PRECESSION THEORY USING THE INVARIABLE PLANE OF THE SOLAR SYSTEM

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ABSTRACT. Standard precession theory builds up the precession matrix $\mathbf{P}$, which rotates coordinates from the mean equator and equinox of epoch to the mean equator and equinox of date, by a sequence of three elementary rotations by the accumulated Euler angles ζ_A, θ_A, and z_A: $\mathbf{P} = \mathbf{R}_3(-z_A)\,\mathbf{R}_2(\theta_A)\,\mathbf{R}_3(-\zeta_A)$. This scheme works well provided both the epoch and the date are within a few centuries of J2000. For long-term applications, the alternative formulation using the accumulated luni-solar and planetary precession, $\mathbf{P} = \mathbf{R}_3(\chi_A)\,\mathbf{R}_1(-\omega_A)\,\mathbf{R}_3(-\psi_A)\,\mathbf{R}_1(\varepsilon)$, is more stable.

Yet another formulation for $\mathbf{P}$ is possible, using the invariable plane of the Solar System as an intermediate plane: $\mathbf{P} = \mathbf{R}_3(-L)\,\mathbf{R}_1(-I)\,\mathbf{R}_3(-\Delta)\,\mathbf{R}_1(I_0)\,\mathbf{R}_3(L_0)$. The angles I_0 and L_0 are the inclination and ascending node of the invariable plane at epoch; I and L are the same quantities at the date. Only the angle Δ is a function of both times. This scheme works for both short-term and long-term applications.

For the short term, polynomial coefficients for I, L, and Δ are derived from the currently-accepted coefficients of the angles ζ_A, θ_A, and z_A. For the long term, these angles are expressed as sums of Chebyshev polynomials obtained from analysis of a million-year numerical integration.

If the intersection of the mean equator and the invariable plane were adopted as the origin of right ascensions, the theory would be simplified further: since L_0 and L would no longer be required, $\mathbf{P}$ would again consist of the minimum three rotations.

1. Introduction

This paper is a brief report of my doctoral research (Owen 1990) into the consequences of using the invariable plane of the Solar System as an intermediate plane in the formulation of precession theory. Space limitations prevent the inclusion of any details.

The work described here comprises three major topics: the determination of the orientation of the invariable plane, the "short-term theory" based on the precession angles of Lieske *et al.* (1977) (hereinafter denoted "L77"), and the "long-term theory" based on numerical integration of Kinoshita's (1977) model for the speed of luni-solar precession and Laskar's (1990) formulation for the ecliptic. A comparison of the long-term results near J2000 with the short-term theory reveals possible improvements to the currently-adopted precession theory, particularly in the motion of the ecliptic and in the rate of change of Newcomb's Precessional Constant.

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2. The Orientation of the Invariable Plane

The invariable plane of the Solar System is rigorously defined as the plane which passes through the Solar System barycenter and is normal to the total angular momentum. The rotational angular momenta are poorly known (especially for the Sun), and the orbital angular momenta of the satellites are subject to precession; these were therefore ignored, and only the orbital angular momenta of the planets and Sun were kept. The planets were thus assumed to be point masses (including the masses of their satellites) located at their respective planet-satellite barycenters.

Positions and velocities for the nine planetary barycenters, the Sun, and five asteroids were interpolated from the M04786 planetary ephemeris (Jacobson *et al.* 1990), the most recent one produced at JPL and the only one so far to use the Voyager 2 determination of Neptune's mass. The total angular momentum vector (after Burkhardt 1982), rotated into J2000 coordinates, is directed toward

$$\alpha_0 = 273^\circ 51' 09''\!.262 \pm 0''\!.038, \quad (1)$$
$$\delta_0 = 66^\circ 59' 28''\!.003 \pm 0''\!.013. \quad (2)$$

The right ascension L_0 of the ascending node of the invariable plane on the mean equator of J2000 and the inclination I_0 of the invariable plane to the mean equator of J2000 are consequently

$$L_0 = 3^\circ 51' 09''\!.262 \pm 0''\!.038, \quad (3)$$
$$I_0 = 23^\circ 00' 31''\!.997 \pm 0''\!.013. \quad (4)$$

It is worth noting that the standard errors above have decreased nearly a hundredfold due solely to the improvements in the planetary masses provided by Voyager 2.

3. Short-term Precession Theory

The precession matrix $\mathbf{P}$ to transform from the mean equator and equinox of J2000 to that of date is given by L77 as

$$\mathbf{P} = \mathbf{R}_3(-\zeta_A)\,\mathbf{R}_2(\theta_A)\,\mathbf{R}_3(z_A), \quad (5)$$

where $\mathbf{R}_i(\alpha)$ is the 3×3 orthogonal matrix which rotates the coordinate axes by the angle α about axis i. The angles in equation (5) are approximated by

$$\zeta_A = \zeta_1 T + \zeta_1' T^2 + \zeta_1'' T^3, \quad (6)$$
$$\theta_A = \theta_1 T + \theta_1' T^2 + \theta_1'' T^3, \quad (7)$$
$$z_A = z_1 T + z_1' T^2 + z_1'' T^3, \quad (8)$$

with T measured in Julian centuries from J2000 (JED 2451545.0).

From Figure 1, $\mathbf{P}$ can also be represented by the sequence of rotations

$$\mathbf{P} = \mathbf{R}_3(-L)\,\mathbf{R}_1(-I)\,\mathbf{R}_3(-\Delta)\,\mathbf{R}_1(I_0)\,\mathbf{R}_3(L_0), \quad (9)$$

where L is the right ascension of the ascending node of the invariable plane on the mean equator of date, I is the inclination of the invariable plane to the mean equator of date, and Δ is the angle in the invariable plane from the mean equator of J2000 to that of date.

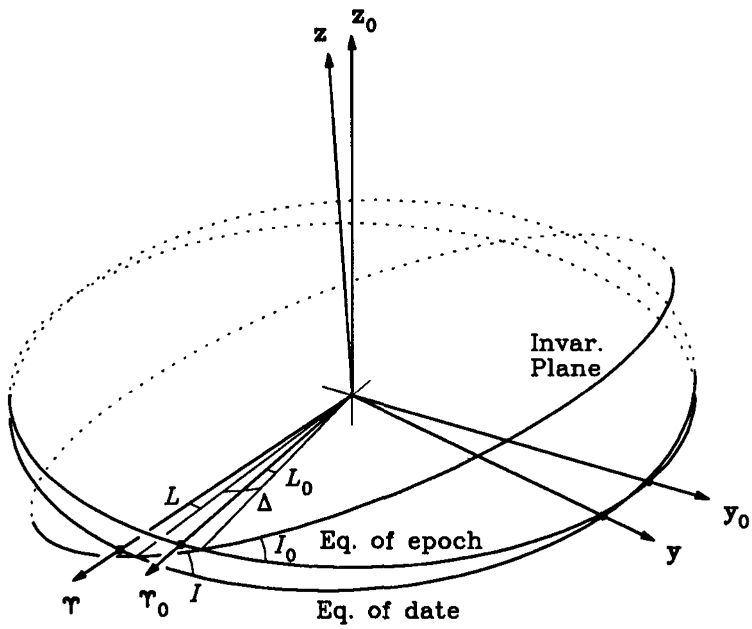

Figure 1. Precession Angles Using the Invariable Plane

Equating the right-hand sides of equations (5) and (9) and expanding the matrix products yields exact expressions for L, I, and Δ:

$$L = \text{plg}[\cos\theta_A \sin(L_0 + \zeta_A)\sin I_0 - \sin\theta_A \cos I_0,\ \cos(L_0 + \zeta_A)\sin I_0] + z_A, \tag{10}$$

$$I = \cos^{-1}[\cos\theta_A \cos I_0 + \sin\theta_A \sin(L_0 + \zeta_A)\sin I_0], \tag{11}$$

$$\Delta = \text{plg}[\sin\theta_A \cos(L_0 + \zeta_A),\ \cos\theta_A \sin I_0 - \sin\theta_A \sin(L_0 + \zeta_A)\cos I_0]. \tag{12}$$

In equations (10) and (12), plg(y, x) is Eichhorn's (1987/88) notation for the four-quadrant arctangent, expressed in Fortran as `ATAN2(Y,X)`.

Equations (10) through (12) can be expanded in powers of T to yield approximation polynomials for the angles L, I, and Δ. The L77 coefficients of the precession angles imply

$$L = 3°51'09''.262 - 96''.7230T - 1''.94824T^2 + 0''.006539T^3, \tag{13}$$

$$I = 23°00'31''.997 - 134''.6685T + 0''.49754T^2 + 0''.006173T^3, \tag{14}$$

$$\Delta = 0''.000 + 5116''.1809T + 2''.92466T^2 - 0''.005636T^3. \tag{15}$$

4. Long-Term Precession Theory

Since the L77 approximations for ζ_A, θ_A, and z_A begin to break down after a few centuries, numerical integration was used to obtain the precession angles over longer time spans. Kinoshita's (1977) model supplied the speed of luni-solar precession, and the orientation of the ecliptic came from Laskar (1990). The integration covered one million years centered at

J2000. The obliquity ε and the precession angles ψ_A, χ_A, ω_A, L, I, and Δ were obtained every century, and Chebyshev polynomials were fit to these results. Computer-readable tables of the Chebyshev coefficients may be obtained from the author.

Two substantial differences are apparent when the long-term results are compared with the short-term ones. First, Laskar's motion of the ecliptic near J2000 differs from that in L77. This changes the speed of planetary precession and therefore ζ_1, θ_1, and z_1: ζ_1 and z_1 decrease from $2306''.2181$/cy to $2306''.2174$/cy while θ_1 increases from $2004''.3109$/cy to $2004''.3141$/cy. The rate of change of the obliquity changes by a greater amount, from $-46''.8150$/cy to $-46''.8065$/cy. Second, Kinoshita's terms containing M_1 and M_3 are absent in the L77 work; their presence causes P_1, the derivative of P at J2000, to change from $-0''.00369$/cy in L77 to $-0''.00393$/cy.

5. Conclusions

One notes several desirable properties in equation (9) for **P**. Foremost among these is that the initial and final times are isolated rather cleanly; when one is precessing between two arbitrary times, **P** takes the form

$$\mathbf{P} = \mathbf{R}_3[-L(T_2)]\,\mathbf{R}_1[-I(T_2)]\,\mathbf{R}_3\{-[\Delta(T_2) - \Delta(T_1)]\}\,\mathbf{R}_1[I(T_1)]\,\mathbf{R}_3[L(T_1)]. \qquad (16)$$

The angles L and I are each functions of only one time, and only $[\Delta(T_2) - \Delta(T_1)]$ would be evaluated using both initial and final times.

Finally, it is obvious that if right ascensions were measured from the ascending node of the invariable plane on the mean equator (instead of from the traditional vernal equinox), the first and last $\mathbf{R}_3$ rotations in equation (9) would vanish, leaving once again a sequence of three rotations. Now, however, the three rotation angles would require only two formulas for their evaluation, and only one of those two would require two arguments. Such a scheme is simpler computationally than that of the L77 paper.

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