Similarly y_3, y_4 are the roots of

$$y^2+\tfrac{1}{2}(1-\sqrt{17})y-1=0 \quad \ldots \quad \ldots \quad (4)$$

Then from equations (3) and (4) attending to the proper signs we have

$$y_1=\{-1-\sqrt{17}+\sqrt{(34+2\sqrt{17})}\}/4 \quad \ldots \quad (5)$$

$$y_3=\{-1+\sqrt{17}+\sqrt{(34-2\sqrt{17})}\}/4 \quad \ldots \quad (6)$$

Substituting these values in equation (1) and solving, we have finally

$$x_1=2\cos 2\pi/17$$

$$=\Big\{\sqrt{17}-1+\sqrt{(34-2\sqrt{17})}$$

$$+\sqrt{\{68+12\sqrt{17}+2(\sqrt{17}-1)\sqrt{(34-2\sqrt{17})}-16\sqrt{(34+2\sqrt{17})}\}}\Big\}/8.$$

A short notice of the additions to the Mathematical Theory of Heat since the transmission of Fourier's Memoir of 1811 to the French Academy.

By George A. Gibson, M.A.

What is here printed contains merely a list of the memoirs and treatises that may perhaps be found useful for one who wishes to trace the progress of the mathematical theory of heat beyond the stage at which Fourier left it. As discussions of the Fourier series and integrals occur in almost every treatise on the Integral Calculus, I have omitted reference to these. Similar considerations have led me to omit references to the discussion of differential equations, except where these specially dealt with the problem of the conduction of heat.

Poisson.—Mémoires sur la Distribution de la Chaleur dans les Corps Solides. (*Journal de l'École Polytechnique*, t. xii., cah. 19, 1823.)

There are two memoirs, the first of which was presented to the Institute in 1815, and the second in 1821.

Poisson.—Théorie Mathématique de la Chaleur. (Paris, 1835.)

The problems Poisson discusses are in the main those of Fourier generalised. The methods he gives of proving the possibility of the expansion of a function in a Fourier series, or in a series of spherical harmonics, are those usually given in English text-books. The treatise contains little of importance that is not to be found in the memoirs.

Laplace (*Additions aux Connaissances des Temps*, 1823; *Mécanique Celeste*, Livre xi.) treated before Poisson, the case of the sphere with arbitrary initial distribution.

Duhamel.—" Mémoire sur la méthode générale relative au mouvement de la chaleur dans les corps solides plongés dans des milieux, dont la température varie avec le temps. (*École Polyt. Journ.*, t. xiv., 1833.) It is proved that the solution for any value of the time can be found by a simple integration if a solution for the time $t = \text{const.}$ can be found.

Sturm.—" Mémoire sur les équations differentielles lineaires du second ordre." (*Liouv. Jour.*, t. i., 1836, pp. 106-186.)

Sturm.—" Mémoire sur les équations aux différences partielles." (*Liouv. Jour.*, t. i., 1836, pp. 373-444.)

Liouville.—" Demonstration d'un Theorème dû à M. Sturm." (*Liouv. Jour*, t. i., 1836, pp. 269-277.)

These memoirs discuss equations of the form $\frac{d}{dx}\left(K\frac{dy}{dx}\right) + Gy = 0$, K,G being functions of x and of an arbitrary parameter r, with the view of tracing the variation of the roots of $y = 0$ due to variation of r. The applications to the theory of heat are very important.

Thomson.—In Sir William Thomson's "Mathematical and Physical Papers" are several important articles on the conduction of heat, reprinted from the original journals. Articles LXXII. and LXXIII., together with Article XIV. of "Papers on Electrostatics and Magnetism," seem to me of special interest. The methods of these articles have been recently applied to several interesting problems by

Hobson.—" Synthetical Solutions in the Conduction of Heat." (*Proc. London Math. Soc.*, 1888, pp. 279-294.)

Lamé.—In his three treatises, " Les Fonctions Inverses des Transcendantes et les surfaces Isothermes" (Paris, 1857), "Coordonnées Curvilignes" (1859), and "Théorie Analytique de la Chaleur" (1861), will be found Lamé's contributions to the subject. His methods differ very considerably from those of his predecessors, and, as Mathieu remarks, he pays too little attention to them; but while it is probable Lamé estimates the value of his own methods too highly, he has nevertheless made important additions to the mathematics of physics generally.

Mathieu.—" Mémoire sur le mouvement de la température dans le corps renfermé entre deux cylindres circulaires excentriques et dans des cylindres lemniscatiques" (*Liouv. Journ.*, t. xiv., 1869). A discussion of cases in which the simple solution cannot be put in the form of a product of factors, each of which contains one, and only one, of the thermometric parameters.

Mathieu.—"Cours de Physique Mathématiques" (Paris, 1873). An excellent treatise, giving a good treatment of the conduction of heat in its present mathematical development.

Boussinesq.—"Sur les problemes des températures stationaires, &c." (*Liouv. Journ.*, t. vi., 1880). A short notice of Lamé's methods.

Neumann, C.—"Über das Gleichgewicht der Wärme und das der Electricät in einem Körper welcher von zwei nicht concentrischen Kugelflächen begrenzt wird" (*Crelle's Journ.*, Bd. LXII.). This is merely a notice of the methods used by the author in a book entitled "Allgemeine Lösung des Problems über den stationären Temperaturzustand eines homogenen Körpers welcher von irgend zwei nicht concentrischen Kugelflächen begrenzt wird" (Halle, 1862). The book itself I have not seen, but the method of solution seems ingenious, though complicated.

Tait.—"On Orthogonal Isothermal Surfaces" (*Edin. Trans.*, vol. xxvii., 1872). An investigation by quaternion methods of properties of isothermal surfaces.

On the conduction of heat in crystalline media, I may refer to the following.

Duhamel.—"Sur les équations générales de la propagation de la chaleur dans les corps solides dont la conductibilité n'est pas la même dans tous les sens" (*Ecole Polyt. Journ.*, t. xiii., 1832).

Duhamel.—"Note sur les surfaces isothermes dans les corps solides dont la conductibilité n'est pas la même dans tous les sens" (*Liouv. Journ.*, t. iv., 1839).

Duhamel.—"Sur la propagation de la chaleur dans les cristaux" (*École Polyt. Journ.*, t. xix., 1848).

Stokes.—"On the conduction of heat in crystals" (*Camb. and Dub. Math. Journ.*, vol. vi., 1851). This article of Stokes establishes all Duhamel's results in a very ingenious, yet simple manner.

Lamé.—"Théorie Analytique de la Chaleur."

Boussinesq.—"Étude sur les surfaces isothermes et les courants de chaleur dans les milieux homogénes échauffés en un de leurs points" (*Liouv. Journ.*, t. xiv., 1869).

The following text-books present a good many of the methods to be found in the articles quoted above in a very useful form:—

Riemann's "Partielle Differentialgleichungen;" Heine's "Kugelfunctionen," vol. ii., pp. 302-332; Todhunter's "Functions of Laplace, Lamé, and Bessel;" Jordan's "Cours d'Analyse, vol. iii., chap. iii., Part iv.

The notice now given has, of course, no pretentions to being exhaustive; but it may perhaps serve a useful purpose in helping one to follow the development of the theory whose basis Fourier so thoroughly established.

Second Meeting, December 14*th*, 1888.

George A. Gibson, Esq., M.A., President, in the Chair.

On the general equation of the second degree representing a pair of straight lines.

By David Munn, M.A.

Kötters synthetic geometry of algebraic curves— Part I., imaginary curves.

By Rev. Norman Fraser, M.A.

[See Index.]

Third Meeting, January 11*th*, 1889.

George A. Gibson, Esq., M.A., President, in the Chair.

Note on a Formula in Quaternions.

By R. E. Allardice, M.A.

The formula referred is the condition for the coplanarity of the extremities of four coinitial vectors; namely, if α, β, γ, δ, are the vectors, then

$$a\alpha + b\beta + c\gamma + d\delta = 0, \text{ where } a + b + c + d = 0.$$

(See Kelland and Tait's Quaternions, p. 62.)

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