

STRUCTURE AND EXISTENCE OF SIDON SETS
ON COMPACT GROUPS

DAVID CHARLES WILSON

Criteria for the existence of infinite Sidon sets in the dual object of a compact group are already known in the cases where the group is abelian [2, 37.18] or connected [1] or totally disconnected [4]. This thesis addresses the existence problem for arbitrary compact groups and provides a complete solution : a compact group admits an infinite Sidon set if, and only if, either it admits infinitely many continuous irreducible representations of the same degree (that is, it is not tall), or it is tall and contains an open subgroup admitting an infinite higher-order Figà-Talamanca - Rider (*FTR*) set.

Membership in these indicators sets, the *FTR* sets (which generalize those introduced in [1]), is defined in terms of allowable images of irreducible representations. Indeed, the essence of our methods lies in the observation that local Sidon behaviour is governed solely by the images of the representations involved. We therefore extend the usual notion of Sidonicity by defining a "Sidon constant" for any closed group of unitary matrices, irreducible or otherwise. Since the allowable *FTR* images are compact linear semisimple Lie groups, we are able to make extensive and explicit use of the representation theory of compact almost simple Lie groups in investigating the Sidonicity of *FTR* sets.

In order to establish our existence criterion we consider the effect

Received 25 July 1986. Thesis submitted to University of Sydney, February 1986. Degree approved July 1986. Supervisor: Dr. J.R. McMullen.

Copyright Clearance Centre, Inc. Serial-fee code: 0004-9727/87
\$A2.00 + 0.00.

on Sidonicity of restriction to and lifting from closed subgroups, especially the connected component of the group. In this context certain projective representations arise naturally, and accordingly we introduce an appropriate concept of local Sidonicity. Building on results of [3] we define and heavily exploit a special kind of lifting from a closed normal subgroup, the Hutchinson lifting, whose peculiar property is that Sidon sets lift to Sidon sets regardless of the index of the subgroup. In the estimation of Sidon constants we adhere to the developments of [1], expending most of our effort in extending results obtained there about sets on product groups to the more general setting of pre-Hutchinson liftings.

As somewhat of a side benefit our methods yield union and structure theorems for Sidon sets on a class of compact groups which includes all connected compact groups. Broadly speaking, every Sidon set on a group in this class is the union of a set of representations of bounded degree with a finite union of sets whose elements satisfy a quasi-independence condition. These results appear in [5].

References

- [1] D.I. Cartwright and J.R. McMullen, "A structural criterion for the existence of infinite Sidon sets", *Pacific J. Math.* 96 (1981), 301-317.
- [2] E. Hewitt and K.A. Ross, *Abstract Harmonic Analysis, Vol. II*, (Springer-Verlag, New York-Heidelberg, Berlin, 1970).
- [3] M.F. Hutchinson, "Lacunary sets for connected and totally disconnected compact groups", Ph.D. thesis, University of Sydney, 1977, *Bull. Austral. Math. Soc.* 18 (1978), 149-151.
- [4] M.F. Hutchinson, "Local Λ sets for profinite groups", *Pacific J. Math.* 89 (1980), 81-88.
- [5] D.C. Wilson, "On the structure of Sidon sets", *Monatsh. Math.* 101 (1986) 67-74.

Department of Pure Mathematics,
University of Sydney,
New South Wales 2006
Australia.