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Some techniques of harmonic analysis on compact Lie groups with applications to lacunarity

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The study of lacunary subsets of the duals of compact abelian groups can be traced back to Sidon and Hadamard, and it is still an area of activity today (see [2], §37). During the last decade, this study has been extended to the compact non-abelian case, which forms the central theme of this thesis.

A notion of lacunary set due to Bożejko and Pytlik [1] is generalized from the abelian to the non-abelian setting, and a study is made of the class of sets thus obtained for compact Lie groups. This class includes most types of lacunary sets previously considered. In the process, it is necessary to develop some new techniques of harmonic analysis on compact Lie groups; the results obtained generalize and improve a number of known results on previously considered notions of lacunarity.

Chapter 1 is a collection of the definitions, notation, and elementary results of harmonic analysis which will be used throughout the thesis, followed by a brief introduction to some aspects of the theory of lacunarity for compact nonabelian groups.

In Chapter 2, I begin the study of sets of type $V(p, q)$ and $\Lambda(p, q)$ $(p, q \in [1, \infty])$ and their central and local central analogues. An important subclass of these sets consists of the p-Sidon and central p-Sidon sets $(p \in [1, 2[)$. Most of the theory of this chapter is valid for arbitrary compact groups.

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Chapter 3 deals with sets of type local central $V(p, q)$ and local central $\Lambda(p, q)$, specializing to the case of a compact Lie group. The theorems are proved by extensive use of the theory of semisimple Lie algebras to obtain estimates for the p-norms of the irreducible characters in terms of their dimensions. It is shown that the dual of a compact Lie group is of type local central $\Lambda(2+\varepsilon)$, but that no infinite subset of the dual of a compact semisimple Lie group is of type local central $\Lambda(4-\delta)$. Here $\varepsilon > 0$ and $\delta > 0$ are constants which depend on the group. It is also a result of this chapter that the dual of a compact semisimple Lie group contains no infinite p-Sidon sets $(1 \leq p < 2)$.

The results of Chapter 4 concern the convolution centres of the classical function and measure algebras of a compact connected Lie group (that is, the space of trigonometric polynomials, the space of continuous functions, the L^p spaces $(1 \leq p \leq \infty)$ and the space of measures). It is shown that the convolution centre of each of these spaces is linearly isometrically isomorphic to a subspace of the same space of functions or measures on the maximal torus, and a formula is derived which makes it possible to calculate the Fourier transform of a central function or measure in terms of the Fourier transform of its image under this isometric isomorphism. One is thus able to reduce many questions concerning convolution centres to questions about abelian groups. Again, important use is made of the representation theory of Lie groups and Lie algebras.

In Chapter 5, the theory of Chapter 4 is applied to sets of type central $V(p, q)$ and central $\Lambda(p, q)$ for compact connected Lie groups. In particular, it is shown that every infinite subset of the dual contains an infinite set of type $\Lambda(2+\varepsilon)$ (where ε is as in Chapter 3), and it is shown that a set of representations is of type central $\Lambda(2)$ if and only if the corresponding set of characters of the maximal torus is of type $\Lambda(2)$. Finally, central p-Sidon sets are considered; central p-sidonicity is related to a simple arithmetic condition - r-boundedness - on sets of characters, a new proof of the Ragozin-Rider result (see [3]) that the dual contains no infinite central Sidon sets is given, and it is shown that the dual of $SU(2)$ contains no infinite central p-Sidon sets $(1 \leq p < 2)$.

I make much wider use of the theory of Lie groups, Lie algebras, and their representations than previous authors in this field: the requisite facts from this theory are outlined in two appendices, Appendix A on Lie algebras, and Appendix B on Lie groups.

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