

that, even when used in underresolved situations, vortex methods will give a correct qualitative answer.

The energy conservation in two-dimensional schemes, which follows from the Hamiltonian nature of the particle motion in the velocity field, is a feature that distinguishes vortex methods from the familiar Eulerian methods. For the three-dimensional schemes that are dealt with in Chapter 3 the conservation of circulation has generally led to a preference for vortex filament methods rather than vortex particle methods. This chapter discusses several ways that are now available for enforcing conservation in the three-dimensional vortex particle methods. Chapter 4 deals with the implementation of boundary conditions for inviscid flow vortex simulations. The kinematic boundary condition (no-through flow) is enforced by means of an extended vorticity field. The concept of a surface vortex sheet is introduced and Poincaré's identity is invoked, relating the values of the velocity field with its values at the boundary. The resulting integral equations are analysed and their approximation by panel methods is discussed.

Early simulations using inviscid vortex methods predicted linear growth in the mixing layer and they were able to predict Strouhal frequency in a variety of bluff-body flow simulations. However, the inviscid approximation of high Reynolds number flows has its limitations. In bluff-body flows viscous effects are responsible for the generation of vorticity at the boundaries, and an approximation of viscous effects is necessary, at least in the neighbourhood of the body. In three dimensions the transfer of energy to small scales produced by vortex stretching produces complex patterns of vortex lines. This complexity increases as time evolves, and viscous effects provide the only limit in the increase of complexity. The simulation of viscous effects in the context of vortex methods is discussed in Chapter 5. It is shown that vortex methods are able to simulate viscous effects accurately, while maintaining the Lagrangian character of the methods. In Chapter 6 vortex methods are discussed for unsteady flows in domains containing solid boundaries. This chapter deals with the no-slip boundary condition and its equivalence with the vorticity boundary condition. Integral methods are presented for the implementation of boundary conditions and the techniques are illustrated by direct simulation of bluff-body flows.

Chapter 7 deals with the problem of particle distortion in Lagrangian methods. In numerical simulation the clustering and spreading of the particles have various consequences, depending on the specific numerical schemes that are being adopted. For two-dimensional inviscid flows the result is a loss of accuracy in the computed velocity, which may lead to the appearance of undesirable small scales. For three-dimensional flows in regions of high strain the depletion of particles becomes fairly severe as the flow is generally associated with vorticity intensification. To overcome these difficulties there are two possible strategies, which may be used independently or in combination. The first approach consists of restarting the particles every few time steps at revised locations where the distortions are well controlled. The second one consists of processing the circulation carried by the particles in order to correct the effect of the distortion of the flow and to allow particles to continue to give an accurate description of the vorticity. Both strategies are considered in this chapter. Finally, Chapter 8 deals with hybrid schemes. A scheme of this type is formed by combining a vortex method and an Eulerian method with a view to by-passing the difficulties inherent in particle methods near boundaries.

This is a well-presented, readable text that illustrates some of the recent advances in these powerful methods for simulating incompressible flows.

D. M. SLOAN

CONSTANDA, C. *Direct and indirect boundary integral equation methods* (Monographs and Surveys in Pure and Applied Mathematics no. 107, Chapman & Hall/CRC, 2000), vi+201 pp., 0 8493 0639 6, £43.99.

The book is a detailed study of boundary integral equation methods in application to three different two-dimensional mathematical models: the Laplace equation, plane strain linearized

elasticity, and bending of linearly elastic plates with transverse shear deformation. Several types of such techniques are analysed in depth for each model: the direct method (based on the Green–Somigliana representation formulae) and the so-called classical, alternative, modified and refined indirect methods, where the solution is assumed to have a convenient integral representation in terms of single layer and double layer potentials. Boundary value problems with Dirichlet, Neumann and Robin conditions for interior and exterior domains are reduced to integral equations on the boundary contour, which are then solved in spaces of Hölder continuous or Hölder continuously differentiable functions. The solution is based on an algebra of operators associated with the boundary values of the potentials and the spectral properties of these operators.

It is shown that two-dimensional problems are more difficult to handle than three-dimensional ones on two counts. First, there are boundary curves for which zero is an eigenvalue of the operator defined by the boundary values of the single layer potential. For the Laplace equation this happens when the logarithmic capacity of the curve is equal to one. The logarithmic capacity is generalized to a characteristic matrix of the boundary curve for the systems of plane strain and bending of plates. The ‘pathology’ occurring on curves for which this characteristic matrix is singular is eliminated through a modification of the fundamental solution (the modified indirect method) or the addition of a constant (rigid displacement) to the potential used in the integral representation of the solution (the refined indirect method). Second, the fundamental solutions do not decay to zero at infinity. To be able to derive Green–Betti and representation formulae, special asymptotic classes of functions are introduced for the far-field of the solution. These classes consist of functions of finite energy and they facilitate the construction of uniqueness theorems.

In the last chapter all direct and indirect techniques described in the book are compared to find out which one is best suited for numerical computations.

The text is written clearly and the proofs are given in detail. For fluency some of the necessary hard-analysis material is gathered in an appendix at the end. Overall, the book offers a comprehensive treatment of the subject matter and constitutes a very useful source of information for mathematicians and other scientists interested in boundary integral equation methods.

M. ARON

SUNDER, V. S. *Functional analysis: spectral theory* (Birkhäuser Advanced Texts, Basel, 1998), ix+241 pp., 3 7643 5892 0 (hardback), DM 78.

The last 50 years have seen the appearance of several notable introductory texts on functional analysis, the emphasis of each reflecting its author’s particular research interests. The author of this book works on von Neumann algebras, part of the more general field of operator algebras, the norm-closed algebras of operators on Hilbert spaces. It is no surprise that the book’s mix of topics has particular relevance to this area. Despite the fact that operator algebras is one of the most active areas of functional analysis, this is, to my knowledge, only the second general text tailored particularly to the needs of those entering this field.

The treatment is mostly conventional, the exposition very clear and well explained. The main part of the book, amounting to three quarters of its length, is divided into five chapters. The first, ‘normed spaces’, is a concise but fairly complete treatment of the elements of Banach space theory. All the fundamental topics are here, including completeness, the dual of a normed space and spaces of bounded linear mappings. Several of the most important examples of Banach spaces are presented, and nice accounts of the Hahn–Banach, closed graph, open mapping and Banach–Steinhaus theorems are given. In the final part of the chapter, there is a short introduction to the theory of locally convex topological vector spaces, which sets the scene for the definition of the weak and weak*-topologies on a Banach space and its dual, and the statement and proof of Alaoglu’s theorem.