of the zero-lower bound on the behavior of the underlying user costs following the GFC and continuing for a number of years and also following the onset of the pandemic as discussed earlier. Another factor, which applies to the long-term yield as well, is the potential stationarity of the dual user cost variable. In any case, given these findings, we do not pursue models including the dual user cost further in our analysis.

3.2.3. Estimation results excluding the opportunity cost variables

Building on our previous results, we now estimate more parsimonious models that include just the stock load variable as an explanatory factor while accounting for CFMA as in Bordo and Duca (2025), although we estimate our models differently as described above. In these models, $p = 2$. As reported in Table 3, for Models 1 and 2, which use the pre-GFC (1985Q3 to 2005Q4) and the 1985Q3 to 2012Q4 subsamples respectively, we find a significant cointegrating vector ($r = 0$ is rejected at the 95% significance level or higher and $r = 1$ cannot be rejected at the 90% significance level). For Model 3, which uses the pre-pandemic subsample (1985Q3 to 2019Q4), $r = 0$ is rejected at the 99% significance level, but $r = 1$ is rejected at the 90% significance level, so there is some ambiguity about whether the system is cointegrated or just stationary for that sample. Setting $r = 1$ and normalizing on velocity yields similar coefficient estimates across all three sub-samples in Models 1–3, with coefficients on the load variable near −0.3 and those on *CFMA* near −0.1. The estimates for these sample periods are all based on VARs with 6 lags. We also estimated VARs with 4 and 8 lags and found similar coefficient estimates.[18]

The estimated impact coefficient on the error-correction term for log velocity is −0.071 with a t-stat of 2.56 for the pre-GFC subsample indicating that velocity significantly error-corrects. The corresponding coefficients are around −0.12 with t-stats above 4 for the other two subsamples indicating faster adjustment of velocity. The model also performs well out-of-sample insofar as the error-correction terms based on the subsample ending in 2012Q4 are very similar to the ones estimated over the pre-pandemic sample period from 2013 on. This is an improvement *vis-à-vis* the models that included the long-term rate. When we included the long-term bond rate, we found that the error-correction terms based on the estimates from the subsample ending in 2012Q4 were higher than the ones estimated over the pre-pandemic sample period from 2013 on. Turning to the full sample (Model 4), we identify a significant cointegrating vector (i.e., $r = 1$) with sensible coefficients on *LSload* (−0.320) and *CFMA* (−0.091) and the impact coefficient on the error-correction term for log velocity (−0.09) is reasonable as well.

To provide more perspective on our results, Figure 5 plots the estimated long-run equilibrium consumption velocities from samples ending in 2012Q4 (Model 2) and 2022Q4 (Model 4). These line up well with each other and with long-run movements in log velocity. The slight lead of the long-run equilibrium relative to actual log velocity reflects the importance of accounting for partial adjustment toward long-run equilibrium for tracking velocity in the short-run and buttresses the credibility of the error-correction models. As explained above, velocity fell initially during the onset of the pandemic when the money supply surged.

Overall, including stock loads without either the bond yield or the dual user cost measure works well for modeling consumption velocity. Our model also performs well if we estimate our specification allowing for an unrestricted broken constant (corresponding to $H_{\ell c}(1)$ rather than $H_c(1)$).

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Table 3. Quarterly models of U.S. consumption velocity excluding long treasury yields.

Long-run relationship: $LV^{*} = \theta_0 + \theta_1 CFMA + \theta_2 LSLoad$				
	Model 1 1985Q3-2005Q4	Model 2 1985Q3-2012Q4	Model 3 1985Q3-2019Q4	Model 4 1985Q3-2022Q4
Constant	2.631 (89.50)	2.630 (91.39)	2.631 (90.84)	2.638 (80.55)
$CFMA_t$	−0.105 (6.16)	−0.100 (7.52)	−0.096 (7.86)	−0.091 (6.45)
$LSload_t$	−0.306 (12.02)	−0.302 (12.33)	−0.304 (12.35)	−0.320 (11.60)
Trace ($r = 0$)	27.56*	32.43**	36.89**	57.17**
Trace ($r = 1$)	9.93	10.45	11.92+	8.39
Rank (90%):	$r = 1$	$r = 1$	$r = 2$	$r = 1$
v_1 or $1 - v_1$	0.239	0.422	0.465	0.433
Lag length (k)	6	6	6	7
Short-Run: $\Delta LV_t = \alpha_1 EC_{t-1} + \Gamma_1(1,1)\Delta LV_{t-1} + \Gamma_1(1,2)\Delta LSload_{t-1} + \ldots$				
EC_{t-1}	−0.071 (2.56)	−0.119 (4.17)	−0.115 (4.33)	−0.091 (5.65)
ΔLV_{t-1}	0.382 (3.54)	0.442 (5.00)	0.401 (4.98)	0.420 (5.47)
$\Delta LSload_{t-1}$	−0.012 (0.15)	−0.069 (0.91)	−0.070 (0.98)	−0.082 (1.14)
$D2020Q1_t$				−0.035 (5.23)
$D2020Q2_t$				−0.204 (27.83)
$D2020Q3_t$				0.138 (7.67)
$D2020Q4_t$				−0.020 (1.01)
Log-Likelihood	829.8	1090.8	1375.2	1503.6

Notes: Significance is determined using critical values for the trace test from Giles and Godwin (2012) for various values of v_1 as described in footnote 13. +, ∗, and ∗∗ denote 90%, 95%, and 99% significance respectively. Long-term coefficient estimates are for $r = 1$. Absolute t-statistics are in parentheses. For the short-run model, all coefficients are for the velocity equation. $\Gamma_1(i,j)$ is the i,j element of Γ_1 in eq. (5). Short run coefficients corresponding to higher order lags ($\Gamma_2, \ldots \Gamma_{k-1}$) are omitted to save space.

Furthermore, we found that estimating real Divisia M3 money demand versions of our model yielded similar results when imposing a single cointegrating vector. These additional results are presented in Appendix 1, parts A and B respectively. Accordingly, we adopt our parsimonious model of long-run equilibrium velocity for our P-Star models of inflation.[19]

3.2.4. Broader levels of aggregation

Finally, we briefly consider the simple velocity model using the broader Divisia M4- and M4 aggregates, both of which include commercial paper. We estimated Models 1–3 excluding opportunity cost variables using both of these aggregates for lag lengths of 4, 6, and 8 similar to Divisia M3 as discussed above. For M4, we found generally similar coefficient estimates for the pre-GFC

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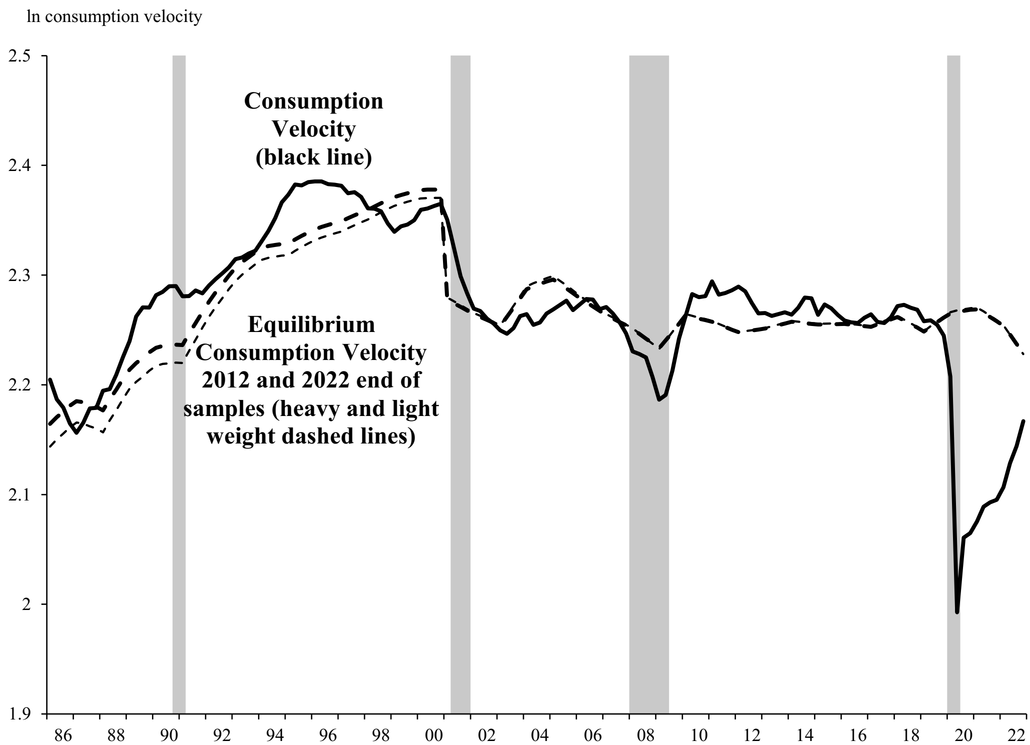

Figure 5. Estimated equilibrium consumption velocity trends with long-run velocity.
Sources: CFS, BEA, and author's calculations.

sample period (Model 1) and the sample period ending in 2012Q4 (Model 2) across the various lag lengths. For these two sample periods, the estimated coefficients on the load variable ranged from −0.25 to −0.21 and the estimated coefficients on *CFMA* ranged from −0.09 to −0.075 (with t-stats above 6.0 in all cases) consistent with our theoretical priors. The estimated coefficients differed for the pre-pandemic sample period (Model 3). At 4 lags, we found that the coefficient on the load variable was −0.27 (t-stat. of 5.93) and the coefficient on *CFMA* was −0.054 (with a t-stat of 2.52). At 6 and 8 lags, the coefficients on the load variable were around −0.3 with t-stats above 5.75 and the coefficients on *CFMA* were −0.038 and −0.021 (with t-stats of 1.49 and 0.97) respectively.[20] Thus, there is evidence of parameter instability especially regarding the impact of CFMA between the sample period ending in 2012Q4 and the pre-pandemic sample period.

For M4-, there is additional evidence of parameter instability. For the pre-GFC sample period (Model 1), the coefficient estimates at 6 and 8 lags are around −0.28 for the load variable and around −0.08 for *CFMA*. At 4 lags, the corresponding estimates are −0.153 for the load variable and −0.103 for *CFMA*. For Model 2, however, the coefficient estimates ranged from −0.322 to −0.275 for the load variable and from −0.056 to −0.03 for *CFMA*. At 6 lags, the coefficient estimate on *CFMA* (−0.03) had a t-stat of just 0.92. For the pre-pandemic sample period (Model 3), the estimated coefficients on the load variable were around −0.37, but the coefficients on *CFMA* were wrongly signed (estimates ranged from 0.002 to 0.024), although the t-stats were below 0.7 for all three lag lengths.[21]

Thus, for Divisia M4, and even more so for Divisia M4-, there is some evidence of parameter instability particularly with respect to the impact of *CFMA* on long-run velocity *vis-à-vis* Models 2 and 3. Throughout the remainder of the paper, our focus will be on prediction via the P-Star

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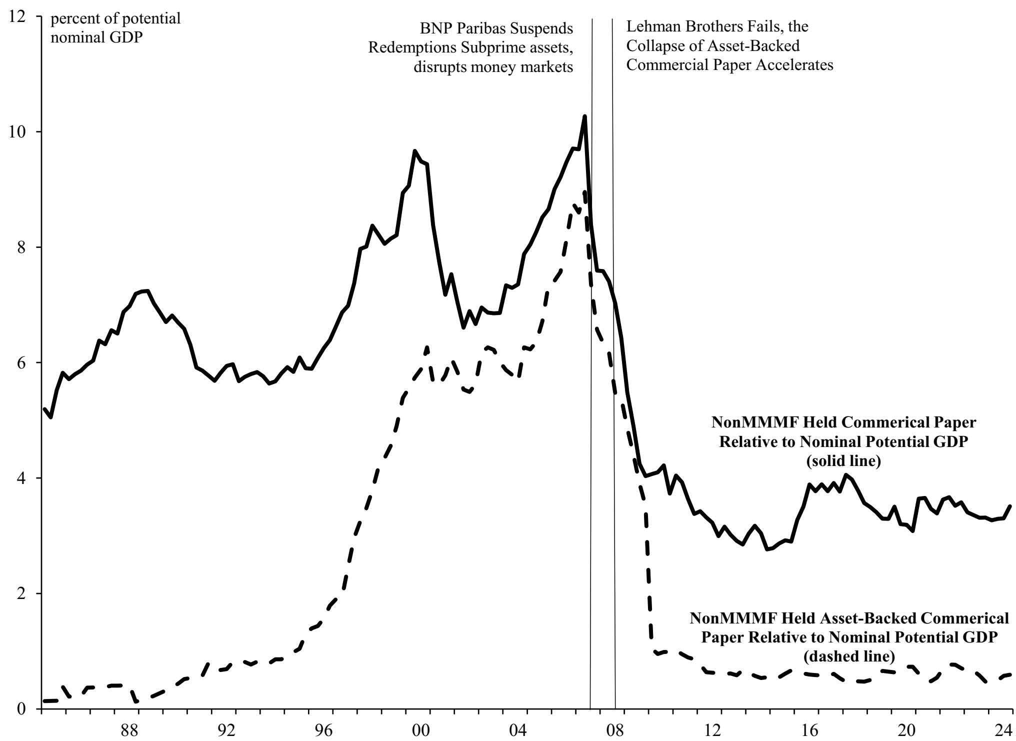

Figure 6. Commercial paper not held by money funds scaled by potential nominal GDP.
Sources: Financial Accounts of the U.S., CBO, and authors' calculations.

model and the relative stability of the parameter estimates for Divisia M3 is an attractive feature in that respect. Consequently, we will focus on Divisia M3 for subsequent analysis.

A plausible explanation for the difficulty in modeling the demand for Divisia M4- and Divisia M4 is that they include the monetary services from assets that have not been transformed by intermediaries into broadly accepted means of payment. As described earlier, Divisia M4- augments Divisia M3 by including commercial paper. When scaled by trend potential nominal GDP, commercial paper holdings shifted up with the rise of asset-backed commercial paper from the mid-1990s to the mid-2000s (Figure 6).[22] This occurred when shadow banks made greater use of leverage in funding their assets by issuing commercial paper much, but not all, of which was held by money funds. This rise unwound sharply starting with the onset of the Global Financial Crisis and amplified after Lehman Brothers failed. Even though the opportunity cost of commercial paper is small, the large swings in asset holdings could have loosened the overall link between Divisia M4- and nominal spending. In addition to including commercial paper, M4 also includes Treasury bills held outside of banks and money funds, which may further affect the ability to model the velocity of M4.

4. Using Divisia M3 in a P-Star framework for predicting inflation

4.1. A P-star model for consumer inflation

As discussed previously, P-Star models are derived from the equation of exchange:

$$P_t Y_t = M_t V_t \tag{14}$$

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where P_t is the price level, Y_t is real income, M_t is nominal money, and V_t is velocity. As in Belongia and Ireland (2015, 2017, 2021) and Ireland (2024), the long-run price-level target is:

$$P_t^* = M_t V_t^* / Y_t^* \tag{15}$$

In natural logs (denoted by lower case letters), eq. (14) takes the following form:

$$p_t + y_t = m_t + v_t \tag{16}$$

and, similarly for eq. (15). If the log of the price level has a unit root (is integrated of order 1), then an error-correction model of inflation ($\pi = \Delta p$) could be specified as:

$$\Delta p_t = \alpha + \beta_1 \Delta p_{t-1} + \cdots + \beta_q \Delta p_{t-q} + \gamma \left(p_{t-1}^* - p_{t-1}\right) + \varepsilon_t \tag{17}$$

where γ is expected to be positive, q is the lag length, and ε is an i.i.d. residual. Equation (17) is the form of P-star estimated by, for example, Kamal (2014). However, for the sample period used by Hallman et al. (1991), π was not stationary and had a unit root. Appealing to the inflation-augmented Phillips Curve, Hallman et al. (1991) cast their P-Star model in terms of the change in inflation ($\Delta^2 p = \Delta\pi$), which was stationary over their sample periods resulting in:

$$\Delta^2 p_t = \alpha + \beta_1 \Delta^2 p_{t-1} + \cdots + \beta_q \Delta^2 p_{t-q} + \gamma \left(p_{t-1}^* - p_{t-1}\right) + \varepsilon_t \tag{18}$$

Their model is equivalent to regressing the change in inflation on its lags and the price gap:

$$\Delta\pi_t = \alpha + \beta_1 \Delta\pi_{t-1} + \cdots + \beta_q \Delta\pi_{t-q} + \gamma \left(p_{t-1}^* - p_{t-1}\right) + \varepsilon_t \tag{19}$$

One can decompose the "price gap" in logs ($p^* - p$) as in Belongia and Ireland (2017) as:

$$p_t^* - p_t = \left(m_t + v_t^* - y_t^*\right) - \left(m_t + v_t - y_t\right) = \left(v_t^* - v_t\right) - \left(y_t^* - y_t\right) \tag{20}$$

which implies that eq. (19) can be transformed into:

$$\Delta\pi_t = \alpha + \beta_1 \Delta\pi_{t-1} + \cdots + \beta_q \Delta\pi_{t-q} + \gamma \left(v_{t-1}^* - v_{t-1}\right) - \gamma \left(y_{t-1}^* - y_{t-1}\right) + \varepsilon_t \tag{21}$$

Ireland (2024) estimates versions of (21) that allow the coefficients on the two gap terms to differ.[23]

Following Ireland (2024), we treat y as real PCE and use the PCE price deflator to measure inflation, so that y^* is trend or equilibrium real consumption; see also Belongia and Ireland (2017). The change in the log of the PCE deflator ($\Delta LPPCE$) is stationary over 1985 to 2024Q3; see Table 1. Consequently, we estimate a specification corresponding to eq. (17) while incorporating the decomposition of the gap term in eq. (20), which results in our general estimating equation:

$$\pi_t = \alpha + \beta_1 \pi_{t-1} + \cdots + \beta_q \pi_{t-q} + \gamma^v \left(v_{t-1}^* - v_{t-1}\right) - \gamma^y \left(y_{t-1}^* - y_{t-1}\right) + \varepsilon_t \tag{22}$$

which allows for the coefficients on the two gap terms to differ as in Ireland (2024). We consider specifications where $\gamma^v = \gamma^y$ is imposed and ones where it is not. In estimating the latter specifications, we report the actual regression coefficient on the consumption gap, which we expect should be negative.

We use our velocity model developed in Section 3.2.3 to estimate long-term equilibrium log velocity, v^*. To implement the P-star model, we treat the 1985Q3 to 2012Q4 period as our "in sample" and then calculate the "velocity gap," $v^* - v$, out of sample from 2013Q1 to 2024Q3. Here, we set the stock load variable at its 2022Q4 value in 2023-24 in order to cover more of the period encompassing the partial winding down of inflation. As Figure 5 shows, velocity tends to be above its long-run equilibrium following the GFC until just before the pandemic. Consequently, the velocity gap is generally negative throughout the 2010s implying downward pressure on inflation due to monetary factors given eq. (22). Although not conclusive, this is certainly suggestive given that core inflation was nearly always below the Fed's 2% target throughout this period. We select the lag length by the SIC criteria following Ireland (2024).

Belongia and Ireland (2015) use the one-sided HP filter (Stock and Watson, 1999) to detrend real income and velocity.[24] We too use a one-sided HP filter to calculate equilibrium log consumption in order to compute $y^* - y$, which we will refer to as *cgap* in the tables. As explained above, however, we modify the Belongia-Ireland (B-I) framework by using our parsimonious model of equilibrium velocity to calculate the velocity gap, $v^* - v$, which we will refer to as *vgap*. Later, we compare results to using the one-sided HP filter to also detrend log velocity. Finally, we use *pgap*, which equals *vgap* minus *cgap*, to refer to the price gap. A potential advantage of our model-based approach for velocity is that some detrending filters may not perform well over the pandemic or in the lead up to it. For example, Kamber et al. (2025) found that for the output gap, "the real-time HP filter estimates in 2018–2019 suggested that the economy was very close to trend, while the estimates for the same time period have been subsequently revised upwards..." In contrast they found that their refined Beveridge-Nelson filter "would have provided a better prediction of the final-vintage HP filter estimates than a one-sided HP filter" and also had "good revision properties during the pandemic."[25] Our model of equilibrium velocity is estimated over an in-sample period that excludes COVID-19 and its out-of-sample performance is impressive as seen in Figure 5. Nevertheless, we are still subject to this criticism as it relates to detrended consumption.

4.2. Modeling overall and core PCE inflation

We use the framework in eq. (22) to model overall and core PCE inflation: $\Delta LPPCE$ and $\Delta LPcorePCE$ respectively. We use the estimated coefficients from Model 2 in Table 3 corresponding to the 1985Q3-2012Q4 sample period, to calculate v^*. Using these coefficients, we calculate a static forecast of v^* out-of-sample from 2013 until the end of our sample period and the corresponding velocity gap term, $v^* - v$ (*vgap*). As previously noted, we extend the out-of-sample period to 2024Q3. The in-sample period ends in 2012Q4, which reflects two factors. First, as Figure 5 shows, this provides ample time for both equilibrium and actual velocity to recover from the large negative shock associated with the subprime and global financial crisis (GFC) of 2007–11. Moreover, the regulatory response to the GFC took the form of the Dodd-Frank Act, which was passed in mid-2010, and which came out in phases and affected money creation. We then estimate the P-Star model eq. (22) from 2013Q1 to 2024Q3. All regressions include separate dummies for 2020Q2, 2020Q3, and 2020Q4 to control for the initial phase of the pandemic.

To eq. (22), we add two variables to control for other unusual shocks. All of our estimated models include *DPEnergy*, which is a dummy equal to 1 in any quarter in which the PCE price index for energy goods and services falls by more than 10 percent relative to a PCE price index which excludes energy goods and services. Including this dummy addresses large outliers that are reflective of global shocks to energy prices while avoiding introducing multicollinearity with smoother energy price variables, such as the continuous percent change in relative energy prices that may partly reflect general trends in overall inflation. The dummy picks up 2020Q2, but we set it to zero in that period to avoid conflating the coefficient on it with the worst quarter of the pandemic.[26] Consequently, the dummy is *de facto* a dummy for 2015Q1, when energy prices sagged during a major downshift in the long-term growth rate of the Chinese economy. Owing to its construction, *DPEnergy* is expected to have a negative coefficient, which is expected to be larger in magnitude for overall than for core PCE inflation. The coefficient in core PCE inflation models may be significant and negative owing to the pass-through of energy price drops into the prices of other consumer items.

To shed light on how disruptions to supply chains affected inflation during the recovery from the COVID Recession, we test versions of eq. (22) that also include the t-1 level of the quarterly average of the FRBNY's Index of Global Supply Chain Pressures (*SupPress*). This index draws information from several indicators of supply pressures and effort is made to remove the influence of demand side factors from the index. During the pandemic, spikes in *SupPress* reflect breakdowns in market functioning associated with supply chain disruptions that are not easily captured by our measure of the consumption gap and are plausibly inflationary in nature. Over a sample

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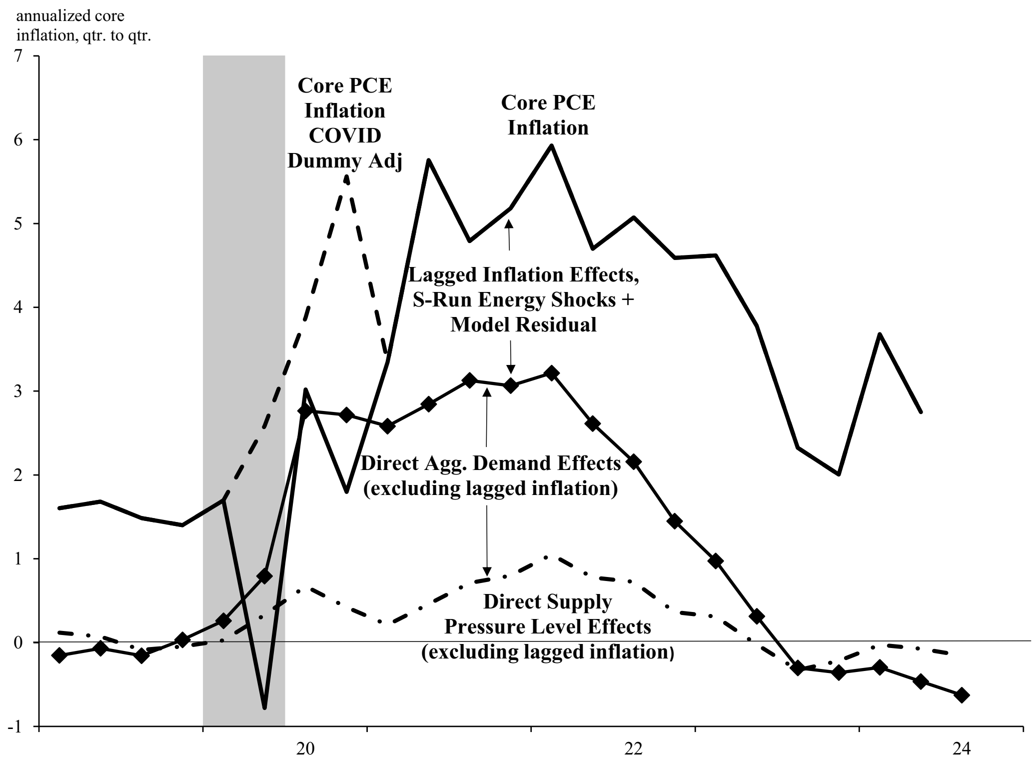

Figure 7. Effects of the price gap and supply chain (level) pressures on U.S. core inflation.
Sources: BEA, FRBNY, and authors' calculations.

from when *SupPress* begins in 1998Q1 to 2024Q3, the unit root tests (Table 1) indicate that it is I(0), implying that the level of *SupPress* could be tested in a regression with other variables that are stationary. We separately add the t-1 change and the t-1 level of the index to different models because it is unclear *a priori* which should enter the P-Star model. For overall PCE inflation, we include both the t-1 and t-2 lags of Δ*SupPress* when testing for the effects of changes.

We estimate six P-Star models for quarterly overall PCE inflation and core PCE inflation (annualized) over a 2013Q1-2024Q3 sample, which are reported in Tables 4 and 5 respectively. By extending the sample through 2024Q3, we are able to bolster the limited number of observations and to pull in more data from the recent period when *SupPress* moves a lot. In each table, Model 1 is a P-Star model that follows eq. (22) allowing for differences in the coefficients on *vgap* and *cgap* as in Ireland (2024). Model 2 combines the velocity (*vgap*) and consumption (*cgap*) gaps into the price gap ($pgap = vgap - cgap$) thus imposing equality of the coefficients (in absolute value) as in the standard formulation of the model (e.g., Belongia and Ireland, 2017). Given the potential errors in measuring consumption gaps in real time during the pandemic, it may be more accurate to focus on the overall price gap rather than its subcomponents. Models 1 and 2 omit *SupPress*. Models 3 and 4 repeat Models 1 and 2, respectively, except that they add the lagged level of *SupPress*, while Models 5 and 6 add the lagged change in *SupPress* to the models of core inflation and both the first and second lags of the change in *SupPress* to the models of overall PCE inflation.

4.3. Estimation results for overall and core PCE inflation

Several notable patterns emerge in Tables 4 and 5. In the even-numbered models, the combined price gap term is statistically significant. In the odd-numbered models that decompose the price gap, the velocity gap terms are significant with the expected positive sign.[27] The real consumption

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Table 4. Quarterly P-star models of U.S. PCE (consumer) inflation, 2013Q1-2024Q3.

$$\pi_t = \alpha + \beta_1 \pi_{t-1} + \ldots + \beta_5 \pi_{t-5} + \gamma^v vgap_{t-1} + \gamma^c cgap_{t-1} + \lambda_1 SupPress_{t-1} + \varepsilon_t$$

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Constant	1.147**	1.074**	1.427**	1.348**	1.217**	1.028**
	(4.63)	(4.75)	(4.97)	(5.02)	(5.49)	(4.76)
π_{t-1}	0.207	0.189	0.109	0.091	0.084	0.071
	(1.58)	(1.47)	(0.79)	(0.66)	(0.70)	(0.56)
π_{t-2}	−0.156	−0.153	−0.194	−0.191	−0.145	−0.142
	(1.21)	(1.20)	(1.53)	(1.51)	(1.25)	(1.16)
π_{t-3}	0.198+	0.192+	0.141	0.136	0.210*	0.194+
	(1.86)	(1.82)	(1.30)	(1.26)	(2.20)	(1.93)
π_{t-4}	0.117	0.114	0.108	0.104	0.138	0.125
	(1.08)	(1.06)	(1.02)	(1.00)	(1.43)	(1.23)
π_{t-5}	−0.026	−0.001	0.045	0.071	0.098	0.126
	(0.26)	(0.01)	(0.42)	(0.69)	(1.02)	(1.24)
$vgap_{t-1}$	0.200**		0.165**		0.239**	
	(6.11)		(4.46)		(7.77)	
$cgap_{t-1}$	−0.038		0.004		0.218	
	(0.19)		(0.02)		(1.14)	
$pgap_{t-1}(vgap\text{-}cgap)$		0.188**		0.153**		0.201**
		(6.65)		(4.55)		(7.47)
$DPEnergy_t$	−2.781**	−3.015**	−2.777**	−3.022**	−2.716**	−3.288**
	(2.97)	(3.44)	(3.05)	(3.55)	(3.31)	(4.00)
$SupPress_{t-1}$ 3,4			0.451+	0.446+	0.391	0.285
$\Delta SupPress_{t-1}$ 5,6			(1.78)	(1.77)	(1.55)	(1.09)
$\Delta SupPress_{t-2}$ 5,6					0.794**	0.600*
					(3.33)	(2.57)
$D2020Q2_t$	−4.303**	−3.869**	−4.858**	−4.400**	−5.819**	−4.403**
	(4.14)	(4.52)	(4.60)	(4.97)	(5.61)	(5.11)
$D2020Q3_t$	−2.904	−1.190	−4.020	−2.218+	−8.462**	−3.001*
	(1.13)	(1.03)	(1.56)	(1.75)	(3.08)	(2.36)
$D2020Q4_t$	−4.671**	−4.175**	−4.762**	−4.241**	−6.206**	−4.667**
	(3.58)	(3.75)	(3.76)	(3.91)	(4.70)	(3.94)
Adjusted R^2	0.831	0.833	0.841	0.843	0.871	0.856
S.E.	0.829	0.824	0.804	0.800	0.725	0.765
LM(1)	0.38	0.68	0.15	0.25	1.30	0.07
LM(4)	14.07**	9.31+	8.13+	5.99	8.81+	2.41

Notes: $^{+}$,*,** denote 90%, 95% & 99% significance. Absolute t-statistics are in parentheses. *vgap* is formed using estimates of v^* from Model 2, Table 3, estimated over 1985Q3-2012Q4. *cgap* is formed from real PCE and an estimate of trend real PCE from a one-sided HP filter.

gaps have insignificant coefficients in all cases with unexpected positive coefficients in two cases. Moreover, the coefficient on the consumption gap is lower in absolute magnitude than the coefficient on the velocity gap across all specifications. The lagged level of the supply chain index has a positive and significant coefficient for all models in Tables 4 and 5, while the coefficient on the

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Table 5. Quarterly P-Star models of U.S. core PCE (consumer) inflation, 2013Q1-2024Q3.

$$\pi_t = \alpha + \beta_1\,\pi_{t-1} + \ldots + \beta_5\,\pi_{t-5} + \gamma^v vgap_{t-1} + \gamma^c cgap_{t-1} + \lambda_1 SupPress_{t-1} + \varepsilon_t$$

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Constant	0.922**	0.905**	1.075**	1.067**	0.808**	0.778**
	(5.64)	(5.82)	(6.23)	(6.40)	(5.15)	(5.04)
π_{t-1}	0.313*	0.298*	0.249+	0.239*	0.346**	0.307**
	(2.51)	(2.54)	(2.02)	(2.08)	(2.98)	(2.80)
π_{t-2}	−0.288*	−0.292*	−0.320*	−0.323**	−0.233+	−0.245*
	(2.37)	(2.43)	(2.73)	(2.79)	(2.03)	(2.15)
π_{t-3}	0.328**	0.321**	0.272**	0.267**	0.273**	0.261**
	(3.61)	(3.65)	(2.99)	(3.05)	(3.15)	(3.04)
π_{t-4}	−0.064	−0.059	−0.052	−0.049	−0.039	−0.029
	(0.69)	(0.64)	(0.58)	(0.56)	(0.46)	(0.34)
π_{t-5}	0.206*	0.223*	0.273**	0.283**	0.228*	0.265**
	(2.10)	(2.54)	(2.75)	(3.21)	(2.50)	(3.16)
$vgap_{t-1}$	0.138**		0.116**		0.135**	
	(7.43)		(5.61)		(7.80)	
$cgap_{t-1}$	−0.087		−0.087		−0.009	
	(0.69)		(0.72)		(0.07)	
$pgap_{t-1}$ (*vgap-cgap*)		0.136**		0.115**		0.130**
		(7.73)		(5.88)		(7.80)
$DPEnergy_t$	−0.822	−0.892+	−0.783	−0.822+	−0.714	−0.886*
	(1.66)	(1.95)	(1.65)	(1.88)	(1.55)	(2.07)
$SupPress_{t-1}$ 3,4			0.247*	0.249*	0.369*	0.337*
$\Delta SupPress_{t-1}$ 5,6			(2.08)	(2.14)	(2.64)	(2.48)
$D2020Q2_t$	−3.194**	−3.066**	−3.434**	−3.364**	−3.781**	−3.432**
	(5.65)	(6.70)	(6.21)	(7.34)	(6.66)	(7.57)
$D2020Q3_t$	−0.875	−0.375	−1.135	−0.854	−1.956	−0.700
	(0.61)	(0.55)	(0.82)	(1.24)	(1.40)	(1.07)
$D2020Q4_t$	−3.872**	−3.746**	−3.835**	−3.764**	−3.400**	−3.149**
	(5.04)	(5.43)	(5.22)	(5.71)	(4.64)	(4.57)
Adjusted R^2	0.906	0.909	0.914	0.917	0.920	0.920
S.E.	0.450	0.445	0.430	0.424	0.416	0.416
LM(1)	0.26	0.12	0.01	0.00	0.46	0.14
LM(4)	10.88*	9.13+'	6.03	6.00	8.02+	6.65

Notes: +,*,** denote 90%, 95% & 99% significance. Absolute t-statistics are in parentheses. *vgap* is formed using estimates of v^* from Model 2, Table 3, estimated over 1985Q3-2012Q4. *cgap* is formed from real PCE and an estimate of trend real PCE from a one-sided HP filter.

lagged change in the index is significant and positive as expected for models of core PCE inflation in Table 5. For overall PCE inflation, the first lag of the change is insignificant, but positive, while the second lag is significant and positive. The evidence that the velocity and price gaps have statistically significant and positive effects, while the coefficients on the level and first difference of *SupPress* are positive and significant, together imply roles for both aggregate demand and supply pressures in driving the rise and partial ebbing of inflation during the COVID recovery. The COVID impact dummies for 2020Q2 and 2020Q4 are highly significant in all specifications.

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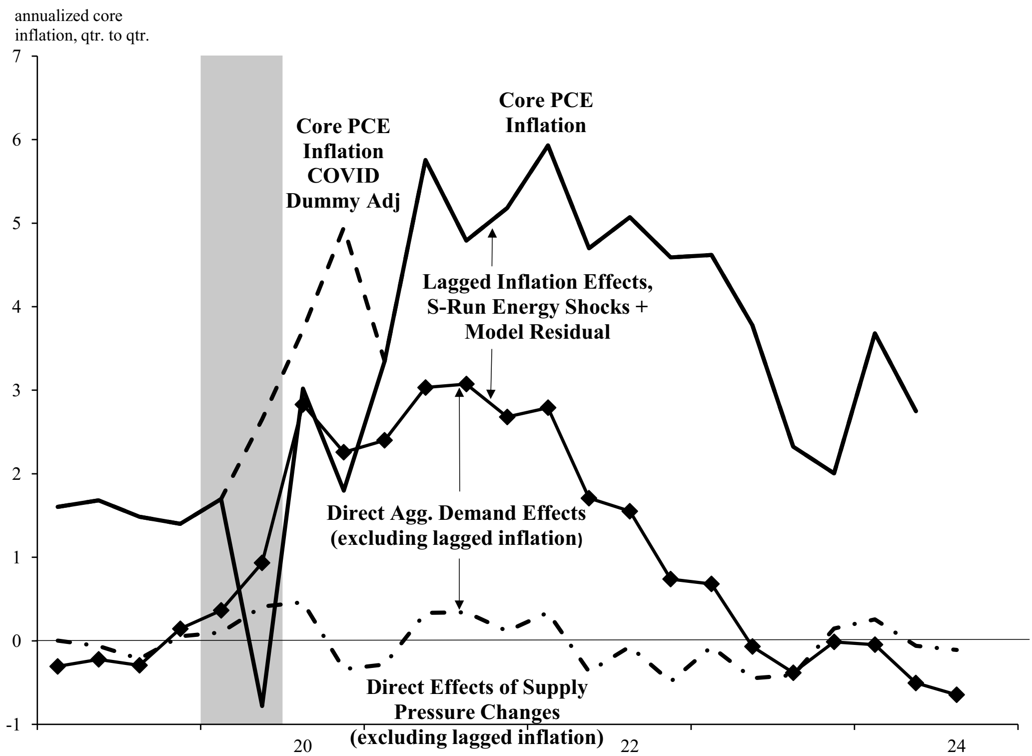

Figure 8. Effects of the price gap and supply chain (changes) pressures on U.S. core inflation.
Sources: BEA, FRBNY, and authors' calculations.

Figure 7 illustrates the estimated role of these factors on core PCE inflation using Model 4 from Table 5. Actual core inflation is plotted along with an adjustment for the COVID dummy effects. The figure plots the estimated effect of global supply pressures and the sum of the estimated effects of the lag of *pgap* and of the level of supply pressures, where the latter is the product of the lag of *SupPress* and its estimated coefficient. The gap between the latter two lines reflects how much the lagged deviation between equilibrium prices and actual prices (*pgap*) affects inflation beyond that captured by including five quarterly lags of inflation (and the energy dummy). It is unclear how much of the lagged changes in inflation reflect past aggregate demand versus supply shocks. The chart illustrates how the standard P-Star price gap and changes in supply conditions are directly linked to recent movements in core inflation rates. While the estimated contribution of supply pressures to the temporary run-up of core inflation is notable, more of it is attributable to aggregate demand pressures reflected in the gap between the two lower plotted lines.

Figure 8 repeats Figure 7 except it uses estimated coefficients from Model 6 in Table 5 and also gauges supply chain pressure effects using the change instead of the level of *SupPress*. The only notable difference in results is that the estimated effect of global supply chain pressures is much smaller, implying that there was an even larger relative role for aggregate demand pressures in explaining the 2021—22 run-up in core PCE inflation and its retreat in 2023 to mid-2024.

4.4. Estimation results using the HP filter to model long-run velocity

Tables 6 and 7 report results from overall and core PCE inflation models that correspond to the models in Tables 4 and 5, respectively, except that they use a one-sided HP filter to calculate the velocity gap, as in Belongia and Ireland (2015, 2017, 2021) and Ireland (2024), and consistent

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Table 6. Quarterly P-star models of U.S. PCE inflation (HP filter), 2013Q1-2024Q3.

$$\pi_t = \alpha + \beta_1\, \pi_{t\text{-}1} + \ldots + \beta_5\, \pi_{t\text{-}5} + \gamma^v vgapHP_{t\text{-}1} + \gamma^c cgap_{t\text{-}1} + \lambda_1 SupPress_{t\text{-}1} + \varepsilon_t$$

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Constant	0.104	0.083	1.364*	1.357*	−0.032	−0.051
	(0.29)	(0.23)	(2.63)	(2.62)	(0.09)	(0.14)
$\pi_{t\text{-}1}$	0.478**	0.531**	0.154	0.198	0.428*	0.443*
	(2.85)	(3.44)	(0.84)	(1.14)	(2.47)	(2.70)
$\pi_{t\text{-}2}$	0.054	0.067	−0.098	−0.088	0.103	0.112
	(0.32)	(0.40)	(0.62)	(0.56)	(0.59)	(0.66)
$\pi_{t\text{-}3}$	0.242	0.264+	0.046	0.063	0.236	0.244
	(1.60)	(1.78)	(0.30)	(0.43)	(1.53)	(1.62)
$\pi_{t\text{-}4}$	0.103	0.115	0.032	0.042	0.142	0.149
	(0.67)	(0.76)	(0.23)	(0.30)	(0.89)	(0.96)
$\pi_{t\text{-}5}$	0.068	0.051	0.118	0.103	0.161	0.163
	(0.44)	(0.33)	(0.85)	(0.75)	(0.94)	(0.96)
$vgapHP_{t\text{-}1}$	0.180+		0.007		0.212*	
	(1.99)		(0.07)		(2.24)	
$cgap_{t\text{-}1}$	−0.416		−0.217		−0.310	
	(1.60)		(0.90)		(1.13)	
$pgapHP_{t\text{-}1}$ (*vgapHP-cgap*)		0.214*		0.036		0.227**
		(2.65)		(0.39)		(2.81)
$DPEnergy_t$	−3.084*	−2.748*	−2.975*	−2.676*	−3.097*	−2.976*
	(2.42)	(2.29)	(2.60)	(2.48)	(2.42)	(2.47)
$SupPress_{t\text{-}1}$ 3,4			1.024**	1.033**	0.449	0.483
$\Delta SupPress_{t\text{-}1}$ 5,6			(3.09)	(3.14)	(1.11)	(1.25)
$\Delta SupPress_{t\text{-}2}$ 5,6					0.259	0.289
					(0.73)	(0.86)
$D2020Q2_t$	−2.826*	−3.431**	−4.201**	−4.751**	−3.807*	−4.108**
	(2.04)	(2.94)	(3.19)	(4.22)	(2.39)	(3.26)
$D2020Q3_t$	3.418	0.951	0.964	−1.252	0.620	−0.524
	(1.00)	(0.60)	(0.31)	(0.79)	(0.15)	(0.28)
$D2020Q4_t$	−1.640	−2.298	−1.950	−2.538+	−1.718	−1.982
	(0.97)	(1.55)	(1.28)	(1.91)	(0.93)	(1.22)
Adjusted R^2	0.686	0.689	0.748	0.751	0.683	0.692
S.E.	1.130	1.124	1.012	1.008	1.136	1.121
LM(1)	8.66**	10.61**	1.88	3.03+	5.48*	5.99*
LM(4)	10.84*	13.42**	1.95	3.40	6.60	6.87

Notes: +,*,** denote 90%, 95% & 99% significance. Absolute t-statistics are in parentheses. *vgapHP* is formed from velocity and an estimate of trend velocity from a one-sided HP filter. *cgap* is formed from real PCE and an estimate of trend real PCE from a one-sided HP filter.

with how we calculated *cgap*. We use *vgapHP* and *pgapHP* to distinguish these from the earlier estimates in Tables 4 and 5. We refer to models in Tables 4 and 5 as our "modified approach."

There are several patterns across the overall PCE inflation results in Tables 4 and 6. First, in the odd numbered models, the velocity gap is either less significant or not at all for the HP filtered

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Table 7. Quarterly P-star models of U.S. core PCE inflation (HP filter), 2013Q1-2024Q3.

$$\pi_t = \alpha + \beta_1 \pi_{t-1} + \ldots + \beta_5 \pi_{t-5} + \gamma^v vgapHP_{t-1} + \gamma^c cgap_{t-1} + \lambda_1 SupPress_{t-1} + \varepsilon_t$$

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Constant	−0.437	−0.415	0.486	0.434	−0.714*	−0.593+
	(1.26)	(1.26)	(1.07)	(1.03)	(2.19)	(1.91)
π_{t-1}	0.677**	0.659**	0.437*	0.466**	0.716**	0.640**
	(4.09)	(4.51)	(2.53)	(3.11)	(4.77)	(4.74)
π_{t-2}	−0.022	−0.026	−0.141	−0.133	0.037	0.014
	(0.14)	(0.16)	(0.92)	(0.89)	(0.25)	(0.10)
π_{t-3}	0.415**	0.405**	0.240+	0.259+	0.354**	0.318*
	(2.99)	(3.12)	(1.71)	(2.01)	(2.78)	(2.57)
π_{t-4}	−0.086	−0.085	−0.100	−0.100	−0.015	−0.020
	(0.62)	(0.62)	(0.79)	(0.80)	(0.11)	(0.15)
π_{t-5}	0.279+	0.287+	0.328*	0.317*	0.352*	0.375*
	(1.80)	(1.91)	(2.30)	(2.31)	(2.47)	(2.64)
$vgapHP_{t-1}$	0.211**		0.093		0.240**	
	(3.10)		(1.25)		(3.85)	
$cgap_{t-1}$	−0.164		−0.157		−0.026	
	(0.92)		(0.97)		(0.15)	
$pgapHP_{t-1}$ (*vgapHP-cgap*)		0.203**		0.106		0.207**
		(3.39)		(1.64)		(3.74)
$DPEnergy_t$	−0.931	−0.990	−0.878	−0.803	−0.717	−0.987
	(1.32)	(1.52)	(1.36)	(1.34)	(1.12)	(1.65)
$SupPress_{t-1}$ 3,4			0.472**	0.460**	0.580**	0.513**
$\Delta SupPress_{t-1}$ 5,6			(2.84)	(2.87)	(2.95)	(2.73)
$D2020Q2_t$	−2.895**	−2.778**	−3.221**	−3.367**	−3.992**	−3.376**
	(3.52)	(4.30)	(4.24)	(5.39)	(4.80)	(5.32)
$D2020Q3_t$	0.383	0.862	0.686	0.042	−2.231	0.093
	(0.17)	(0.88)	(0.33)	(0.04)	(0.99)	(0.10)
$D2020Q4_t$	−2.809*	−2.673*	−2.478*	−2.667**	−2.537*	−1.995*
	(2.43)	(2.72)	(2.34)	(2.97)	(2.42)	(2.12)
Adjusted R^2	0.811	0.816	0.842	0.846	0.845	0.844
S.E.	0.640	0.632	0.584	0.577	0.579	0.582
LM(1)	5.83*	6.06*	6.98**	7.11**	0.85	1.45
LM(4)	11.20*	11.37*	8.02+	8.37+	3.16	4.41

Notes: +,*,** denote 90%, 95% & 99% significance. Absolute t-statistics are in parentheses. *vgapHP* is formed from velocity and an estimate of trend velocity from a one-sided HP filter. *cgap* is formed from real PCE and an estimate of trend real PCE from a one-sided HP filter.

velocity gaps (Table 6), whereas *vgap* is always significant at the 99% level in our modified models (Table 4). Second, and similarly, the price gap terms in the even-numbered models are more significant in the modified models in Table 4 than in the corresponding models in Table 6 except for Model 6. Third, the fit of the modified models in Table 4 is uniformly better (including for Model 6) as evidenced by the higher adjusted R^2's across the board that range from 0.092 higher for Model 4 (0.751 vs. 0.843) to 0.188 higher for Model 5 (0.683 vs. 0.871).[28]

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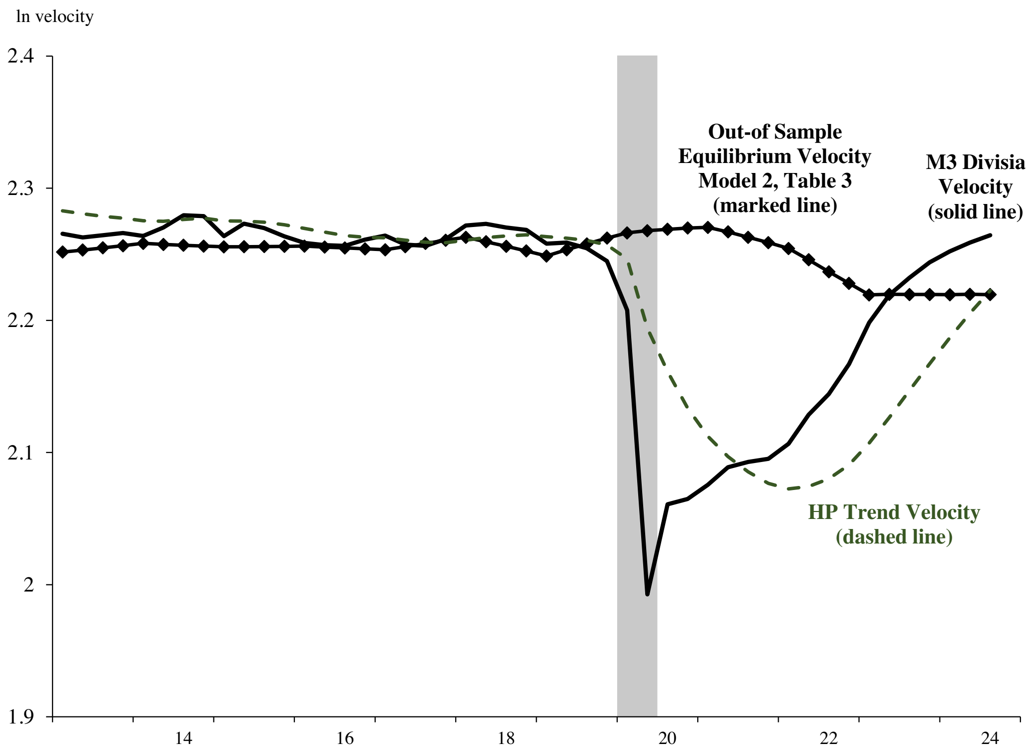

Figure 9. HP trend velocity lags actual U.S. velocity in the early-2020s.
Sources: CFS, Federal Reserve, BEA, and author's calculations.

The differences in significance of the price and velocity gaps and overall fit of the model arise from the differences in how the velocity gap is calculated. These are compared in Figures 9 and 10. During much of the period from 2010 to 2019, the one-sided HP filter trend is above velocity leading to small, but often positive, velocity gaps, indicating upward pressure on inflation. In contrast, the velocity gap from our model is generally slightly negative over 2010-19 (since velocity is usually above its long-run equilibrium over this period), implying that velocity was putting downward pressure on inflation in a period when inflation was generally slightly below the Fed's 2 percent target.

From 2020 through 2024, the one-sided HP trend lags behind the actual u-shaped velocity trajectory as shown in Figure 9. This produces a short-lived upward spike in the velocity gap as shown in Figure 10 after which the velocity gap turns negative in mid-2021 and bottoms out in early 2023—suggesting downward pressure on inflation precisely when inflation was surging. In contrast, our velocity gap also spikes and then declines, but much more gradually, and remains positive through early 2023. Then, small negative velocity gaps and negative money growth accompany the sharp deceleration of inflation since early 2023. Thus, our velocity gap measure appears more consistent with actual inflation.

Similar qualitative differences arise when modeling core PCE inflation, Tables 5 and 7, except that they are smaller in magnitude in several ways. While the modified models in Table 5 fit better across the board, the improvement in fit relative to the corresponding models in Table 7 is not as large for core inflation. This partly reflects the fact that *vgapHP* and *pgapHP* are both statistically significant at the 99 percent level in two of core PCE inflation models in Table 7 and have notably higher t-statistics than in the corresponding models of overall inflation in Table 6. Despite some

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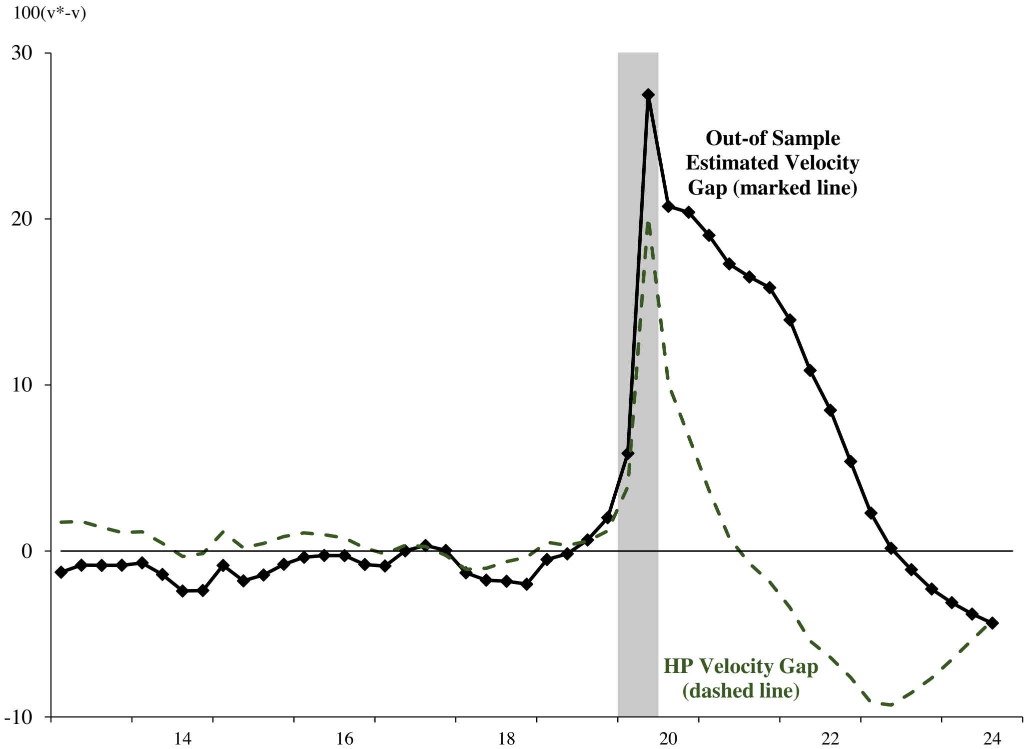

Figure 10. Estimated velocity gap implies more plausible swings in U.S. inflationary pressures than an HP filter-based measure.
Sources: CFS, Federal Reserve, BEA, and author's calculations.

of these nuanced differences, however, the model fits are better, the velocity and price gap terms are more statistically significant, and the magnitude of the effects of supply pressures is smaller for core inflation when the velocity gap is calculated out-of-sample from our error-correction model versus when it is calculated via a one-sided HP filter.

Taken together, these results suggest that our modified model enhances the ability of the P-star model to gauge inflationary pressures over this period and that by doing so, it helps us to avoid overstating the role of supply pressures in contributing to the rise and the partial ebbing of inflation over 2021-2024.

5. Conclusion

Broad Divisia measures of money services were ignored by many macroeconomists during the COVID Recession and the recovery from it because of past instability of the demand for simple-sum monetary aggregates. We show that the velocity of U.S. broad Divisia M3 is well modeled by, and is a stable function of, stock mutual fund loads and the shift in derivatives legislation since 1985. We also demonstrate that deviations of actual from equilibrium velocity help explain recent inflation in the U.S. We estimate a parsimonious error-correction model for the consumption velocity of Divisia M3 and use it to calculate the deviation of velocity from its long-run equilibrium out of sample. We then incorporate the corresponding velocity gap into the P-star model advocated by Belongia and Ireland (2015, 2017) and Ireland (2024) among others allowing us to

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incorporate structural factors altering the consumption velocity of Divisia M3. We find that our approach enhances the ability of the P-star model to gauge inflation pressures resulting from the velocity gap. In our modified framework, we find that only some of the 2021−22 surge in U.S. inflation is attributable to global supply pressures, with a larger contribution coming from aggregate demand. From a broader perspective, our findings are consistent with recent studies, which imply a role for both strong aggregate demand growth and negative supply factors in driving up U.S. core inflation during 2021−22 (e.g., Bernanke and Blanchard, 2025; Di Giovanni et al. 2022).

Movements in Divisia M3 and its short-run determinants plausibly reflect the confluence of macroeconomic factors. These can include some fiscal policy actions that were indirectly supported by Fed balance sheet actions, as well as shifts in risk aversion associated with the unfolding of the pandemic. Indeed, there are strong parallels during and after the pandemic between the build-up and unwinding of excess saving in response to tax cuts and changes in risk aversion from a Keynesian perspective, and a rapid rise and then fall of real monetary services from a more monetary view. This argues against a simple and overly reductive monetarist interpretation of our findings. Instead, as Nelson (2003) argued, "a spectrum of yields matters for the determination of aggregate demand *and* money demand" so that "money conveys information about monetary conditions not summarized by the short-term interest rate." Nelson's (2003) point is strengthened by the subsequent use of unconventional monetary policy tools in the Great and COVID Recessions that affected long-term Treasury yields and of private bond backstops that capped private credit risk premia in the COVID Recession (see Bordo and Duca, 2021, 2022). Further, Divisia money measures are better suited for an indicator role than simple sum monetary aggregates that assume perfect substitutability and which make no use of monetary asset user costs in their construction. We argue that broad Divisia measures are more reflective of variation in overall aggregate demand pressures and their lagged effects on the macroeconomy. From this perspective, the usefulness of broad Divisia money as an important U.S. macroeconomic indicator is consistent with the central themes and the title of Don Patinkin's (1956) classic book, *Money, Interest, and Prices*, in which prices, quantities, and lagged adjustment play key roles in the macroeconomy.[29]

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Notes

1 Despite similar Fed QE of $4 Trillion in the Great and COVID Recessions, Divisia money growth and inflation were much slower following the Great Recession when banks were impaired and less fiscal stimulus was provided.

2 Cronin (2018) employs the standard two-sided HP filter. Other recent studies based on the P-Star framework include El Shagi and Giesen (2013), El Shagi *et al.* (2015), Moosa and Al-Nakeeb (2020), and Ireland (2023, 2024).

3 Divisia monetary aggregates were developed by Barnett (1978, 1980), Barnett and Spindt (1982), and Barnett et al. (1984). See Anderson and Jones (2011) and Barnett et al. (2013) for overviews of U.S. Divisia data and see Barnett (2012), Barnett and Chauvet (2011), and Anderson et al. (2015) regarding the empirical significance of these measures.

4 Using VAR models, Hall et al. (2023) find that the main drivers of inflation for the U.S. were simple-sum M2 and large increases in government spending.

5 Barnett et al. (2013) describes these data. Divisia M4- adds commercial paper to Divisia M3. To Divisia M4-, Divisia M4 also adds Treasury bills. For corresponding historical Divisia series, see Anderson et al. (2019).

6 Commercial sweep programs also transferred funds from demand deposit accounts to mutual funds and offshore and overnight instruments; See, for further discussion, Jones et al. (2005) and Cynamon et al. (2006).

7 As stressed by Bolton and Oehmke (2015) and Stout (2011), CFMA made CDS contracts enforceable nationwide and gave them priority over other claims in bankruptcy.

8 See Bissoondeeal *et al.*, 2010) and Binner et al. (2025) for applications to the UK. Fleissig et al. (2023) analyze the Euro area.

9 Chen and Valcarcel (2024) test for cointegration between the velocity of Divisia and simple sum monetary aggregates and either the 3-month Treasury rate or the dual user costs for Divisia money; See also Chowdhury and Serletis (2024).

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10 See also Brill et al. (2021) for analysis of the Euro area.
11 Recall that $E_{1,t}$ is equal to one over the entire sample and $E_{2,t} = 1$ in the post-*CFMA* period as detailed above.
12 They also estimate cointegration models over pre- and post-1980 samples and over pre- and post-GFC samples. Post 1980, they find correctly signed relationships for Divisia M2 and M3 using their user cost measures, but not for simple sum M2. Similarly, they find correctly signed relationships for Divisia M3 and M4 in their post-GFC sample.
13 Our application has only one breakpoint, which is associated with CFMA. Giles and Godwin (2012, Table 1) provide critical values for the trace test. The test statistics depend on the relative breakpoint, v_1, calculated as the length of the sample up to 2000Q3 divided by the full sample size. For 1985Q3-2005Q4, we estimate the model using four quarterly lags, so that (including the four initial observations), $v_1 = 65/86 = 0.756$ and $1 - v_1 = 0.244$. With a single breakpoint, the critical values for v_1 and $1 - v_1$ are the same. Giles and Godwin compile statistics for $v_1 = 0.1, 0.2, 0.3, 0.4, 0.5$, which we use to bracket the actual values of the relative breakpoints for each of our samples. Thus, for 1985Q3-2005Q4, the corresponding critical values are for $v_1 = 0.2$ and 0.3. The bracketing values produce identical inference in most cases.
14 For this subsample, the estimated impact coefficient for the error-correction term for the equation for log velocity (α_1) is positive with a very low t-statistic providing further evidence that the model does not perform as expected.
15 We also estimated the model from 1985Q3 to 2013Q4 using the same lag length as for the sample ending in 2012Q4. Extending the sample by one year caused the sign of the coefficient on the bond rate to switch from being positive (0.011 with a *t*-stat of 0.38) to being negative (−0.035 with a *t*-stat of 1.6), which is more in line with the pre-pandemic sample period results.
16 Anderson and Jones (2011) and Anderson et al. (2019) constructed Divisia aggregates using alternative benchmark rates, one of which was a long-term rate.
17 For 2 lags, setting $r = 1$ yielded similar estimates as 4 lags for both samples. The coefficient estimates seemed unreasonable using 8 lags for both sample periods.
18 Trace tests are more decisive at 8 lags, rejecting $r = 0$ at the 99% level and not rejecting $r = 1$ at the 90% level for all three samples. Using 4 lags, $r = 0$ is rejected at the 95% level and $r = 1$ is not rejected at the 90% level for the 1985Q3 to 2012Q4 sample. In contrast, for the pre-GFC sample, $r = 0$ and $r = 1$ are both rejected at the 90% level using 4 lags, while $r = 0$ is rejected at the 99% level and $r = 1$ is rejected at the 95% level in the pre-COVID sample.
19 Note that in the post-CFMA period, the econometric model dummies out the observations corresponding to the number of lags in the VAR, but in the figure we show the broken constant kicking in immediately in 2000Q4.
20 Our lag length selection procedure described above would have chosen a lag length of 6 for Model 1 and lag lengths of 5 for Models 2 and 3. The estimates using that lag length were similar to what we reported for Model 2 except that the coefficient on *CFMA* was closer to −0.1, but the corresponding estimate had the wrong sign for Model 3.
21 Our lag length selection procedure described above would have chosen a lag length of 6 for Models 1 and 2 but would have chosen 7 for Model 3. At 7 lags, the estimates for Model 3 were −0.35 for the load variable and −0.004 for *CFMA* (with a *t*-stat of just 0.12).
22 The scaling controls for the overall size of the economy and using potential abstracts from the business cycle.
23 An alternative is El Shagi *et al.* (2015) who define adjusted (log) velocity as $v_t^a = p_t + y_t^* - m_t = p_t + y_t - m_t + (y_t^* - y_t) = v_t + (y_t^* - y_t)$, implying that $p_t^* - p_t = v_t^* - (v_t + (y_t^* - y_t)) = v_t^* - v_t^a$. El Shagi and Geeson (2013) examine an adjusted velocity that does not impose long-run unit income elasticity of money demand.
24 Hallman et al. (1991) assumed that trend velocity was constant and equal to its sample average, while Orphanides and Porter (2000) estimated it using a forecasting equation.
25 Regarding the Hamilton filter, they find that it "is more reliable than the HP filter and this holds during the pandemic" but it "suffers from base effects that produce a mechanical spike in the estimated output gap exactly two years after the onset of the pandemic, in line with the filter horizon."
26 The larger absolute size of coefficients on *D2020Q2* versus *DPEnergy* supports having two separate variables.
27 Because the velocity gap is based on an estimated variable, v^*, the standard errors should be corrected. Nevertheless, the standard t-statistics on the *vgap* and *pgap* variables in Tables 4 and 5 are very high.
28 In addition, the modified models (Table 4) fit somewhat better when Δ*SupPress* is used, while Table 6 models favor using the level of *SupPress*. The difference in fit is also much larger in Table 6.
29 See Keating et al (2014), Keating et al (2019), and Belongia and Ireland (2016, 2018) for empirical support for including Divisia monetary aggregates in structural VAR models of monetary policy.

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Appendix 1: Robustness checks on the preferred model of consumption velocity

A. Unrestricted constant version of the consumption velocity model

As a robustness check, we estimate a model corresponding to eq. (5), with Π being of reduced rank, but leaving μ unrestricted as in $H_{\ell c}(r)$. This results in a model of the form:

$$\Delta X_t = \alpha\beta' X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \sum_{i=1}^{k} \sum_{j=2}^{q} \kappa_{j,i} D_{j,t-i} + \mu E_t + \varepsilon_t \tag{5'}$$

We estimate eq. (5’) with $r = 1$ for the samples ending in 2012Q4 and 2019Q4. For this specification, the constant in the cointegration relations equals the estimated constants in eq. (5’) multiplied by $\alpha(\alpha\alpha')^{-1}$ following Johansen (1991, p. 1553). For both samples, the coefficient on *LSload* is about −0.29, the impact coefficient for velocity is around −0.15, and the coefficients on *CFMA* are −0.107 and −0.101 for the samples ending in 2012Q4 and 2019Q4, respectively. The error-correction terms are also very similar to those estimated from the less general model.

B. Estimating an equivalent money demand model of real Divisia M3

The consumption velocity models effectively impose a unitary income elasticity of money demand. As a robustness check, one can instead estimate real money demand models by separating the consumption velocity into real Divisia M3 (*LDM*) and real PCE (*LPCE*), where real Divisia M3 equals nominal Divisia M3 divided by the implicit PCE price deflator. In this case, we would be interested in the long-run money demand function corresponding to eq. (1’) in the main text, which we repeat here for the reader’s convenience:

$$LDM^* = \gamma_0 + \gamma_1 CFMA + \gamma_2 LSLoad + \gamma_3 LPCE \tag{1'}$$

We estimate this specification over the 1985Q3-2012Q4 and 1985Q3-2019Q4 sample periods using the same number of lags as for the corresponding velocity models for consistency. For the pre-pandemic sample period, $r = 0$ is rejected at the 99% significance level, $r = 1$ is rejected at

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the 95% significance level, and $r = 2$ cannot be rejected at the 90% significance level. Despite these results, we are primarily interested in the case where there is a single cointegrating vector. Setting $r = 1$, we normalized the estimated cointegration vector on *LDM* and represent the other coefficients in terms of a long-run real money demand function as in eq. (1'). If the estimated consumption elasticity is close to one, then we would expect the estimated coefficients on *LSLoad* and *CFMA* to be similar to their estimated values in the velocity models, but with opposite signs, since $LV = LPCE - LDM$ as explained in the main text. We found that the estimated consumption elasticity is 1.004, the estimated coefficient on the stock load is 0.285, and the estimated coefficient on *CFMA* is 0.124. We also estimated the specification corresponding to eq. (5'), as described in part A of this Appendix. Setting $r = 1$, we found nearly identical estimated coefficients for *LPCE* and *LSLoad* and an estimated coefficient on *CFMA* of 0.113. Thus, despite the results regarding the rank of the system, if we impose a single cointegrating vector, we find that the model is very consistent with our velocity models presented in the main text.

For the subsample ending in 2012Q4, $r = 0$ is rejected at the 99% significance level, $r = 1$ is rejected at the 90% significance level, but not at the 95% level, and $r = 2$ cannot be rejected at the 90% significance level. Again setting $r = 1$, we found that the estimated consumption elasticity is 1.16, the estimated coefficient on the stock load is 0.364, and the estimated coefficient on *CFMA* is 0.097. For the specification corresponding to eq. (5') over this sample period with $r = 1$, we found estimated coefficients of 1.13 on *LPCE*, 0.351 on *LSLoad* and 0.093 on *CFMA*. Once again, the estimated coefficients are generally in line with those of the velocity model when a single cointegrating vector is imposed, although we find a consumption elasticity of money demand that is close to but slightly higher than unity making the results somewhat less compelling.

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