

## 'Columbus's Method of Determining Longitude: An Analytical View'

Arne B. Molander *writes*

1. INTRODUCTION. The detailed *Diario* Christopher Columbus recorded during his first voyage lacks any description of his methods for mapping the latitudes and longitudes he had promised to his sovereigns.<sup>1</sup> To rationalize a landfall 90 miles south of Columbus's measured latitude, historians have speculated that he relied entirely upon inaccurate dead reckoning navigation to obtain both coordinates, despite considerable opinion from navigation experts that transoceanic pilots then commonly maintained latitude by celestial means.<sup>2</sup> On the other hand, navigators and historians agree that lunar distance, a celestial technique for measuring longitude, was not conceived until 1514 – eight years after Columbus's death. However, Amerigo Vespucci recorded a 1499 longitude measurement on the east coast of South America using a lunar conjunction with Mars.<sup>3</sup> If Amerigo's controversial claim is true, then his good friend, Columbus, might have actually pioneered the technique in 1492. Strong circumstantial evidence supporting this possibility was published by the author four years ago in a *Journal for the History of Discoveries* addressed mainly to historians and geographers.<sup>4</sup>

Keith A. Pickering brought this navigation issue to the attention of appropriate experts in a critique of my article published in the January issue of the *Journal of Navigation*.<sup>5</sup> Unfortunately, Pickering's article contained a number of errors and misinterpretations of fact, some of which may have resulted from my ambiguous presentation. Thus, before responding to Pickering's critique, it would be useful to clarify the lunar longitude method I have postulated for Columbus and his contemporaries.

2. THE POSTULATED TECHNIQUE. With the possible exception of infrequent occultations, any useful 15th century system for measuring longitude from lunar-planetary conjunctions would have to be based on angle measurements made near the horizon, much as circumpolar stars culminating near the northern horizon were then used to maintain latitude.<sup>6</sup> Thus, the optimum lunar conjunctions of Mars, Saturn and Jupiter would have been those which occurred below the horizon (BTH.) These counter-intuitive conjunction locations would have been the most useful to the early explorers because they define horizon geometries unique to the viewing location with the planet usually just a few degrees above the horizon at both moonrise and moonset, either of which was generally visible. (Venus conjunctions may be useful whether they occur above or below the horizon because that planet is usually visible at the horizon.) Accordingly, on average, there should be about 2.5 optimum conjunctions of the four visible planets every lunar cycle. Even after adding the infrequently visible Mercury conjunctions, it is reasonable to conclude that the probability of one or more optimum conjunctions occurring within any randomly-selected 24-hour period is about 0.1.

Pickering missed the point when describing my postulated longitude technique for Columbus as 'one dependent not on measurement of angular distances, but on the timing of lunar-planetary conjunctions'.<sup>7</sup> My paper actually pointed out that Columbus could have used either visible or BTH conjunctions, but the latter had the advantage that 'no time measurements are required'.<sup>8</sup>

3. **VESPUCCI'S CLAIM.** On the night of 23 August 1499, Amerigo Vespucci was anchored off the east coast of South America when he made history's first recorded claim for using conjunctions to measure longitude. His estimate that 'the Moon was one degree and several minutes east of Mars'<sup>9</sup> at moonrise was impressively accurate if his location was then the Bay of Oyapoc, as can reasonably be inferred from his writings. Fig. 1 shows the Mars horizon geometries he would have observed shortly after this **WTH** conjunction for a 10-degree grid of Earth locations centred on Oyapoc, where the actual azimuth angle difference measured to the Moon's centre was one degree and seven minutes at moonrise!

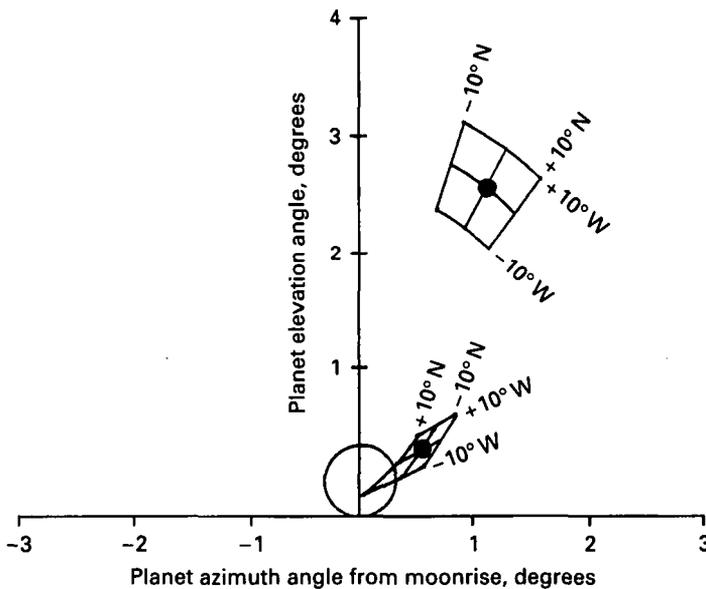


Fig. 1. Horizon geometries for two **WTH** conjunctions during Amerigo Vespucci's first voyage. **UPPER**: Mars position relative to evening moonrise on 23 August 1499 for 10-degree viewing-location grid centred at Oyapac. **LOWER**: Saturn's relative position at 4:45 AM moonrise on 26 May 1500 for 10-degree grid centred at 25° N – 90° W.

Upon returning to Spain, Vespucci's 18 July 1500 letter to Lorenzo Di Pierfrancesco De' Medici claimed his voyage extended to 90 degrees west longitude.<sup>10</sup> His supposedly outrageous claim might actually have been based on a measurement of Saturn's 26 May 1500 lunar occultation – possibly the most favourable conjunction geometry of the decade. Figure 1 shows Saturn's close proximity at moonrise that morning for a 10-degree grid centred at 25° N–90° W. In this case, a simple sketch of the lunar-Saturn geometry could determine longitude to within a degree or so after *ephemeris* corrections were applied by astronomers who had followed the same event in Spain. (This accuracy would be slightly degraded if Saturn did not become visible until the Moon was a degree or so above the horizon.) The 1502 Cantino Map supports my conjecture by depicting a landmass northwest of Cuba having many place names that match up with modern features of the US coastline extending from the Mississippi Delta to Long Island's Montauk Point.<sup>11</sup>

Pickering appears to dismiss Vespucci's Mars observation as coincidence while deploring the shortage of 'historical context'<sup>12</sup> supporting use of the Moon for longitude measurement. He ignores Jaime Ferrer, the court cosmographer, whose interest in the Line of Tordesillas longitude led to a 1495 audience with the Queen where he described 'two methods, theoretical and practical, for determining the much-discussed line and showed sincere appreciation of Columbus and his accomplishments'.<sup>13</sup> The 'theoretical' method proposed by this leading cosmographer was probably the well-known use of occasional lunar eclipses, while the 'practical' one might well have advocated the more frequent BTH horizon geometries.

4. COLUMBUS' SAILING DATE. In his Appendix II, Pickering purported to show that June was a more favourable sailing date than September for celestial longitude measurements.<sup>14</sup> This argument isn't supported by the 26 lunar-planetary conjunctions listed in his table 1 for June through October of 1492. My Fig. 2 shows the optimum

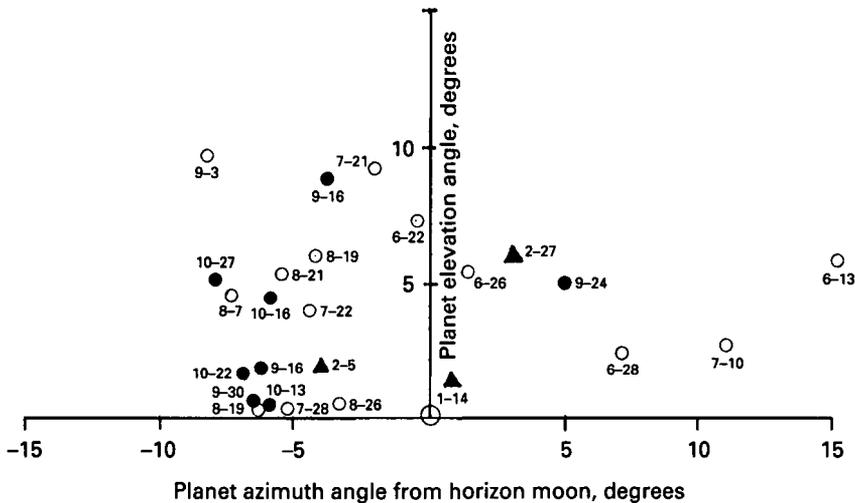


Fig. 2. Optimum horizon geometries predicted by *Ephemerides* for conjunctions from Pickering's Table 1. Empty circles are for first  $\frac{2}{3}$ 's of Pickering's five-month interval. Solid circles are for final  $\frac{1}{3}$  when Columbus planned to be in western Atlantic. Solid triangles are predictions for conjunctions he may have utilized on his return voyage.

geometries for 22 of these conjunctions relative to the Moon on a *darkened* horizon. (Pickering's 6-25 Jupiter, 7-23 Mercury and 9-22 Mercury conjunctions, all within eight degrees of the Sun, were probably invisible. His 10-21 Venus conjunction was certainly invisible with the planet then only two degrees from an *inferior* solar conjunction.) The temporal distribution of these remaining 22 conjunctions supports neither my claim nor Pickering's, because 14 (empty circles) occurred during the first two-thirds (June 1-Sept. 10) of his 153-day interval while eight (solid circles) occurred during the final third (Sept. 11-Oct. 31) when they might have been useful on Columbus's Atlantic crossing. However, several of the 14 resulted from visible conjunctions yielding weaker horizon geometries than BTH. If an acceptance threshold is set at 10 degrees angular separation from the Moon, the counts for the two time intervals are reduced to eleven and eight, respectively, while a more useful eight-degree threshold results in a count of nine and six. Thus, Pickering's own table shows the

density of optimum conjunctions was actually larger during the time-period Columbus selected for an ocean crossing.

Two obvious conclusions follow from Fig. 2. First, my assumption of a 0.1 probability of a BTH conjunction on any given day is a reasonable approximation. Secondly, Pickering's contention that a June transatlantic sailing date would have been preferable to Columbus's September choice is simply not true.

5. COLUMBUS'S OUTBOUND VOYAGE. Now examine Columbus's *Diario* for evidence of longitude estimates following each conjunction in Pickering's Table 1 listed between his departure date of 3 August and his landfall on 12 October.

*Saturn (8-7)*. The Moon set at 4:17 AM on August 7th with Saturn 5.5 degrees above the horizon and 8.5 degrees to its north, following the only lunar-planetary conjunction on the first leg of his journey. Columbus's only position estimate for this leg was delayed 24 hours (possibly to collect estimates from the other ships) until his 8 August entry as:

There were among the pilots of the three caravels diverse opinions about where they were, and the Admiral came out nearest the truth.<sup>15</sup>

Picking confirms that the *Ephemerides* data were almost error-free on 7 August<sup>16</sup>, but the weakest horizon geometry of Columbus's entire voyage might have been responsible for his cautious performance claim. Pickering relies on an ambiguous *Diario* entry to conclude that this 8 August position estimate was actually 'triggered by arrival in the Canaries',<sup>17</sup> but Ferdinand's biography of his father removes all doubt with 'at daybreak on Thursday, August 9th, they came in sight of the Canaries'.<sup>18</sup>

*Mars (8-19)*; *Jupiter (8-19)*; *Venus (8-26)* and *Saturn (9-3)*. As can be seen in Fig. 2, this set of conjunctions was not as favourable as those in the following lunar cycle. However, Columbus could have utilized them to test his method at known longitudes, because none of them occurred during the five days he was shuttling between harbours in the Canaries.<sup>19</sup>

*Jupiter (9-16)* and *Mars (9-16)*. When the Moon rose at 2:12 AM on 17 September, a pair of planets, Mars and Jupiter, stood just above the eastern horizon. Fifty-two hours after that moonrise, Columbus finally recorded the fleet's estimates as:

the pilot of the *Niña* found himself 440 leagues from the Canaries, the pilot of *Pinta*, 420, and the pilot of the ship in which the Admiral sailed, an even 400.<sup>20</sup>

Once again, Columbus seems to have delayed recording his position estimate until he had collected corresponding ones from the *Niña* and *Pinta*.

*Venus (9-24)*. When Venus (at maximum brightness) set at 7:06 PM on 24 September, the Moon was only 4.5 degrees above the western horizon. The following day Columbus's third position fix came to light when he:

said to send the said chart to him. And it having been sent over by means of some cord, the Admiral began to plot their position on it with his pilots....<sup>21</sup>

His unreliable chart must have indicated islands in the vicinity because 25 September initiated a week of hopeful search for land. The timing of this *Diario* position entry clearly falls within the 24-hour correlation window.

*Saturn (9-30)*. The Moon set at 1:05 AM on 1 October when Saturn was within one degree of the western horizon, and five hours later Columbus recorded yet another correlated position fix as:

The Admiral's pilot held at dawn today that they had made up to this point 578 leagues from the island of Hierro.... But the true account that the Admiral figured and kept to himself was 707.<sup>22</sup>

In his Table 2, Pickering converts this 707-league estimate into degrees to 'demonstrate' its correspondence with a longitude 50 percent greater than would have been predicted by the *Ephemerides*.<sup>23</sup> Unfortunately, this 'demonstration' is achieved by using an artificial league of 4:22 n.m., 50 percent longer than Pickering elsewhere assumes Columbus used to measure distance. His contrived argument conceals a possible influence of Saturn's conjunction on this position estimate. Table 2 also includes an irrelevant 15 February longitude estimate obviously triggered by the sighting of land following a terrible storm, rather than derived by dead reckoning or celestial means, which is the question at hand.

Pickering goes on to argue that the 3 October entry is an uncorrelated position fix<sup>24</sup> excluded from my analysis. Actually, this entry merely terminates his week-long island search, a decision amplified in Fernando's biography with 'he reflected [that day] that he would lose respect and credit for his voyage if he beat aimlessly about from place to place looking for lands whose position he had claimed to know most accurately'.<sup>25</sup> Coincidentally, this excerpt from Fernando also validates my assumption that multi-ship position estimates were sometimes delayed when it compares 3 October readings from the *Pinta* and *Niña* with the *Santa Maria*'s 1 October reading.<sup>26</sup>

6. COLUMBUS' RETURN VOYAGE. Almost a fortnight after Columbus had lost the *Santa Maria* to Hispaniola's reefs, he was rejoined by the wayward Pinzón returning the *Pinta* from an unauthorized two-month excursion to parts unknown. Now, almost ready to return to Spain, the *Niña* and *Pinta* sailed eastward for six days along Hispaniola's northern coast, past several fine harbours all opening towards the north. On 12 January 1493 they found anchorage along the north shore of Samaná Bay, an excellent vantage point from which to view the eastern horizon when the Moon and Venus rose together at 1:21 AM on the 14th.

*Venus (1-14)*. If Columbus used this fine horizon geometry (the voyage's best, although 1.5 degrees further from the horizon than predicted by the *Ephemerides*) to estimate his longitude, he would have erred badly in this vital position fix. As luck would have it, the *Ephemerides* error for that date was the largest of his voyage, as demonstrated in Pickering's Fig. 1.<sup>27</sup> If he had corrected for lunar parallax, as 'optimistically' assumed by Pickering,<sup>28</sup> these *Ephemerides* errors would have generated a longitude 37 degrees east of his actual location! This huge error could account for this surprising *Diario* entry the following day:

And he says that if this is so, these Indies are very near the Canary Islands, and for this reason he believed that they were less than 400 leagues distant.<sup>29</sup>

While he was then actually 3000 n.m. from Hierro, a longitude computed from the *Ephemerides* would have reduced his apparent separation to 900 n.m., a reasonable agreement with his *Diario* entry, given the quality of his charts. It is likely that Columbus was stunned to find such a large discrepancy with his pilotage records from the preceding three months, but he seems to have accepted this erroneous celestial estimate, because he confirmed it the next morning by sailing for home on (an erroneous) course:

...straight to Spain, northeast by east...<sup>30</sup>

which intersects his 19-degree latitude near his erroneous estimate rather than his true longitude. A third indication that he favoured the erroneous celestial estimate over his pilotage occurred when he reached the Canary Islands latitude. Apparently accepting its indication that the islands were just to his east on 26 January (when they were actually 2000 n.m. away) he altered his north-easterly course after sunrise to steer for 12 hours

...sometimes east-southeast, and sometimes southeast...<sup>31</sup>

This entry tells me he was making a daylight search for Canary Island peaks, rather than taking a close-hauled port tack with the wind East by North. With that assumed wind direction he could have reached home sooner by taking a close-hauled starboard tack to the north-northeast. A fourth, and stronger, indication occurred on 30 January when he interrupted his east-northeast course at Madeira's latitude to sail 'south by east'<sup>32</sup> during the twelve search hours provided by daylight.

*Mars* (2–5). At 8:40 PM on 5 February, the day after he started sailing eastward along the Azores latitude, Columbus would have seen the waning full Moon rising when Mars was less than two degrees above the eastern horizon. Pickering's Fig. 1 demonstrates the *Ephemerides* error on this date was about a third of the 14 January error, and of opposite polarity.<sup>33</sup> If Columbus had utilized this conjunction to estimate his longitude he would have suddenly shifted his perceived location almost 3000 n.m. westward! This shock might have reverberated throughout the fleet for several days while they tried to resolve the contradictory longitude indications. Thus, it is not surprising that the next day's entry, 6 February, initiated a debate when:

Vincente Anes [figured] that today in the morning the island of Flores lay to the north, and that of Madeira, to the east. Roldán said that the island of Fayal or that of San Gregorio lay to the north-northeast and Puerto Santo, to the east.<sup>34</sup>

This longitude debate seems to have carried over to the next day with:

This morning the Admiral [figured that he] was 75 leagues south of the island of Flores, and the pilot Pero Alonso [figured that] going [i.e. if he went] north he would pass between Terceira and Santa Maria....<sup>35</sup>

I believe the debate continued off and on until 10 February because there had been no celestial or terrestrial triggers since the 5th. On that date the position estimates of all the *Niña's* pilots were specified as:

In the Admiral's caravel Vincente Anes and the two pilots, Sancho Ruyz and Pero Alonso Niño, and Roldán charted their position; and all of them, according to their charts, [figured they] were passing much to the east of the islands of the Azores and, steering north, none would encounter the island of Santa Maria, which is the last of all of the islands of the Azores, rather, [they] would be beyond it by five leagues... but the Admiral found himself much off his route, and far behind them, because he figured that tonight the island of Flores lay to the north... So that they were nearer Castile than the Admiral by 150 leagues... (when) they see land, it will be known who figured most correctly.

*Jupiter* (2–27). The Moon set on 27 February at 3:50 AM with Jupiter visible six degrees above the western horizon. That day's *Diario* entry includes his only position estimate (following the only conjunction) after the Azores on his final leg. It reads:

And he reckoned himself 125 leagues from the Cabo de San Vicente, and 80 from the island of Madeira; and 106 from that of Santa Maria.<sup>36</sup>

Although this position estimate correlates with the Jupiter conjunction, Columbus probably wouldn't have given the *Ephemerides* much weight just two days out of Santa Maria in light of the large errors revealed by his recent celestial efforts.

7. FIRST VOYAGE CORRELATION RESULTS. Pickering nullifies the many celestial correlations by suggesting that the *Diario* dates the event recordations, rather than the events themselves.<sup>37</sup> Extending Pickering's argument to the *Diario's* first entry, he

would date Columbus' departure from Spain as 2 August rather than the 3 August date celebrated each year in Palos, Spain. While the celestial correlations hold true for any arbitrary shifting of the time axis, they are a lot easier to explain when the position estimates follow the conjunctions within 24 hours.

Nine position estimates were recorded in the *Diario* for:

August 8th (multiple ships)	26 hours after a darkened moonset
September 19th (multiple ships)	52 hours after a darkened moonrise
September 25th (single ship)	11 hours after a darkened moonset
October 1st (single ship)	5 hours after a darkened moonset
January 15th (single ship?)	30 hours after a darkened moonrise
February 6th (single ship)	10 hours after a darkened moonrise
February 7th (single ship)	34 hours after a darkened moonrise
February 10th (single ship)	106 hours after a darkened moonrise
February 27th (single ship)	3 hours after a darkened moonset.

For a worst-case analysis, assume all nine *Diario* position entries are independent. In this case, the joint probability of both multi-ship position estimates falling within any given 72-hour window, and four out of seven single-ship estimates within any given 24-hour interval is roughly one in 400. For a better-case analysis, assume the February 7th and 10th entries merely extend the February 6th position debate, thereby reducing the probability to about one in 24,000. For a best-case analysis, also elevate the shocking longitude finding of 15 January to a multi-ship conference. Then the joint probability that all three multi-ship estimates fall within a 72-hour window and all four single-ship estimates within a 24-hour window is reduced to one in 370,000.

8. EVIDENCE FROM OTHER VOYAGES. In addition to the first voyages of Columbus and Vespucci, two others have enough recorded detail to suggest possible usage of lunar distance techniques:

(a) *Third Voyage of Columbus*. According to Samuel Eliot Morison, the *primary* purpose of this 1498 voyage was to test whether a great continent lay on the Portuguese side of the Line of Tordesillas.<sup>38</sup> On 30 June Columbus departed Boavista's best anchorage in the Cape Verde Islands for São Tiago, closer to the Tordesilla's reference line. Early on 4 July he sailed westward from this uncomfortable harbour just 30 hours after a midnight occultation of Jupiter high in the night sky. On 29 July, Jupiter had a near-occultation high in the night sky, and 30 hours later 'Columbus correctly estimated that the fleet lay almost due south of the Caribbee islands'<sup>39</sup> Columbus crossed the Atlantic too far to the north, so detection of Brazil's continental bulge would have to wait for Vespucci's voyage a year later.

(b) *Cabral's Voyage to Brazil*. 'According to the treaty [of Tordesillas], they [Spain and Portugal] should have appointed a commission of pilots and navigators to meet at the Cape de Verde Islands, where they would sail in company due west until they found land, or agree that the proper meridian had been reached; in the latter event, they would change course 90 degrees to due south and, where they first made land, would set up a stone pillar. Considering that nobody then had the foggiest notion of how to ascertain longitude, it is probably fortunate for international relations that this voyage never came off.'<sup>40</sup>

Despite Morison's nay-saying, the Portuguese navigator, Pedro Álvares Cabral, seems to have followed a similar route on his 'discovery' voyage to Brazil in 1500. Morison claims Cabral sighted São Nicolau (longitude 24° 30' W) on 21 or 22 March and two days later proceeded due south along this Tordesillas reference line, trying to cross the Equator at that longitude.<sup>41</sup> While on this reference meridian he could have observed

a fine BTH conjunction of Jupiter on 27 March with the planet within five degrees of the Moon when it rose at 4:30 AM. On 22 April he anchored in the shallows 20 miles from Brazil's coast near 17° S latitude<sup>42</sup> (the mirror latitude of São Nicolau). The following day he re-anchored on the coast at the same latitude,<sup>43</sup> better positioned to measure coastal longitude with Jupiter six degrees above the eastern (unobscured) horizon when the Moon rose at 3:00 AM that night.

9. CONCLUSIONS. Strong circumstantial evidence suggests some form of lunar distance technique was employed by European explorers prior to Werner's supposed development of the method in 1514. This evidence includes:

- (a) Correlation of Columbus' 1492-93 position estimates with BTH conjunctions,
- (b) Correlation of Columbus' January 1493 erroneous distance estimate to Canary Islands, erroneous sailing direction to Spain, and otherwise inexplicable course changes with the actual longitude error that would have resulted from his proper use of the erroneous *Ephemerides* data,
- (c) Vespucci's accurate description of the Mars azimuth at 1499 moonrise,
- (d) Coincidence of Vespucci's 1499 longitude claim with mapping of Mississippi Delta on the Cantino Map,
- (e) Visible occultations at both ends of the 1498 Columbus attempt to measure the Line of Tordesillas,
- (f) Good BTH conjunctions at both ends of a possible 1500 Cabral attempt to measure the Line of Tordesillas.

Although historians might contribute additional data points for evaluation, most do not have the technical background to evaluate their significance. For example, they know Columbus used an early evening lunar eclipse to scare Jamaica's natives into submission in 1504. What most don't understand is that Columbus's *Ephemerides* predicted the eclipse for 1:36 AM the following morning in Nuremburg, so he certainly would not have staged his demonstration if he had believed himself more than seven hours west of there. Despite this simple scientific evidence, most Columbus scholars continue to believe he went to his deathbed believing Jamaica to be off the coast of Asia!

What is needed now is an *objective* assessment of the preliminary results I have presented here and in my 1992 paper. Pickering's critique contains some useful data, but its negative assessment depends on:

- (1) a fictitious league length which conceals distance correlations,
- (2) distorted *Diario* interpretations which conceal time correlations, and
- (3) misinterpretations which conceal the motivation for a September sailing date.

The readers of this *Journal* are well-qualified to assess the validity of my preliminary results. If deemed valid, their next step might be an error analysis defining the actual navigational capabilities of Columbus for historians.

#### REFERENCES

- <sup>1</sup> Dunn, O. and Kelley, J. E. Jr (1989). *The Diario of Christopher Columbus's First Voyage to America, 1492-1493*. Norman and London, University of Oklahoma Press, p. 21.
- <sup>2</sup> Marcus, G. J. (1981). *The Conquest of the North Atlantic*. New York, Oxford University Press, p. 118.
- <sup>3</sup> Arcienagas, G. (1955). *Amerigo and the New World*. Alfred A. Knopf, New York, p. 192.
- <sup>4</sup> Molander, A. B. (1992). Columbus and the method of lunar distance. *Terrae Incognitae* 24, 65-78.

- <sup>5</sup> Pickering, K. A. (1996). Columbus's method of determining longitude: an analytic view. *This Journal* 49, 95–111.
- <sup>6</sup> Tibbetts, G. R. (1971). *Arab Navigation in the Indian Ocean before the Coming of the Portuguese*. The Royal Asiatic Society of Great Britain and Ireland, London, pp. 121–157.
- <sup>7</sup> Pickering (1996), 95–96.
- <sup>8</sup> Molander (1992), 69.
- <sup>9</sup> Arciniegas (1955), 92.
- <sup>10</sup> Formisano, L. (1992). *Letters from a New World; Amerigo Vespucci's Discovery of America*. Marsilio, New York, p. 15.
- <sup>11</sup> Molander, A. B. (1989). The mystery of the Cantino Map. *The Portolan* 15, 15–26.
- <sup>12</sup> Pickering (1996), 99.
- <sup>13</sup> Taviani, E. P. (1984). *The Voyages of Columbus, The Great Discovery*. Translated by Beckwith, M. A. and Farina, L. F. Instituto Geografico De Agostino, Novara, Vol. 1, p. 183.
- <sup>14</sup> Pickering (1996), 106–108.
- <sup>15</sup> Dunn and Kelley (1989), 25.
- <sup>16</sup> Pickering (1996), Fig. 1, p. 102.
- <sup>17</sup> Pickering (1996), 98.
- <sup>18</sup> Colon, F. (1992). *The Life of the Admiral Christopher Columbus by his son, Ferdinand*. Translated by Keen, B. Rutgers University Press, New Brunswick, p. 46.
- <sup>19</sup> Colon (1992), 47–48.
- <sup>20</sup> Dunn and Kelley (1989), 37.
- <sup>21</sup> Dunn and Kelley (1989), 43.
- <sup>22</sup> Dunn and Kelley (1989), 49.
- <sup>23</sup> Pickering (1996), 109.
- <sup>24</sup> Pickering (1996), 97.
- <sup>25</sup> Colon (1992), 55.
- <sup>26</sup> Colon (1992), 55.
- <sup>27</sup> Pickering (1996), 109.
- <sup>28</sup> Pickering (1996), 111.
- <sup>29</sup> Dunn and Kelley (1989), 341.
- <sup>30</sup> Dunn and Kelley (1989), 343.
- <sup>31</sup> Dunn and Kelley (1989), 353.
- <sup>32</sup> Dunn and Kelley (1989), 355.
- <sup>33</sup> Pickering (1996), 109.
- <sup>34</sup> Dunn and Kelley (1989), 359.
- <sup>35</sup> Dunn and Kelley (1989), 359.
- <sup>36</sup> Dunn and Kelley (1989), 389.
- <sup>37</sup> Pickering (1996), 99.
- <sup>38</sup> Morison, S. E. (1942). *Admiral of the Ocean Sea*. Little, Brown and Company, Boston, p. 515.
- <sup>39</sup> Morison (1942), 528.
- <sup>40</sup> Morison, S. E. (1974). *The European Discovery of America, The Southern Voyages*. Oxford University Press, New York, p. 217.
- <sup>41</sup> Morison, S. E. (1974), p. 222.
- <sup>42</sup> Morison, S. E. (1974), p. 223.
- <sup>43</sup> Morison, S. E. (1974), p. 223.

## KEY WORDS

1. History.
2. Astronomical navigation.