

and are generally the holders of County Council and other scholarships. If the early methods of these boys are not up to date there is waste all the way round—waste that can be prevented if those in authority will see to it.—Yours faithfully,

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### THE PILLORY.

DEAR SIR,—I send the following question from the last *Responsions* Arithmetic paper. You may find it worth printing in the "pillory."

The hands of a clock are together at 22 minutes past 4. Is the clock slow or fast, and how much does it lose or gain in an hour of true time? The italics are mine.—Yours faithfully,

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### ANSWER TO QUERY.

[67, p. 330, vol. v.] This theorem is due to Mr. V. Ramaswami Aiyar. It appears in an article contributed to the *Proc. Ed. M. Soc.*, 1896-7.

My proof of the theorem is given in the *Ed. Times Reprint*, Vol. 17, June 1910. It depends on the fact that the common pedal circle of  $F, F'$  (the auxiliary circle of the in-conic) cuts the medial or N.P. circle in  $\omega, \omega'$ , the orthopoles of  $OF, OF'$ ; i.e. the points whose Simson lines (in the medial circle) are parallel to  $OF, OF'$ . When  $\omega'$  coincides with  $\omega$ , then  $OF'$  falls on  $OF$ , and hence the well-known theorem: If  $FF'$  passes through  $O$ , the pedal circle of  $FF'$  touches the medial circle.

Feuerbach's theorem is, of course, a particular case.

May I be permitted to state what is known at present of this very curious and interesting 'orthopole'?

(The initial N. stands for Professor J. Neuberg; G. for the present writer.)

1. If  $Ap, Bq, Cr$  be  $\perp$ rs on a given line  $L$ , then the  $\perp$ rs from  $p, q, r$  on  $BC, CA, AB$  respectively are concurrent at a point  $\omega$  called the orthopole of  $L$ . (N.)

2.  $L$  being  $px + qy + rz = 0$  (in barycentric coordinates),  $\omega$  is given by

$$2\Delta x = q(r-p)ca \cos B - r(p-q)ab \cos C + a^2bc \cos B \cos C. \quad (G.)$$

3. If  $L$  cuts the circle  $ABC$  in  $T, T'$ , then  $\omega$  is the point of intersection of the Simson lines of  $T, T'$ . (N.)

If  $\theta_1, \theta_2, \theta_3; \lambda, \mu, \nu; \lambda', \mu', \nu'$  are the direction angles of  $TT'$  and the Simson lines, then for  $\omega$ ,

$$a = 2R \cos \theta \sin \lambda \sin \lambda', \text{ etc.} \quad (G.)$$

4. For the quadrilateral formed by  $L$  and the sides of  $ABC$ , the common R.A. of the three diameter circles passes through  $\omega$ . (N.)

The power of  $\omega$  for these three circles =  $2d\delta$ , where  $d, \delta$  are  $\perp$ rs from  $O$  and  $\omega$  on  $L$ ; also  $\delta = 2R \cos \theta \cos \theta_2 \cos \theta_3$ . (G.)

5. The most remarkable of all the properties of the orthopole is that discovered by M. T. Lemoine.

*Lemoine's Theorem.* The power of  $\omega$  for the pedal circle of every point  $F$  on  $L$  is constant. The three diameter circles are pedal circles, so that this common power =  $2d\delta$ .

6. Another noteworthy property has been recently published by Prof. Neuberg.

*Neuberg's Theorem.* If parallel forces  $\cos \theta_2 \cos \theta_3 \sin A$ , etc., be applied at the vertices of the pedal triangle of any point  $F$  on  $L$ , then their centre is a fixed point, the orthopole.