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BOOK REVIEWS

KIGAMI, J. *Analysis on fractals* (Cambridge Tracts in Mathematics, no. 143, Cambridge, 2001), viii+226 pp., 0 521 79321 1 (hardback), £35 (US$54.95).

Over the last decade there has been a concerted effort to develop a mathematical analysis of the dynamics of wave propagation in fractal structures. To date such investigations have been limited to linear field equations over deterministic structures, with the central result being a rigorous definition of the Laplacian in such domains. This self-contained text is a milestone in this pursuit, providing a detailed account of one of the main methodologies. With very few prerequisites, the book manages to act as both an introductory text to this burgeoning field and as an essential resource for researchers.

Fractals have come a long way since their early role of providing counterexamples in areas such as measure theory. In fact the name itself and their widespread prominence only arose in the 1970s when their ability to capture the geometry of naturally occurring phenomena was recognized. In this first stage of development, geometric measure theory and the notions of Hausdorff measure and Hausdorff dimension were utilized to further our understanding of, essentially, the static aspects of these naturally occurring objects. It seems instinctive therefore, and a natural progression, to move into a second stage and examine the dynamics of processes in self-similar geometries. In particular, the manner in which waves propagate in fractal substrates. Such an undertaking can of course look to the classical development of wave propagation to provide a strategy to achieve this aim.

The first steps in a mathematical formulation were in fact to construct a Brownian motion on a fractal substrate. The main archetypal structure used in developmental work is the Sierpinski gasket. A series of random walks are considered on graphs which act as the pre-fractals for the Sierpinski gasket. By appropriate scaling arguments these random walks converge to a diffusion process which can then be associated with a Laplacian.

In contrast to this probabilistic approach, this book proposes a direct definition of the Laplacian. By considering a series of discrete Laplacians (or Dirichlet forms) on the pre-fractal graphs, an appropriate scaling argument can be used to derive a Laplacian operator rigorously. This approach is also very accessible, as the development is conducted in a classical analytic manner with essentially no reliance on probability theory. The definition also affords, for instance, a description of harmonic functions, Green's functions and solutions of Poisson's equation.

At present the analysis only applies to deterministic self-similar sets. Thus from a practical perspective there is still a lot of work to be done before the analysis can be directly applied to a random or statistically self-similar structure. Also, the fractal sets must be finitely ramified (or post-critically finite), which puts a very restrictive constraint on the topology of the graphs and perhaps more importantly on the boundary conditions. For instance, in the Sierpinski gasket it is only the three corner nodes in the outer triangular shell which can transfer information to and from its surroundings. However, this does provide a context in which to develop ideas and to conjecture about what would happen in the general case.

The book starts by providing an introduction to the geometry of self-similar sets. This involves defining what is meant by a self-similar set, describing their topological features and how we can define an appropriate measure on such sets. The analysis then develops by recognizing the close analogy of these graphs and the effective resistance of their electrical network counterparts. This leads to the study of the convergence of Dirichlet forms on a sequence of finite sets and finally to the definition of the Laplacian. This then naturally leads to the study of eigenvalues and eigenfunctions of both Dirichlet and Neumann type Laplacians. It can be seen even at this stage that their nature is quite distinct from those obtained on bounded real domains: for instance, the existence of localized eigenfunctions. It is also possible by defining an eigenvalue-counting function to study the asymptotic behaviour and derive a result of Weyl's theorem type. The last chapter in the book studies solutions to the heat equation and develops useful tools in this regard, such as a maximum principle.

In truth, we are only at the beginning of what this analysis can accomplish. A wide vista of opportunities has opened up for the study of linear field equations in fractal media. Some direction has already come from the physics literature where, for instance, the wave equation and reaction–diffusion equations have been studied. Anyone with a background in the analysis of linear field equations, with an interest in heterogeneous media, or who is looking to breathe new life into their research, should read this book.

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ROBINSON, J. C. *Infinite-dimensional dynamical systems* (Cambridge University Press, 2001) 461pp., 0 521 63564 0 (paperback) £24.95, 0 521 63204 8 (hardback), £70.

This impressive book offers an excellent, self-contained introduction to many important aspects of infinite-dimensional dynamical systems. The title of the book is accompanied by two different subtitles, namely 'From Basic Concepts to Actual Calculations', which appears on the front cover, and then, several pages later, 'An Introduction to Dissipative Parabolic PDEs and the Theory of Global Attractors'. Each gives a clear indication of the author's intention not only to develop the basic theory of infinite-dimensional dynamical systems but also to highlight applications to partial differential equations. The end result is a highly readable and beautifully explained account of a variety of topics ranging from fundamental concepts in functional analysis to the application of compactness methods for establishing the existence and uniqueness of solutions to partial differential equations, and concluding with the importance of the global attractor, and its fractal dimension, in determining the asymptotic dynamics of a dissipative system. The style of presentation is particularly appealing, due in no small part to the author's success in providing an informative insight into the thought processes behind the mathematics.

Following the introduction, where a simple example involving the one-dimensional heat equation nicely motivates the need for an infinite-dimensional theory of dynamical systems, the book is divided into four main parts. These are entitled 'Functional Analysis', 'Existence and Uniqueness Theory', 'Finite-Dimensional Global Attractors' and 'Finite-Dimensional Dynamics'.

The author devotes Part I to presenting a rigorous treatment of topics and results from functional analysis and operator theory that are essential for a full understanding of the later material. Chapters on Banach and Hilbert spaces, ordinary differential equations, linear operators, dual spaces and Sobolev spaces are included. This prerequisite material is developed in a user-friendly manner which should make it accessible to most readers. For example, in the section on spectral theory for unbounded symmetric operators, the desired result on the existence of an orthonormal basis of eigenfunctions is derived using a clever approach that avoids